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ARX-Model based Model Predictive Control with Offset-Free Tracking.

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Abstract

ARX models, is a suitable model class for linear control implementations. The parameter estimation problem is convex and easily handed for both SISO and MIMO system in contrast to ARMAX or State Space model. Model predictive control implementations insuring offset-free tracking are discussed and related. Special attention is given to an adaptive disturbance estimation method with time-varying forgetting which is shown to be less sensitive to the nature of the disturbance.

Keywords: Model Predictive Control, Offset-Free Tracking, Adaptive Estimation,

1. Introduction

Model Predictive Control (MPC) is a state of the art control technology which utilizes a model of the system in order to predict the process output over some future horizon. It solves an open loop quadratic optimization problem with the manipulated variable as decision variable. The first of the controls is implemented and after retrieving the next process output, the problem is solved again for the next control to achieve feedback. Inequality constraints can be formulated for both manipulated variables and the process outputs.

Early achievements and industrial implementations in MPC include IDCOM by Richalet et al. (1978) and Dynamic Matrix Control by Cutler and Ramaker (1980). These early algorithms were based on step or impulse response models. More general linear input-output models structure were used by Clarke et al. (1987) in Generalized Predictive Control, but an interest in MPC implementations based on state space models were created by the seminal paper by Muske and Rawling (1993). The state space approach provides a unified framework for discussion of the various predictive control algorithms and is well suited for stability analysis (Mayne et al. 2000). Other types of linear model representations, which may be convenient for system identification, can be converted to state space form for the MPC implementation. This paper will focus on the following linear, discrete time, single input/single output ARX model representation

$$A(q^{-1})y(t) = B(q^{-1})u(t) + d + \varepsilon(t), \qquad \varepsilon \in N(0,\sigma^2) \tag{1}$$

Where *A* and *B* are polynomials of order *n* in the backwards shift operator q^{-1}.

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}$$
$$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \cdots + b_n q^{-n}$$

This paper advocates the advantages in MPC based on ARX models and discus closed loop performance of the controller in case of unmeasured step disturbances. In order to reject such types of disturbances, the basic MPC formulation needs to be expanded or designed including integrators to achieve offset-free closed loop performance. The paper is organized with an introduction to ARX MPC as a control paradigm in section 2. Implementations insuring offset-free tracking are presented in Section 3. A basic simulation case in Section 4 demonstrates the performance of the controllers and conclusions are drawn in Section 5.

2. ARX MPC as a Control Paradigm

Most industrial MPC implementations are currently based on linear model representations of the underlying system dynamics. Linear MPC is becoming a mature control technology with well established conditions for stability and robustness (Rawlings and Mayne 2009). Since MPC calculate the control based on an optimization over a prediction horizon, closed loop performance will depend strongly on the predictive capabilities of the system model. Hence typically model parameters are regressed based on prediction error methods of either the one-step-ahead prediction error or as a multistep method (Rossiter and Kouvaritakis 2001, Haber et al 2003). Estimation of parameters in ARMAX or State Space models by prediction error methods is a nonlinear and non-convex optimization problem. Parameter estimation in the ARX model structure is a convex problem. Furthermore the ARX model structure provides a much simpler estimation problem of multivariable system than the ARMAX model. Zhu (1998) and Hjalmarsson (2003) identify high order ARX models that are reduced before used in control design. Qin and Badgwell (2003) survey vendor MPC implementations and report that Honeywell's Robust Model Predictive Control Technology (RMPCT or profit-controller) as well as Invensys' model predictive control technology (Connoisseur) are based on ARX models.

Implementation of an MPC with input constraints based on an ARX model of the system is fairly simple as outlined in Huusom et al. (2009a). The ARX model is written as a State Space system on innovation form and optimal predictions of future output is given by the stationary Kalman filter where the data update is based on the innovation

$$\varepsilon_k = y_k - \hat{y}_{k|k-1} \tag{2}$$

The performance is given as the following quadratic cost

$$\phi = \frac{1}{2}\sum_{j=0}^{N-1} \left\| \hat{y}_{k+1+j|k} - r_{k+1+j|k} \right\|_2^2 + \rho \left\| \Delta u_{k+j|k} \right\|_2^2 \tag{3}$$

with penalty on the tracking error and the control move. Based on the observability matrix and the impulse response matrices a vector/matrix description the process output for the entire prediction horizon can be written. This can be used to formulate the minimization of the performance cost over the horizon, *N,* as a quadratic program which can be handled by a standard solver.

3. Offset-Free Tracking

A requirement which has to be provided by any industrial control implementation is offset-free tracking. Offset from the set point may occur for an MPC implementation if an unmeasured sustained disturbance is entering the system. It may also be the result of a mismatch between the true system and the model used for predictions by the MPC. A classical approach to avoid offset is by introducing an integrator in the control loop, as in case of PI control. In integrator can be included in the ARX-model based MPC, if the noise term in Eq. (1) is modeled as integrated white noise. This approach was used in GPC (Bitmead et al. 1990). If we model the random noise and a constant disturbance as integrated white noise we get

$$\frac{1}{(1-q^{-1})}e_k = d + \varepsilon_k \Rightarrow e_k = (1-q^{-1})(d+\varepsilon_k) = \varepsilon_k - \varepsilon_{k-1}$$

where the constant disturbance term disappears. It is seen that the price is that the variance of *e(t)* is twice that of the actual noise. We use the following model, labeled the ΔARX model, in the MPC design, thereby introducing integration and eliminating offset. The term $(1-q^{-1})$ is also known as a Δ operator.

$$\begin{aligned} A(q^{-1})y(t) &= B(q^{-1})u(t) + \frac{1}{1-q^{-1}}e(t) \Rightarrow \\ (1-q^{-1})A(q^{-1})y(t) &= (1-q^{-1})B(q^{-1})u(t) + e(t) \end{aligned} \tag{4}$$

The system order in this model is one order higher than the original system. An alternative approach which also extents the model order is by augmenting the system model with a disturbance state model as

$$d_k = d_{k-1} + \xi, \qquad \xi \in N(0, Q_\xi) \tag{5}$$

In this way the unknown disturbance can be estimated together with the system states by the state observer. This method was introduced by Davison and Smith (1971) and analyzed for use in linear MPC design by Pannocchica and Rawlings (2003) with conditions for detectability of the augmented system. This approach to offset-free MPC offers the disturbance state variance as a tuning parameter. If the variance approaches zero no ability to detect the disturbance is given in the state estimator design. Choosing the variance very large gives a high sensitivity to the prediction error in the disturbance state update. In the limit this approach is equivalent to having the integration. Choosing an appropriate value for the disturbance variance is not a trivial task, also in view of this tuning parameter being unbounded. An alternative approach advocated in Huusom et al (2009a) models the noise as an integrated moving average process with one lag

$$\frac{1-\alpha q^{-1}}{(1-q^{-1})}e_k = d + \varepsilon_k \Rightarrow (1-\alpha q^{-1})e_k = (1-q^{-1})(d+\varepsilon_k) = \varepsilon_k - \varepsilon_{k-1}$$

This lead to the EΔARX model used for the MPC design

$$(1-q^{-1})A(q^{-1})y(t) = (1-q^{-1})B(q^{-1})u(t) + (1-\alpha q^{-1})e(t) \tag{6}$$

It is clear that when the tuning parameter α is changes between 0 and 1 this approach also have the nominal ARX model and the ΔARX model as the extremes. Huusom et al. (2009a) show that this approach is equivalent to augmenting the system with a disturbance state since a state transformation will bring one formulation into the other. The advantage of tuning α rather than the variance of the disturbance state, is that this parameter is bounded between 0 and 1. Furthermore the variance of the disturbance state dependents on, where the disturbance is modelled to enter the system, i.e. as input, output or state disturbance. All the approaches discussed so fare suffers from a tradeoff between fast disturbance estimation versus noise sensitivity which is affected by the tuning. An attempt to get the best of both worlds was presented in Huusom et al. (2009b). Here the disturbance is estimated using adaptive techniques discounting old measurements. The forgetting is time-varying according to the prediction error and its variance as proposed in Fortescue et al. (1981). The idea is that when the level of the disturbance is known, the estimation uses a small gain from the prediction error in the disturbance estimation, making it insensitive to noise. When large prediction errors are observed, the method increases the gain and adapt faster to the new level. In the MPC formulation the disturbance level is used in the predictions and when optimizing the control signal. The recursive algorithm is based in the following set of equations

$$\begin{aligned} e_k &= y - C\hat{x}_{k|k-1} \\ \hat{d}_k &= \hat{d}_{k-1} + \kappa_k e_k \\ \kappa_k &= P_{k-1}(\lambda_k + P_{k-1})^{-1} \\ \lambda_k &= \max\left\{1 - \frac{e_k^2}{N_\infty \sigma^2 (1 + P_{k-1})}, \lambda_{\min}\right\} \\ P_k &= (1-\kappa_k)P_{k-1}\lambda_k^{-1} \end{aligned} \tag{7}$$

Where κ_k is the gain from the prediction error e_k in the disturbance update, λ_k is the forgetting factor which is bounded from below, σ^2 is the process noise variance of the model in Eq. (1) and P_k is an approximation of the variance of the prediction error which is distributed as

$$e_k \in N(0, \sigma^2(1+P_k))$$

N_∞ is the equivalent horizon, which is the tuning parameter of the method. It is seen from Eq. (7) that when the system know the disturbance level, the forgetting factor is approximately $1-1/N_\infty$ and P_k approximates the noise variance. I.e. the gain gets very small and reduces the effect of noise in the prediction error on the update of the disturbance. If the disturbance changes abruptly to a different level, the forgetting factor will decrease, making both P_k and κ_k larger, and render the method able to follow the change. The main result in the analysis in Huusom et al. (2009b) is that closed loop performance is less sensitive with respect to the nature of the disturbance by this method than the classical approach, augmenting the system with an extra state. Hence tuning is less dependent on knowing the true size and frequency of a series of step disturbances.

4. An Example

A simulation study is performed to show the characteristics of the methods for offset-free MPC based on ARX models. The model in Eq (1) is simulated with the following set of parameters which gives a pole in 0.9 and a pair of complex poles in 0.75±0.37i.

$$a_1 = -2.4, \quad a_2 = 2.05, \quad a_3 = -0.63, \quad b_1 = 0.5, \quad \sigma^2 = 0.1$$

For a simulation horizon of 10.000 samples and an unconstraint implementation of the input in the MPC, a series of simulations are performed for a range of the tuning parameters, ρ in Eq. (3) and α and Q_ξ for the EΔARX model and the disturbance model respectively. The following implementations are tested with different models: The true ARX model, the ΔARX model, the EΔARX and finally the augmented system with a disturbance model. The results are shown in a Pareto plot for the input and output variance on Fig. 1where $\alpha \in \{0, 0.1, .., 1\}$ for the EΔARX model and $Q_\xi \in [10^{-7}; 10^2]$ in the disturbance modeling approach. It is clearly seen that the ARX and ΔARX MPC implementations is achieved in the limit for the two other methods.

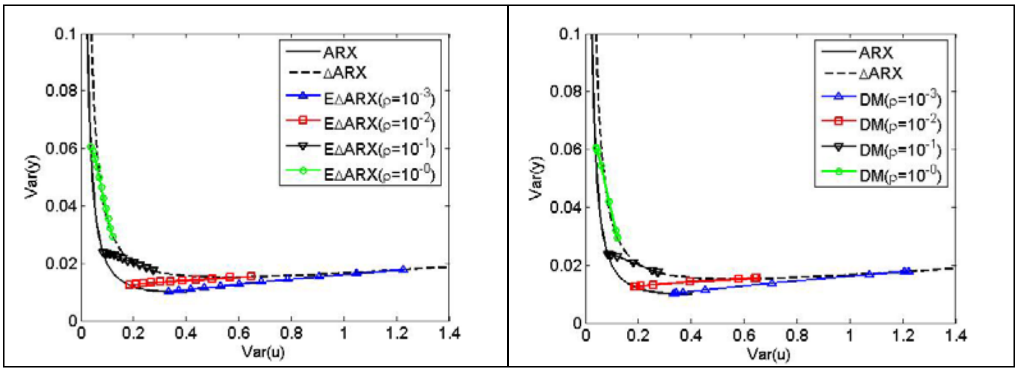

Figure 1. Pareto plot for four MPC implementations of ARX model based MPC.

In order to see the advantage of using the ARX MPC with a disturbance estimation algorithm, a set of simulation was performed over a horizon of 1000 samples. In the Base case no disturbance enters the system. For the case Small step and Large step a sustained disturbance enters at time 50 with a magnitude of 0.25 and 1 respectively. Finally a disturbance which drifts as integrated white noise with the same variance as the process noise is used. The results are shown in Fig. 2 as the closed loop performance versus the sensitivity to the prediction error in the disturbance update. By converting the actual tuning parameters like this, the plots are more easily compared. L_d is the observer gain to disturbance state which is a function of the variance for the disturbance model.

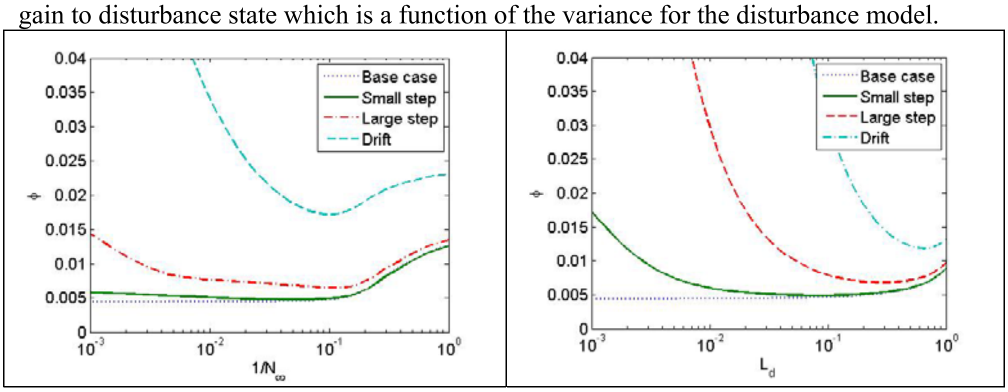

Figure 2. MPC performance versus sensitivity to the prediction error in the disturbance update.

While the minimum value for the performance cost for the four cases are very close for both methods, disturbance estimation with time-varying forgetting clearly provides better performance for different types of disturbances over a broad range of the tuning parameter. This is not the case for the disturbance modelling, which is clearly superior when the disturbance drifts, but this method requires detailed knowledge of the disturbance to provide good performance.

5. Conclusion

ARX models are well suited for control design since this linear model class is associated with a convex parameter estimation problem for both SISO and MIMO systems. Several methods for ensuring offset-free tracking by manipulation of the system model is presented and related in a simulation study. The disturbance model approach or the EΔARX model gives an extra degree of freedom for tuning, compared to including an integrator as in the ΔARX model. This tuning needs to balance fast convergence of the disturbance state against high noise sensitivity in the estimate. These methods are identical in performance, but the tuning parameter, α, in the EΔARX model is bounded. Adaptive disturbance estimation with time-varying forgetting adjusts the speed of adaptation for the disturbance estimate, according to the prediction errors. Hence it provides fast estimation when needed, while low noise sensitivity when the disturbance is known. It was shown in a simulation study that this formulation is less sensitive to the nature of the disturbance and must be preferred if the disturbance consist of infrequent steps of changing size.

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