Abstract—In this paper a prominent class of iterative learning control (ILC) algorithm is reformulated in the framework of estimation-based multiple model switched adaptive control (EMMSAC). The resulting control scheme uses a bank of Kalman filters to assess the performance of a set of candidate plant models, and the ILC update at the end of each trial is constructed using the plant model with smallest residual. The underlying EMMSAC framework provides rigorous bounds for robust performance for unstructured uncertainties and without placing constraints on the underlying controllers. This paper hence addresses current limitations in ILC approaches for uncertain systems with experimental results from a highly relevant application of ILC in stroke rehabilitation confirming efficacy and scope.

I. INTRODUCTION

Iterative learning control (ILC) is a methodology suitable for systems that perform the same tracking operation over a fixed finite time interval, with a reset period between executions in which the system is re-initialised. Each execution is termed a trial, and the key concept is to successively improve tracking accuracy by employing data from previous trials to update the control input applied in the subsequent trial. Over the course of 30 years a rich framework has emerged, combining well-developed theoretical results with a wide variety of application domain examples [1]. A prominent member of the class of optimisation based ILC is termed norm optimal ILC (NOILC) which embeds precise control over error and input norm evolution [2], [3], [4], [5].

One successful application of ILC is in upper limb stroke rehabilitation using functional electrical stimulation (FES). To enable patients to practice functional tasks, FES is applied to their muscles and promotes cortical reorganisation through neuroplastic changes in the brain, addressing the loss of motor function on one side of the body which affects around 80% of stroke survivors [6]. Clinical evidence [7] supported by neuroscience and motor learning results [8], [9] shows that the therapeutic action of FES is closely associated with the accuracy with which FES supports the patient’s movement over repeated attempts at the task. While most clinical FES systems employ open-loop or triggered FES control, recent clinical trials using ILC have confirmed its significant potential to provide far more effective therapy [10]. In this application NOILC tightly regulates the amount and change in FES, ensuring the patient applies maximum voluntary effort which promotes the therapeutic effect. Although ILC has established feasibility of model-based FES control, the performance of standard ILC algorithms degrades quickly due to model uncertainty caused by rapidly changing physiological effects, such as muscle fatigue and spasticity.

ILC schemes exist that are able to adapt to plant uncertainty, however such uncertainty is generally highly parametrised, placing restrictive assumptions on the underlying plant structure. At the same time, the structure and performance of the controller are prescriptive, limiting control over performance and convergence properties. Heuristic approaches involve identification methods that adjust the model used by ILC [11], permitting more general plant uncertainty. Unfortunately a lack of theoretical basis means there is no guarantee of performance or stability.

This paper addresses this problem by re-casting the ILC updating structure in the powerful framework of estimation-based multiple model switched adaptive control (EMMSAC) [12], [13]. The version of EMMSAC utilised here comprises a bank of Kalman filters to assess the performance of a set of candidate plant models, and the ILC controller corresponding to the most suitable plant model is then switched into the closed-loop. Distinct from other switched multiple model approaches, robust performance bounds for EMMSAC are invariant to the size of the plant model set [12], [13]. The combination of multiple models and ILC developed in this paper within the rigorous analytical framework of EMMSAC places no restriction on control structure or plant uncertainty form. Experimental results using FES confirm feasibility and high tracking performance in the presence of rapidly changing levels of muscle fatigue.

II. CONTROL ALGORITHMS

In this section the structure of EMMSAC is summarized, together with an optimal tracking control example. NOILC is introduced and reformulated in the EMMSAC framework.

A. EMMSAC

EMMSAC is a method that utilizes optimal disturbance estimation to measure the performance of candidate plants, \( \{P_1, \ldots, P_n\} \). Given the feedback system shown in Figure 1 where \( P_0 \) represents the true plant and \( C \) a stabilizing controller, the performance of plant candidates is determined by the size of the smallest disturbance estimates, \( \omega_0 = (u_0, y_0) \), that explain the observed plant input-output signals, \( \omega_2 = (u_2, y_2) \). For each plant candidate, \( P_p, 1 \leq p \leq n \), an estimator, \( E_p \), is designed, which is a member of the estimator set, \( \{E_1, \ldots, E_n\} \). The estimator, \( E_p \), produces the smallest disturbance estimate, \( (u_p^0, y_p^0) \), corresponding to its
associated plant, $P_p$, and observed signals, $(u_2, y_2)$. The 2-norm of the disturbance estimate results in the assignment of a scalar residual to each of the candidate plant models. Whilst the determination of the smallest disturbance estimate is computationally intractable, in the $l^2$ setting, Kalman Filters can be used to recursively determine the required residual.

For plant model $p$, the corresponding Kalman filter, is denoted by $KF_p$ (where $p$ will be replaced by $\tilde{p}$ in the ‘lifted’ ILC case). $KF_p$ is designed using the state space matrices, $A_p, B_p$, and $C_p$ for plant $p$ and takes the measured input/output signals $u_2, y_2 - r$ as inputs. Recall that $C_p$ is also used to represent the controller for plant $p$; the meaning of $C_p$ should be clear from the context in which it is used. For plant model $p$ at sample $T$, the residual is given by

$$r_p(T) = \left[ \sum_{i=0}^{T} \lambda^{T-i} \| y_2(i) - \hat{y}_p(i) \|_{C_p\Sigma_p(i)C_p^T + \lambda^{-1}} \right]^{1/2}$$

in which $\lambda \leq 1$ is a forgetting factor. $y_2$ is the measured plant output, $\hat{y}_p$ is the Kalman filter estimate of the plant output, $y_1$, before disturbance $y_0$, and $\Sigma_p(i)$ is the covariance of the updated state estimate.

The residual indicates the extent to which the associated model explains the observed signals; the smaller the size of the residual, the smaller the estimated disturbances, the ‘better’ the model matches the true plant. Prior to practical implementation, a controller design procedure is used to assign a stabilizing controller, $C_p$, $1 \leq p \leq n$ to each of the plant candidates. The resulting controllers form the controller set, $\{C_1, \ldots, C_n\}$. At a specified decision rate (in the case of ILC, periodically at the end of each trial, or in the lifted representation used below, at every time step), the controller corresponding to the plant with minimal residual is switched into closed-loop with zero initial conditions.

The switching signal $q(i)$ indicates the plant $P_p$, $1 \leq p \leq n$, that has minimal residual and index (given an ordered plant model set) at time $i$. It is given by

$$q(i) := \arg\min_{1 \leq p \leq n} r_p[i], \ \forall k \in \mathbb{N}$$

As $q$ varies with time, different controllers are switched into closed-loop operation. The controllers are switched in with zero initial conditions i.e.

$$C : y_2 \rightarrow u_2, \ u_2(i) = C_{q(i)} \begin{cases} 0 & \text{if } t < i_s \\ y_2(t) & \text{if } t \geq i_s \end{cases}$$

where $i_s$ is the sample at which the switch occurs:

$$i_s = \max \{ t \leq i | q(t) \neq q(t-1) \}$$

For a comprehensive description of the estimators, residual calculation, and switching algorithm, see [14], [15]. For full robust stability and performance proofs, see [12], [13].

### B. Underlying Controller A: Optimal Tracking Controller

Suppose the objective is to track a output reference $r(i)$ over samples $0 \leq i \leq N$, and let candidate plant $P_p$ have state-space representation $p = (A, B, C, 0)$, such that

$$x(i+1) = Ax(i) + Bu_1(i)$$
$$y_1(i) = Cx(i)$$

A suitable strategy is to select controller $C_p$ to minimise a linear quadratic (LQ) cost comprising control effort and error norm terms

$$\min_{u_2} \sum_{i=0}^{N} \left\{ u_2(i)^\top R u_2(i) + y_2(i)^\top Q y_2(i) \right\}$$

where $y_2 = y_0 + r - y_1$ and $R = R^\top > 0$, $Q = Q^\top > 0$. $C_p$ can be realised by combined state feedback and a predictive feedforward term, given by

$$u_2(i) = \{B^T K(i) B + R\}^{-1} B^T K(i) A \hat{x}(i) - R^{-1} B^T \xi(i)$$

where $\hat{x}$ is generated by the stable estimator $E_p$

$$\hat{x}(i+1) = A \hat{x}(i) - B u_2(i) + L_p(i) (y_2(i) - r(i) - C \hat{x}(i))$$

The feedforward term is

$$\xi(i) = \{I + K(i) B R^{-1} B^T \}^{-1} \{A^\top \xi(i+1) - C^\top Q r(i+1)\}$$

with terminal condition $\xi(N) = 0$, and the Riccati equation

$$K(i) = -A^\top K(i+1) B [B^T K(i+1) B + R]^{-1} B^T \times K(i+1) A + A^\top K(i+1) A + C^\top Q C$$

has terminal condition $K(N) = 0$. To provide an explicit state-space representation of $C_{K(p)}$, note that

$$\hat{x}(i+1) = A_c(i) \hat{x}(i) + B_c(i) (y_2(i) - r(i)) + BR^{-1} B^T \xi(i)$$
$$u_2(i) = C_c(i) \hat{x}(i) - R^{-1} B^T \xi(i)$$

with $c = K(p)$ denoting matrices $(A_c, B_c, C_c, 0)$ given by

$$A_c(i) = A - L_p(i) C - B [B^T K(i) B + R]^{-1} B^T K(i) A,$$
$$B_c(i) = L_p(i), \ \ C_c(i) = \{B^T K(i) B + R\}^{-1} B^T K(i) A$$

It is easily verified that $[P_p, C_p]$ has the properties of (i) stability and (ii) linear growth in the presence of disturbances $(u_0, y_0)$. This enables the extensive EMMSAC framework to provide comprehensive robust performance results as a function of the choice of the plant model set, estimator and controller properties, and switching delay [14], [15]. Results using the optimal tracking controller appear in Section IV and comprise an experimental validation of EMMSAC.

To enable it to correspond with the later ILC structure, suppose now that the plant states are reset after sample $N$,
and the operation is then repeated. Each execution of the task is termed a ‘trial’, and the trial number \( k \) is an additional argument. The plant \( P_p \) given by (1) is hence replaced by

\[
\begin{align*}
\hat{x}(k, i + 1) &= A\hat{x}(k, i) + B\hat{u}_1(k, i) \\
y_1(k, i) &= C\hat{x}(k, i)
\end{align*}
\tag{8}
\]

and where

\[ u_1(k)(i) = \hat{u}_1(k, i) \] (similarly for \( u_0, u_2, y_0, y_1, y_2 \)). Resetting between trials means that \( \hat{x}(k, 0) = x_0 \) for all \( k \). Over the \( k^{th} \) trial the plant (8) can be represented in ‘lifted’ form

\[ y_1(k) = P_p u_1(k), \quad k \geq 1 \tag{9} \]

using the supervector matrix

\[
P_p = \begin{bmatrix}
CB & 0 & \cdots & 0 \\
CAB & CB & 0 & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
CAN^{−1}B & CAN^{−2}B & \cdots & CB
\end{bmatrix}
\]

where

\[
\begin{align*}
u_1(k) &= \begin{bmatrix} \hat{u}_1(k, 0)^T & \hat{u}_1(k, 1)^T & \cdots & \hat{u}_1(k, N)^T \end{bmatrix}^T \\
y_1(k) &= \begin{bmatrix} \hat{y}_1(k, 1)^T & \hat{y}_1(k, 2)^T & \cdots & \hat{y}_1(k, N+1)^T \end{bmatrix}^T.
\end{align*}
\]

Through algebraic manipulation of (7), the system can then be represented as in Figure 2 where \( P^* = \tilde{R}^{-1}P\check{Q} \).

![Figure 2](image)

The update is represented in lifted form as

\[ u_2(k+1) = u_2(k) + \Xi_c y_2(k+1) - (P^* + \Xi_c)(I + P\tilde{P})^{-1}y_2(k) \tag{13} \]

As \( P_p \) describes a linear system, it can be represented by a state space system in the lifted space, \( \tilde{p} = (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) \). Likewise the controller \( \tilde{c} = K(\tilde{p}) \) can be represented by a further state space system in the lifted space, \( \tilde{c} = (\tilde{A}_p, \tilde{B}_p, \tilde{C}_p, \tilde{D}_p) \). Hence (13) corresponds to the state-space system in lifted space \( \tilde{A} = 0, \tilde{C}B = -(P^* + \Xi_c)(I + P\tilde{P})^{-1}, \tilde{D} = \Xi_c \), e.g. \( \tilde{p} = (0, (I + P\tilde{P})^{-1}, -(P^* + \Xi_c), \Xi_c) \).

Note that similarly to the previous section, the Kalman Filter bank is implemented based on the signals \( u_2 \) and \( y_2 \), since the reference \( r \) in Figure 3 has been incorporated into the control relation between \( u_2 \) and \( y_2 \).

### III. Experimental Algorithm Application

The issues now relevant to implementation are selection of: i) the size of the the uncertainty in the true plant, ii) the number of plants that should be used to give a suitable ‘cover’ for this uncertainty, and iii) their distribution within the set. These issues are now addressed in the following sections.

#### A. Description of True Plant

To illustrate feasibility of the suggested control approach, isometric elbow extension is considered, in which the stimulated muscle is held at a fixed length. The experimental setup is illustrated in Figure 4. The participant is seated at the table with their forearm placed in a passive support that is able to rotate freely about its attachment point. A 6-axis force/moment sensor measures the forces produced at the end effector. A high back chair is used to fix the position of the participant’s shoulder, and is positioned such that the angle at the elbow joint is approximately ninety degrees. Fixing the positions of the hand and shoulder ensures that the triceps is maintained at a fixed length. Two surface
electrodes are attached to the triceps, and connected to a commercial stimulator. The control input to the stimulator is an 80 Hz rectangular pulse signal with amplitude fixed at a maximal comfortable level at the beginning of the session. The pulsewidth is the controlled variable. During experiments, participants are asked to provide zero voluntary effort. Stimulation is applied to the triceps and the resulting forces are measured. A kinematic model of the upper limb is then used to calculate the torque developed at the elbow joint, $T_d = [F_x \ F_y \ F_z]^\top \cdot (\hat{v}_r \times \hat{v}_f) \cdot l_1$ in which $l_1$ is the length of the forearm and $\hat{v}_e = \hat{v}_f \times \hat{v}_u$. The vectors $\hat{v}_f$ and $\hat{v}_u$ are aligned with the forearm and upper arm, respectively, and are calculated using the subject’s arm lengths and a range of movement test [14].

**B. Plant Model Structure**

The Hammerstein structure illustrated in Figure 5 is used to model the true plant, where $f$ is a nonlinear function representing the isometric recruitment of electrically stimulated muscle and $G$ is a transfer function representing linear contraction dynamics. The input, $u_2$, is the stimulation pulsewidth, and $u_0$ and $y_0$ are the disturbances acting on the plant. The input disturbance, $u_0$, is assumed to appear after the nonlinear component, $f$, allowing the nonlinearity to be cancelled by applying an appropriate inverse function, as detailed in Section III-D. The nonlinear component is given by

$$f_p : u_2 \mapsto w$$

$$w = a_1 \left( e^{a_2 u_2} - 1 \right) / \left( e^{a_2 u_2} + a_3 \right), \quad 0 \leq u_2 \leq 300 \quad (14)$$

in which $a_1, a_2$ and $a_3$ must be identified. The limits on the input, $u_2$, are approximate values at which saturation occurs.

The contraction dynamics are represented by

$$G_p : u_1 \mapsto y_1$$

$$Y_1(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} U_1(s)$$

in which, $\omega_n$, is the uncertain parameter. The state space representation of this transfer function takes the observable canonical form. The resulting matrices $(A_p, B_p, C_p)$ are discretised using zero order hold sampling at 160 Hz. The resulting discrete-time state space matrices are used to construct the estimators and controllers.

**C. Plant Model Identification**

The ramp deconvolution identification procedure is used to identify values of the uncertain parameters $[\omega_n, a_1, a_2, a_3]$. The experiment is set up as described in Section III-A and a triangular ramp stimulation pattern applied to the participant’s triceps in open-loop. The measured torque about the elbow is input into a least squares optimisation procedure, which yields $\omega_n, a_1, a_2$, and $a_3$. See [18] for full details.

To achieve a constant level of controller performance, the models within the plant model set should be distributed to accurately reflect the various stages of fatigue. Fatigue can be induced by repeated application of the triangular ramp input signal, where a single application is known as a trial. For each trial, the measured torque response is used to identify a model representing the true plant. Each session consists of thirty consecutive trials with a rest period in between to allow the muscle to recover. For a single participant, five sessions were performed per day over four consecutive days, yielding a set of six hundred plant models $(30 \times 5 \times 4)$. The resulting set of nonlinearly distributed plant models represents the uncertain true plant under the prescribed experimental conditions. Specification of the number, $m$, of linear models, and the number, $n$, of nonlinear models, allows this set to be sampled based on the distribution of models within the set. See [15] for full details and analysis of the obtained fatigue data. Although the set of candidate plants is identified using data from a single subject, an important result is that it can be used to represent multiple subjects of similar age and fitness without significant performance reduction [15].

**D. Linearisation**

The plant is first linearised by applying the inverse, $f^{-1}$, of (14) to the output from the controller before applying it to the true plant. $f^{-1}$ is given by

$$f_p^{-1} : u^* \mapsto u_2$$

$$u_2 = \frac{1}{a_2} \ln \left( \frac{a_1 + a_2 u^*}{a_1 - u^*} \right), \quad T_a \leq u^* \leq T_b$$

in which $u^*$ represents the iterative learning controller output. $T_a$ and $T_b$ are model-dependent limits on the inverse function, corresponding to the pulse width limits imposed on (14), $0 \leq u_2 \leq 300$.

Once the plant has been linearised, the LQ optimal controller or the iterative learning controller of Sections II-B
and II-C, respectively, can be applied to control the linear component, $G$, of the plant model.

IV. EXPERIMENTAL RESULTS

The optimal tracking controller and the NOILC schemes of Section II have been applied and each has been implemented both with and without switching between plant models. This hence enables evaluation of the EMMSAC framework, and role of learning from past executions of the task via ILC.

The optimal tracking controller is implemented using (3), and switching between models is possible at any point over the duration of the experiment. To implement this controller experimentally, the time-series of the matrix gain, $K$, and the computed feedforward term, $\xi$, for each of the models in the plant model set is computed off-line. The bank of Kalman filters then determine which controller is switched into closed loop, and the state estimation corresponding to the minimum residual is used in (3). NOILC is implemented using the lifted representation, with the updating of the plant model at the end of each trial. The control input is provided by (11), and necessitates that the control input, $u_2(k)$, from the previous trial also be fed to the control block. The same bank of Kalman filters as in the optimal tracking case are used, but now only the residual recorded at the last sample in the trial is used to update the controller that is used to calculate the next ILC increment. For the first trial, the control input and state trajectory for the previous trial are assumed to be zero.

Initial experimental results are shown in Figure 6 for the optimal tracking controller without switching, and correspond to a total root mean square error (RMSE) of 1.139, as shown in Table I. Weights of $Q = 10$ and $R = 1$ are used, and degradation in tracking accuracy is clear, due to fatigue and other physiological effects. With the inclusion of switching, significant improvement occurs as the controller switches in plants that better capture the changing muscle dynamics. This is illustrated in Figure 7, and gives rise to an rmse of 0.812.

Turning to NOILC, weights of $Q = 10$ and $R = 1$ are again selected, and results are shown in Figure 8 with no plant switching. Degradation in accuracy is evident as physiological changes take effect, but inherent robustness enables NOILC to compensate provided they are not excessive. Consequently, the resultant RMSE improves upon that of the

<table>
<thead>
<tr>
<th></th>
<th>no switching</th>
<th>switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal</td>
<td>1.1390</td>
<td>0.8120</td>
</tr>
<tr>
<td>NOILC</td>
<td>0.8727</td>
<td>0.8074</td>
</tr>
</tbody>
</table>

TABLE I
RMSE TRACKING RESULTS
linear tracking controller without switching, but is inferior to
the same controller with switching.
Lastly NOILC with plant model updating is applied, with
results appearing in Figure 9. The combination of a controller

![Figure 9. Norm Optimal ILC with switching.](image)

that learns from past experience, together with the ability to
embed plant switching in a robust framework, means that
accurate tracking control is maintained despite significant
physiological changes. These results hence confirm the ef-
ficacy of the proposed control structure, and its potential
within the demanding rehabilitation domain.

V. CONCLUSIONS AND FUTURE WORK

This paper has presented a class of algorithms which
combine multiple models for iterative learning control. The
EMMSAC framework leads to the potential for strong robust
stability guarantees, and the combination of ILC and multiple
models seeks to overcome the dependence on a good nom-
inal model for model based ILC. The experimental results
presented show the efficacy of the approach in the key area
of FES-based stroke rehabilitation, in particular on the isometric
arm. Future work will extend practical evaluation to the non-
isometric case, unlocking the full potential of the approach.

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