EFFECTS OF RESONANCES ON HALO FORMATION IN HIGH INTENSITY RINGS*

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Abstract

Numerical calculations for the Spallation Neutron Source accumulator ring indicate that lattice resonances excited by the space-charge potential can increase a mismatch significantly by deforming the beam distribution in phase space. Hence increased mismatch leads to enhanced envelope oscillations that are driving the 2:1 parametric resonance leading to halo formation, even for initially matched beams. We have observed this behaviour for the $2\nu_x-2\nu_y=0$ resonance and for the $4\nu_y=23$ resonance [1]. This mechanism for halo formation peculiar to rings through resonance driven mismatch is very sensitive to the tunes, which emphasises the importance of a careful choice of operating point in tune space.

1 INTRODUCTION

The acceleration of intense beams has become very relevant due to a number of important applications. These include neutron source, the transmutation of nuclear waste, tritium production, and accelerator-driven fusion. For example, the accelerator system of the Spallation Neutron Source (SNS) [2] will deliver a 1 GeV pulsed proton beam to a liquid Hg target at 60 Hz. The accumulator ring is being designed to support 2 MW of beam, which implies that it must be capable of storing more than 2×10^{14} protons in each pulse. There are stringent requirements for uncontrolled losses, namely about one part in million per meter. Because of this combination of high beam intensity and low uncontrolled loss requirements, space charge contributions to beam loss and halo formation are of crucial importance to the SNS and other high intensity projects.

Space charge effects have been studied for many years, particularly for linear accelerators. Since the development of KV distribution and envelope equation [3-6], numerical simulations, mostly for linacs, and the particle core model indicate that the principal cause of space-charge-induced halo is the 2:1 parametric resonance excited by the envelope oscillations of mismatched beams [4-10].

It was shown for the first time by D. Jeon *et al* [1] that lattice resonances such as $2\nu_x-2\nu_y=0$ induce a mismatch and excite the 2:1 parametric resonance even for initially well-matched beams by deforming beam distribution. This previously unknown resonance mechanism can form appreciable halos in rings even for a very small tune depression of about 1% unlike tens of percents of tune depression usually considered in linacs.

The study is carried out using the lattice and beam parameters of the 2 MW SNS accumulator ring where a 1 GeV beam from the linac is injected into a dispersion free straight section. The tune depression due to space charge effects is $\Delta\nu\approx0.08$, or about 1.4%. In this study, no injection process is simulated. We tracked beams for many turns starting from the initial distribution of 2 MW beams at 0 turn. The ACCSIM code with 2 Dimensional Particle-In-Cell (2D PIC) space charge evaluation is used [11]. The ACCSIM code treats the lattice using linear transfer matrices generated by the DIMAD code. Any higher order multipoles are not included in numerical simulations. Also any skew lattice elements are not included such as skew quadrupoles. Consequently, the space charge potential provides the only nonlinearities in the Hamiltonian. We treat the beam dynamics in transverse phase space assuming a coasting beam with energy spread ±9.4 MeV.

2 DIFFERENCE RESONANCES

Hofmann [12] showed, using self-consistent Vlasov-Poisson equations, that the space charge potential can support an even mode space charge potential

$$\Phi_{4,e}=a_0x^4+a_1x^2y^2+a_2y^4 \qquad (1)$$

inside the beam. We examine now the effect of the $2\nu_x-2\nu_y=0$ resonance. To do this we consider two cases with bare tunes $\nu_y=5.82$ and $\nu_y=5.67$, respectively. Here, $\nu_x=5.82$ for both cases. In the first case, the tune separation is 0 and we anticipate strong excitation of the difference resonance, while in the second case the tune separation is 0.15 and the excitation should be less. In both cases the initial beams are given a radial mismatch of 3%, which is quite small compared with the mismatch considered in linacs to observe halo formation (generally a few tens of percent). 20,000 macro particles are used for numerical simulations except for a few cases where more resolution in phase space distributions is required. In this case 100,000 macro particles are used. Simulations by varying the number of macro particles show that 20,000 macro particles are quite enough.

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* Work supported by Division of Material Science, DOE Contract No. DE-AC05-96OR22464

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Figure 1 shows the rms emittances and the second moments $<\Delta x^2>$ and $<\Delta y^2>$ in mm^2 for the two cases. The oscillation amplitude of the second moments is a measure of beam envelope oscillations. When the tunes are equal, a sudden and massive emittance growth is induced simultaneously in both planes at about 900 turns. This is preceded by a rapid growth in the oscillation amplitudes of $<\Delta x^2>$ and $<\Delta y^2>$ at about 850 turns. Following the emittance growth, the oscillations of the second moments subside, reflecting the existence of a new relaxed beam equilibrium with halo. In contrast, in the case having unequal tunes the beam is fairly stable, with little emittance growth, even though the beam has the same initial mismatch as the case of $\nu_x=5.82$ and $\nu_y=5.82$.

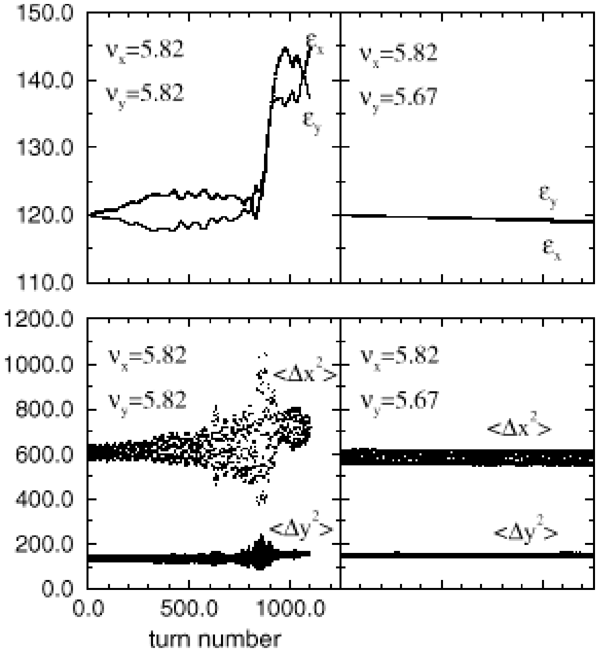

Figure 1. Plots of rms emittances (top plots) in πmmmrad and the second moments $<\Delta x^2>$ and $<\Delta y^2>$ (bottom plots) in mm^2 for two cases. Left plots are for $\nu_x=5.82$ and $\nu_y=5.82$, and right ones for $\nu_x=5.82$ and $\nu_y=5.67$.

Figure 2 shows that the $<\Delta x^2\Delta y^2>$ moment in mm^4, for the case $\nu_x=5.82$ and $\nu_y=5.82$, increases dramatically reaching peak at around 850 turns prior to the sudden emittance growth at around 900 turns, much the same as the second moments. This particular moment reflects the coupling term in Eq. (1) that can excite $2\nu_x-2\nu_y=0$ resonance and its behavior supports the involvement of this resonance in the emittance growth in the case having equal tunes. During the emittance blow-up, asymmetry in the particle distribution is also induced generating weak odd mode space charge potentials. Figure 2 also shows that a sudden growth of the asymmetric $<\Delta x\Delta y>$ moment is induced at about 900 turns when the emittance growth takes place. This shows an inducement of small asymmetry in the particle distribution that can induce the second order odd mode potential $\Phi_{2,o}=a_1xy$ [11]. This potential can excite the $\nu_x-\nu_y=n$ resonance. However, it should be noted that the maximum oscillation amplitude of $<\Delta x^2\Delta y^2>$ moment is at about 850 turns while that of $<\Delta x\Delta y>$ moment occurs at about 940 turns. It is clear that changes in the $<\Delta x^2\Delta y^2>$ moment is leading all the process. Besides, even mode particle distributions which are symmetric with respect to the mid-plane are strongly favored as equilibrium particle distributions rather than odd mode potentials due to the mid-plane symmetry of the accelerator. So the dominant lattice resonance responsible for the emittance blow-up is the $2\nu_x-2\nu_y=0$ resonance rather than the $\nu_x-\nu_y=0$ resonance.

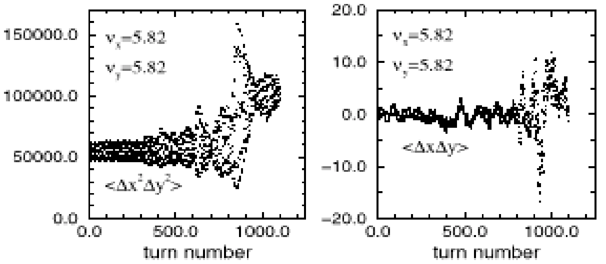

Figure 2. Plots of the fourth moment $<\Delta x^2\Delta y^2>$ in mm^4 and the second moment $<\Delta x\Delta y>$ in mm^2 for the beam in Fig. 1 for $\nu_x=5.82$ and $\nu_y=5.82$.

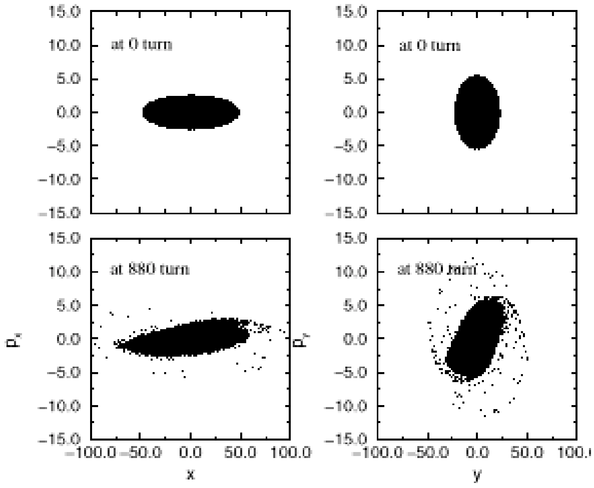

Figure 3. Plots of phase space beam distribution at 0 turn and at 880 turn right before emittance blow-up. These plots are for $\nu_x=5.82$ and $\nu_y=5.82$.

Figure 3 shows the induced deformation in the particle distribution due to the difference resonance at 880 turns in both phase spaces when $\nu_x=5.82$ and $\nu_y=5.82$. The width of the beam has grown in both dimensions x and y. The beam distribution is severely deformed. The point is that the beam distribution changes as time goes on due to the $2\nu_x-2\nu_y=0$ resonance. This deformation of the beam distributions induces an even larger mismatch and envelope oscillations evidenced clearly by the dramatic increase in the oscillation amplitude of second moments shown in the left lower plot of Fig. 1. On the contrary, nothing happens for $\nu_x=5.82$ and $\nu_y=5.67$, being far from the difference resonance. Numerical simulations indicate that the width of the resonance is $|\nu_x-\nu_y|<0.04$.

Figure 4 shows the case of an initially well matched beam (radially with less than 1% mismatch) having $\nu_x=5.82$ and $\nu_y=5.82$. The difference resonance is excited and this leads to a further mismatch, as shown by the increasing oscillation amplitudes of second moments. Even in the case of a well-matched beam, the $2\nu_x-2\nu_y=0$ resonance is excited and ultimately strongly excites a strong envelope oscillations indicated by the huge oscillation amplitude of second moments at around 2300 turns. This well-matched case is observed to remain stable for over 2000 turns. The emittance blow-up is driven by the mismatch induced by the difference resonance $2\nu_x-2\nu_y=0$. Similar effects may be expected for intense beam linacs when the phase advances are equal.

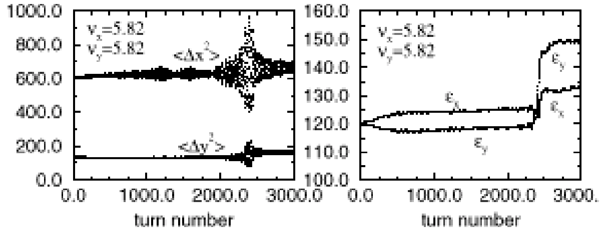

Figure 4. Plots of second order moments and rms emittances for an initially well matched beam when $\nu_x=5.82$ and $\nu_y=5.82$.

3 OTHER RESONACES

Figure 5 illustrates this process for a matched beam having bare tunes $\nu_x=5.82$ and $\nu_y=5.77$. The first moderate emittance increase near 200 turns which is due to the excitation of the fourth order $4\nu_y=23$ resonance. Obviously this is not a structural resonance considering the fourfold symmetry of the lattice. However, we think this is related with the initial beam distribution. The left plot in Fig. 6 is a snapshot of particle distributions in y phase space taken at 250 turns immediately after the first y emittance increase. It shows the structure of this resonance very clearly.

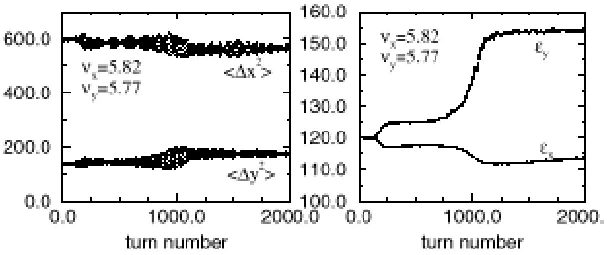

Figure 5. Plots of second moments and rms emittances.

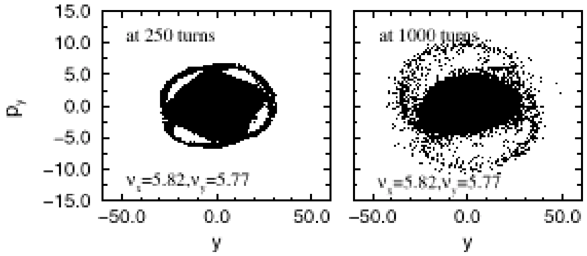

Figure 6. Plots of beam distribution at 250 turns and at 1000 turns for the beam in Fig. 5.

4 CONCLUSIONS

This new mechanism for halo is of considerable interest by itself, but its crucial impact is that it can lead to a better design to minimize beam-loss halo in high intensity rings. All of the dynamics studied in this work are very sensitive to the choice of operating point in tune space. A detailed study of halo generation in tune space will be required to find an operating point that minimizes halo.

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