Application of MMSE Multi-user Detection to CDMA Unslotted ALOHA System

Hiraku Okada†, Peter M. Grant‡, Ian W. Band§, and Akira Ogawa††

†Center for Information Media Studies, ‡Dept. of Electronics & Electrical Engineering, ††Dept. of Information Sciences, The University of Edinburgh, Meijo University, Nagoya University, Nagoya, 464-8603, Japan. Edinburgh, EH9 3JL, UK. Nagoya, 468-8502, Japan.
okada@media.nagoya-u.ac.jp

Abstract—A CDMA unslotted ALOHA system using MMSE multi-user detection is proposed. In CDMA unslotted ALOHA system, a user station can transmit a packet asynchronously, and so the number of simultaneously transmitted packets fluctuates moment by moment. This fluctuation affects the performance of the MMSE multi-user detection. At first, we employ the Wiener filter detector assuming perfect information as an upper bound of the performance of the proposed system. Then, we apply an adaptive filter to CDMA unslotted ALOHA to compensate for the birth/death of a packet.

I. INTRODUCTION
A Code-Division Multiple-Access (CDMA) ALOHA, which is a connectionless-type of CDMA packet communication system, has drawn much attention for wireless data communications because of features such as random access capability, the potential for high throughput performance and low peak power transmission. Moreover, CDMA unslotted ALOHA (CDMA U-ALOHA) systems have the advantage of not requiring synchronisation of the packet transmissions since initialisation occurs at the beginning of a slot in slotted systems. Many authors (e.g. [1]–[3]) have investigated improving the system performance beyond that of the conventional receiver [3].

In a CDMA system, multiple access interference (MAI) is an important obstacle to overcome. In order to reduce the effect of MAI, several multi-user detection techniques have been investigated [4], [5]. Most of these, however, focus on non-packet data, and multi-user detection techniques are seldom applied to CDMA U-ALOHA because other problems arise. One of the most important problems is the birth/death scenario because packets are transmitted randomly and intermittently.

In this paper, we propose to apply the minimum mean square error (MMSE) multi-user detection technique to CDMA U-ALOHA and evaluate the throughput performance of our proposed system. At first, we employ the Wiener filter receiver under the perfect information assumption, which means that all information required for the Wiener filter is known even if a packet birth/death occurs. This corresponds to the upper bound of the performance of a CDMA U-ALOHA system using MMSE multi-user detection. Then, we apply the adaptive finite impulse response (FIR) filter receiver to CDMA U-ALOHA to compensate for the birth/death of a packet. The use of adaptive algorithms is not only motivated by the time-varying channel but also by the dynamic user profile [6]. We also evaluate the throughput performance taking into account the possible birth/death of a packet.

In Section II, we describe the system model. In Section III, the MMSE multi-user receiver structure at the hub station is explained. In Section IV, we evaluate the throughput performance and discuss the results. Finally some conclusions are presented in Section V.

II. THE SYSTEM MODEL

Figure 1 shows the system model of the CDMA U-ALOHA system. This consists of a single hub station and an unspecified number of user stations. Each user station transmits a packet to the hub station by one hop, and we only consider up-link packet access. The hub station receives packets at a rate which follows a Poisson process with a birth rate \( \lambda \). The length of each packet is fixed. Each packet consists of a preamble sequence of \( L_p \) bits and a data block sequence of \( L_d \) bits, so the whole packet length is \( L = L_p + L_d \) bits. The preamble is employed as a training sequence for the adaptive filter. The offered load \( G \) is defined as the average number of packet generated during one data duration \( T_d = L_d / R \), where \( R \) is the data rate, and may be expressed as \( G = \lambda \cdot T_d \). The offered load corresponds to the traffic intensity of generated data. The throughput \( S \) is also defined as the average number of successful packets during one data duration, and is our main performance measure.

Binary phase shift keying (BPSK) is assumed as the modulation scheme. Each packet’s data is then spread with a uniquely assigned random signature sequence of length \( N \) chips. We assume that all packets are received with equal power and all data bit errors are caused by the effect of MAI and additive white Gaussian noise (AWGN). The received waveform of the \( k \)th user may be expressed
The filter receiver structure is shown in Figure 2. After down-converting the received signal to baseband, it is passed where

\[ R_{b}(t) = \sqrt{2P} \sum_{i=-\infty}^{\infty} b_{k}(i)q_{k}(t - iT - \tau_{k}) \cos(\omega_{t}t + \theta_{k}). \]

where \( P \) is the received power of each user’s signal \((P = E_{b}/T)\), where \( E_{b} \) is the bit energy and \( T \) is the bit interval, \( b_{k}(i) \in \{+1,-1\} \) is the \( i \)th bit of the \( k \)th user, \( a_{k}(t) \) is a binary spreading waveform, \( \omega_{t} \) is the carrier frequency, \( \tau_{k} \) is the transmission delay, and \( \theta_{k} \) is the carrier phase. The transmission delay \( \tau_{k} \) and the carrier phase \( \theta_{k} \) are taken to be independent and uniformly distributed over \( 0 \leq \tau_{k} < T \) and \( 0 \leq \theta_{k} < 2\pi \), respectively. These values can be assumed to be constant during reception of the packet because packet length is generally very short. Without loss of generality, we can assume \( \tau_{1} = 0 \) and \( \theta_{1} = 0 \). The spreading waveform may be expressed as

\[ a_{k}(t) = \sum_{j=0}^{N-1} a_{k,j}PT_{c}(t - jT_{c}), \]

where \( a_{k,j} \in \{+1,-1\} \) is the \( j \)th element of the spreading sequence for the \( k \)th user, \( T_{c} \) is the chip interval, and \( PT_{c}(t) \) is the rectangular chip waveform expressed as

\[ PT_{c}(t) = \begin{cases} 1 & (0 \leq t < T_{c}) \\ 0 & \text{otherwise} \end{cases}. \]

Then the received signal at the hub station may be expressed as

\[ R(t) = \sum_{k=1}^{K} R_{k}(t) + n(t), \]

where \( K \) is the number of simultaneously transmitted signals, \( n(t) \) is the AWGN signal with power spectral density \( N_{0}/2 \).

### III. MMSE Multi-User Detector

In this section, two types of MMSE multi-user detectors are explained. One is the Wiener filter receiver, which is employed to obtain an upper bound on the performance of the CDMA U-ALOHA system using an MMSE multi-user detector. The other is the adaptive filter receiver, which is employed to compensate for the birth/death of a packet.

#### A. Wiener Filter (Perfect Information Case)

We focus on the 1st user. A chip-level Wiener FIR filter receiver structure is shown in Figure 2. After down-converting the received signal to baseband, it is passed through a chip matched filter and sampled at the end of every chip interval \( T_{c} \). The \( i \)th normalized sample of the \( i \)th bit at the output of the chip matched filter of the 1st user may be expressed as

\[ r[0]_{i}(t) = \sqrt{\frac{1}{P_{c}T_{c}}} \int_{T_{c}/T+1}^{T_{c}/T+T} R(t)PT_{c}(t) \cos(\omega_{t}t)dt. \]

Let \( \mathbf{r}(i) = (r[0]_{i}(i), \ldots, r[N-1]_{i}(i)) \) be the vector of received samples of the \( i \)th bit, and \( \mathbf{a}_{k} = (a_{k,0}, \ldots, a_{k,N-1})^{T} \) be the vector of spreading sequence of the \( k \)th user. The vector of the received samples may be expressed as [7]

\[ \mathbf{r}(i) = \mathbf{b}_{1}(i)\mathbf{a}_{1} + \sum_{k=2}^{K} \mathbf{b}_{k}(i)\mathbf{b}_{k}^{T} + \mathbf{n}(i), \]

where

\[ \mathbf{b}_{1}(i) = \left[ \frac{\delta_{k}}{T_{c}} \mathbf{a}_{k}(p_{k}+1) + \left(1 - \frac{\delta_{k}}{T_{c}}\right)\mathbf{a}_{k}(p_{k}) \right], \]

\[ \mathbf{b}_{k}(i) = \left[ \frac{\delta_{k}}{T_{c}} \mathbf{a}_{k}(p_{k}+1) + \left(1 - \frac{\delta_{k}}{T_{c}}\right)\mathbf{a}_{k}(p_{k}) \right], \]

\[ \mathbf{a}_{k}^{(m)} = (a_{k,N-m}, a_{k,N-m+1}, \ldots, a_{k,N-1}, a_{k,0}, a_{k,1}, \ldots, a_{k,N-m-1}), \]

\[ \mathbf{a}_{k}^{(m)} = (-a_{k,N-m}, -a_{k,N-m+1}, \ldots, -a_{k,N-1}, a_{k,0}, a_{k,1}, \ldots, a_{k,N-m-1}). \]

and \( \mathbf{n}(i) \) is the AWGN vector, whose element is independent Gaussian random variables with zero mean and variance of \( \sigma^{2} = (2E_{b}/N_{0}T)^{-1} \). Also the delay \( \tau_{k} \) is written as \( \tau_{k} = p_{k}T_{c} + \delta_{k} \), where \( p_{k} \) is an integer and \( 0 \leq \delta_{k} < T_{c} \).

The received samples are fed into the chip-level Wiener FIR filter, which has \( N \) taps. The tap weight vector \( \mathbf{w} \) of this filter is calculated according to the MMSE criterion. It is given by the solution to the Wiener equation [8]

\[ \mathbf{Rw} = \mathbf{p}, \]

where \( \mathbf{R} \) is the autocorrelation \( N \times N \)-matrix of the input signal, and \( \mathbf{p} \) is the crosscorrelation \( N \)-vector of the input signal with the desired symbol. The crosscorrelation vector is given by [7]

\[ \mathbf{p} = \mathbf{a}_{1}. \]

The autocorrelation matrix is given by [7]

\[ \mathbf{R} = \mathbf{a}_{1}\mathbf{a}_{1}^{T} + \sum_{k=2}^{K} \left( \cos \theta_{k} \right)^{2} E[\mathbf{I}_{k}(i)\mathbf{I}_{k}(i)^{T}] + \sigma^{2}\mathbf{I}_{N \times N}, \]

where

\[ E[\mathbf{I}_{k}(i)\mathbf{I}_{k}(i)^{T}] = \frac{1}{2} \left( \frac{\delta_{k}}{T_{c}} \mathbf{a}_{k}(p_{k}+1) + \left(1 - \frac{\delta_{k}}{T_{c}}\right)\mathbf{a}_{k}(p_{k}) \right) \]

\[ + \frac{1}{2} \frac{\delta_{k}}{T_{c}} \mathbf{a}_{k}(p_{k}+1) + \left(1 - \frac{\delta_{k}}{T_{c}}\right)\mathbf{a}_{k}(p_{k}) \]

\[ \left( \frac{\delta_{k}}{T_{c}} \mathbf{a}_{k}(p_{k}+1) + \left(1 - \frac{\delta_{k}}{T_{c}}\right)\mathbf{a}_{k}(p_{k}) \right)^{T} \]
and $I_{N \times N}$ is an $N \times N$ unit matrix.

The outputs of the FIR Wiener filter are sampled at the end of each bit interval $T$, and demodulated by the threshold device.

In order to obtain the tap weights of the Wiener filter, information such as the spreading sequences, carrier phase, and transmission delay of each user must be known. We further assume that all this information is known even if a packet birth/death occurs. Under this assumption, we can obtain the upper bound of the proposed system.

### B. Adaptive Filter

An adaptive FIR filter receiver structure is shown in Figure 3. In the same way as the case of the Wiener filter, received signals are passed through a chip matched filter and sampled. These samples are fed into an $N$-tap adaptive filter. The tap weights of this filter are adjusted according to an adaptive algorithm. We use the two most popular adaptive algorithms, least mean squares (LMS) and recursive least squares (RLS) [8]. During reception of the preamble (training sequence), the outputs of the training sequence generator are used as reference signals. After this training period, the outputs of the threshold device are used to adjust the tap weights (decision direction).

We assume that only the information required for the adaptive algorithm such as the timing of the desired packet and the training sequence is known. The birth/death of interfering packets is therefore unknown. Even so, the adaptive filter has a possibility to compensate for the birth/death of a packet.

### IV. RESULTS

In this section, the system performance of the proposed system is evaluated by Monte Carlo simulation.

Figure 4 shows the throughput performance of CDMA unslotted ALOHA using the Wiener filter receiver under perfect information assumption with parameters $N = 60$, $E_b/N_0 = 10$ dB, $L = L_d = 500$ bits. For comparison, the throughput of a conventional system, in which a matched filter is used to despread the received signal, is also shown. From this figure, it can be seen that the throughput of a system using a Wiener filter is about 4 times as high as that of the conventional matched filter detection system. By using the MMSE multi-user detector, a significant improvement in throughput can be obtained.

Next, we discuss the performance of our proposed system taking into account the birth/death scenario.

Figure 5 shows the squared error (ensemble-averaged over 1,000 trials) of various algorithms for $E_b/N_0 = 10$ dB. Convergence curves of the RLS algorithm with a forgetting factor $\alpha = 0.99$ and the LMS algorithm with several step sizes $\mu$ are shown. Also shown in the lower part of the figure is the pattern of packet generation. In this pattern, there is one desired packet with a training sequence of 50 bits, 5 constant interfering packets, one interfering packet generated at the 100th iteration, and one interfering packet which finishes at the 200th iteration. Except for the case of LMS with $\mu = 0.0001$, the ensemble-averaged squared error converges during the training sequence. Because the effect of AWGN is larger than that of a birth/death of a packet, the convergence is not strongly affected by a birth/death event.

Figure 6 shows the throughput performance of CDMA unslotted ALOHA using an adaptive filter receiver with parameters $N = 60$, $E_b/N_0 = 10$ dB, $L_d = 500$ bits, and $L_t = 50$ bits. As can be seen, the throughput of most of
the adaptive filters is better than that of the conventional system. Moreover, the throughput of the RLS is only slightly better than that of the LMS. The throughput of all the adaptive filters is, however, much worse than that of the Wiener filter, but the parameters to calculate the Wiener filter solution are not readily accessible.

In the case of $E_b/N_0 = 10$ dB, the effect of a packet birth/death on the convergence properties is not clearly seen, so we next consider convergence properties in a low noise environment ($E_b/N_0 = 20$ dB), shown in Figures 7 and 8. The same pattern of packet generation as in Figure 5 is used in Figure 7, from which the effect of a packet birth/death can be seen more clearly. To emphasise, consider Figure 8, in which there are 5 interfering packets generated at the 100th iteration, and 5 interfering packets which finish at the 200th iteration. The effect of a packet birth can easily be seen in the figure, since the ensemble-averaged squared error increases suddenly at the point of packet birth. On the other hand, packet death does not have as strong an influence on the convergence properties, and ensemble-averaged squared error decreases only slightly, as observed previously in [6] and [9]. This observation may be explained by considering the point of packet birth. There, a new source of interference to the adaptive filter is generated and thus the the ensemble-averaged squared-error increases. On the other hand, at the point of packet death, this interference is removed, and thus the ensemble-averaged squared error is not increased. Moreover, use of an adaptive filter can compensate for the death of a packet, and so the ensemble-averaged squared-error is decreased.

Figure 9 shows the corresponding throughput performance for $E_b/N_0 = 20$ dB. The throughput of the RLS is now almost 3 times as high as that of the conventional (matched filter) system, while the throughput of the LMS is almost twice that of the conventional system. As compared with the case of $E_b/N_0 = 10$ dB, significant improvement is obtained at the high SNR.
V. CONCLUSIONS

We have proposed the application of the MMSE multi-user detection to the CDMA U-ALOHA system, and evaluated system performance under two situations. The first is the perfect information case and the other is using an adaptive algorithm to adjust the filter tap weights to compensate for a packet birth/death. As a result, significant improvements in throughput have been achieved with the proposed system under the perfect information assumption. Even employing an adaptive filter and considering the effect of birth/death events, the throughput performance is improved although not to the level of that for perfect information. We have also shown that the ensemble-averaged squared error increases at the point of packet birth, and decreases at the point of packet death.

As a future extension of this work, the use of interleaving and forward error correction coding techniques [10] may prove beneficial, since such techniques may be expected to mitigate the momentary increase in ensemble-averaged squared error caused by packet birth.

ACKNOWLEDGMENT

This work was supported in part by Ministry of Education, Science, Sports and Culture, Government of Japan under Grant-in-Aid for Scientific Research, IDO Corp., and YRP Mobile Telecommunications Key Technology Research Laboratories Co. Ltd.

REFERENCES