Efficient and good Delaunay meshes from random points

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Collaborators

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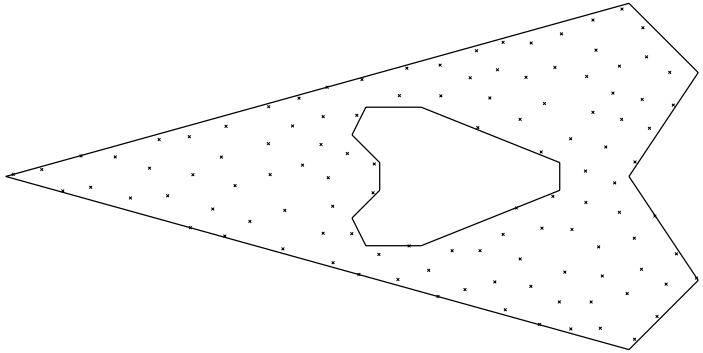
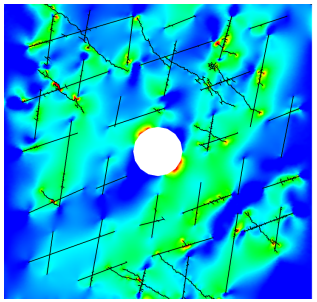
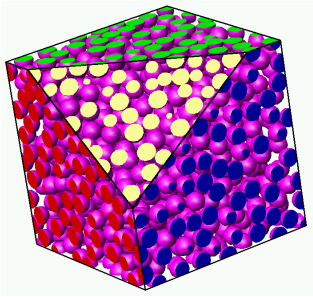
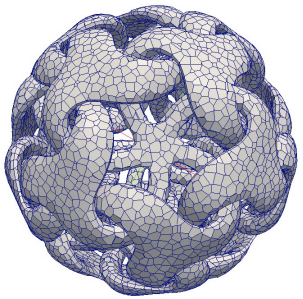
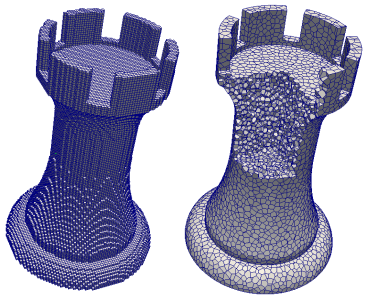










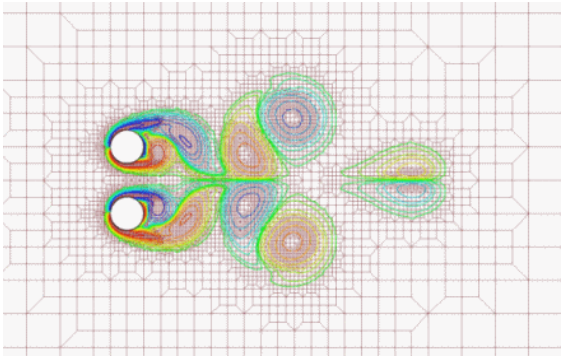
Overview

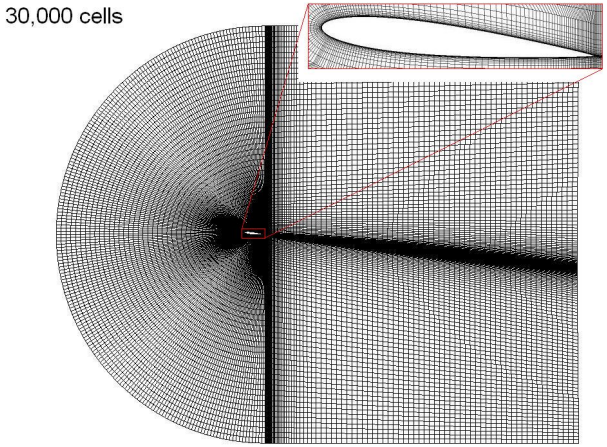
General Principles in MeshingGenie

... part of Trilinos open source library (http://trilinos.sandia.gov/)

Quality

- Point Clouds should adequately represent the associated geometry and physics.
- Extra points needed to improve quality / maintain certain connectivity should be kept at a minimum level.
- Moving points to non-deterministic locations should also be avoided for generating a provably good mesh.





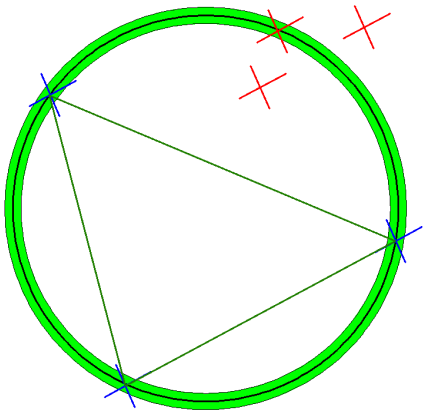
NACA0012 Image courtesy of http://www.cfd-online.com

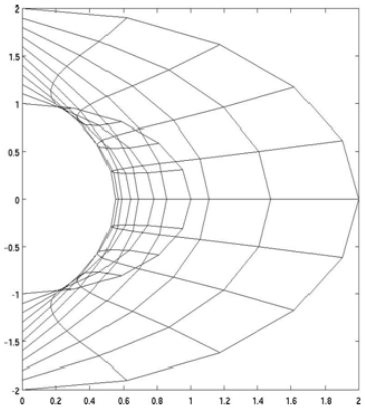
General Principles in MeshingGenie

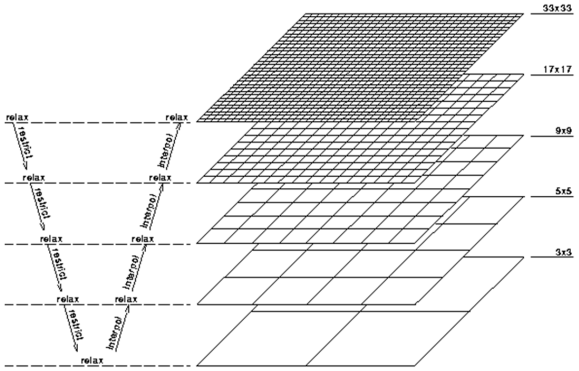
... part of Trilinos open source library (http://trilinos.sandia.gov/)

Efficiency, robustness and ... Multi-Grid!

- Local meshing operations is easier for parallel applications
- Provable robustness (meshing failure is NOT ACCEPTABLE in dynamic simulations).
- Meshing Operations should have minimum storage requirement.
- Geometric MG need to be considered.







What is Maximal Poisson-disk sampling?

Creating a point cloud in given domain satisfying three conditions:

- Each point is a centre of a disk, with radius r, that contains no other points:

$$\forall x_i \in x_j \in X, x_i \neq x_j : ||x_i - x_j|| \geq r$$

- The point distribution should be bias-free:

$$\forall x_i \in X, \forall \Omega \subset \mathcal{D} : P\left(x_i \in \Omega\right) = \int_{\Omega} \mathrm{d}\omega$$

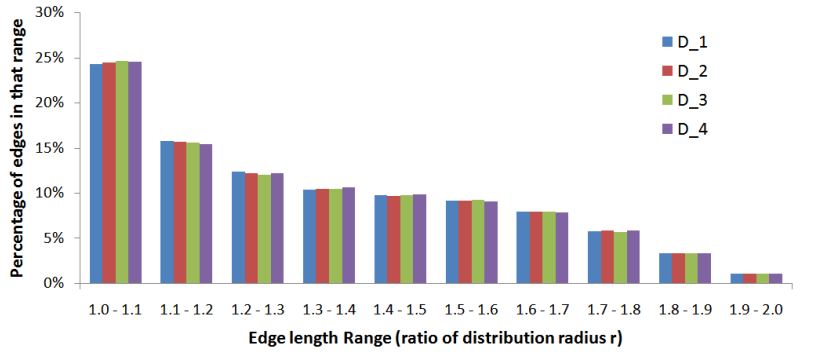
- Termination is achieved when the domain is completely saturated:

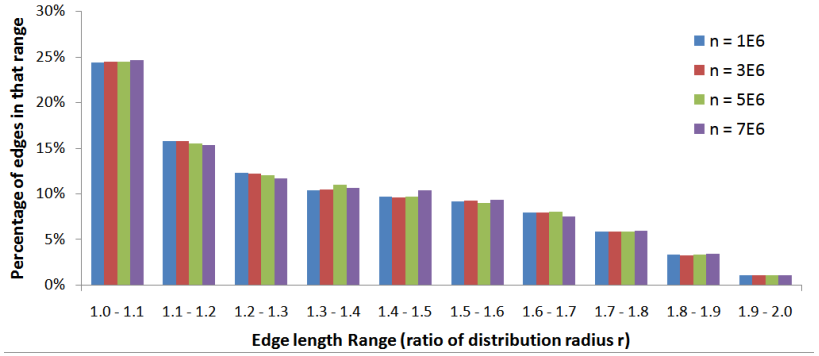
$$\forall x \in \mathcal{D}, \exists x_i \in X : ||x - x_i|| < r$$

How does Maximal Poisson-disk sampling affect meshing algorithms?

Delaunay Edge length

- bounded between r and $2r$
- Connectivity can be retrieved locally
- Linear time complexity
- Easier parallel implementation
- Nice distribution almost independent of the domain / no. of points

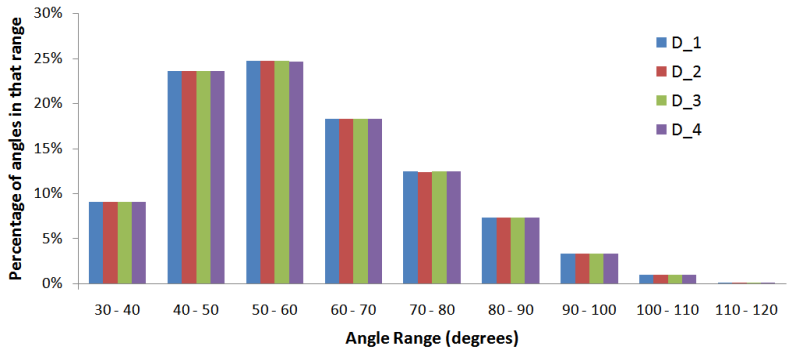


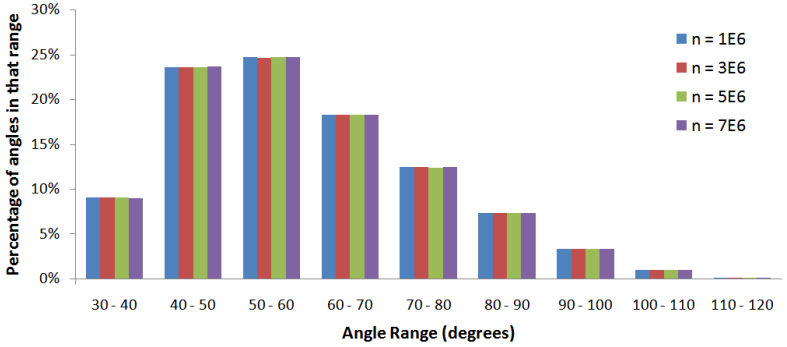


How does Maximal Poisson-disk sampling affect meshing algorithms?

Moreover

- Angles between 30° and 120°
- Nice distribution almost independent of the domain / no. of points
- Easier handling of constrained input.
- Communication is only required in case of non-unique solutions.

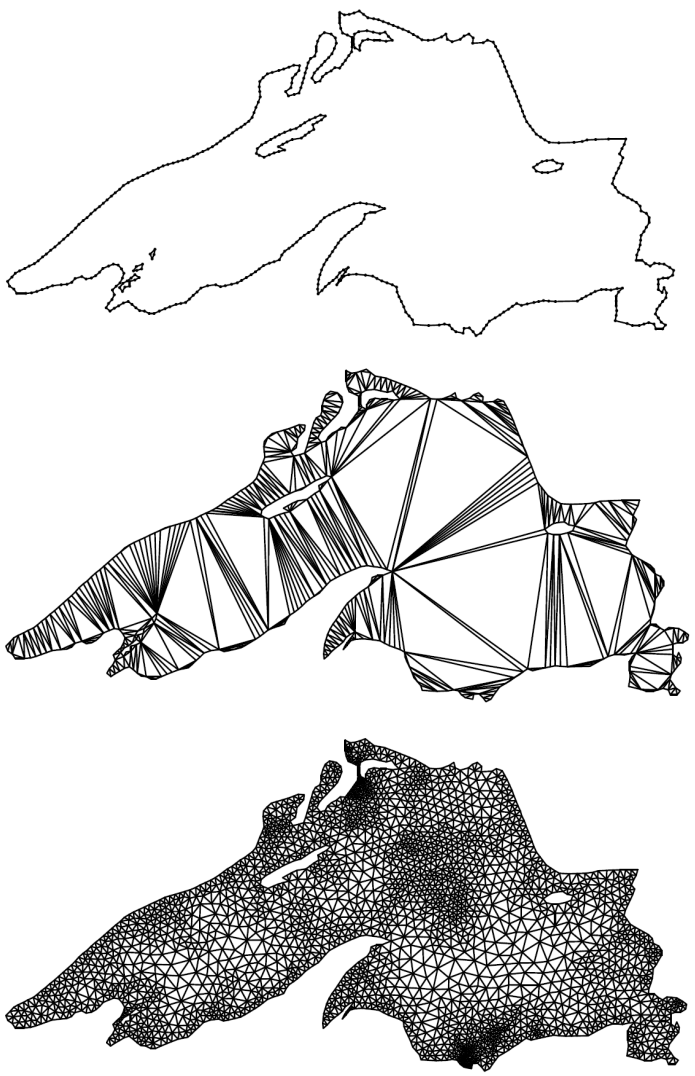




Improving the quality via deterministic point insertion:

Chew's method for example

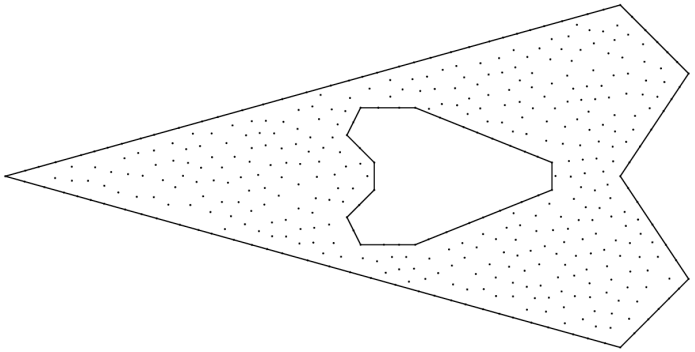
- Ignores the information associated with the input point cloud.
- Physics is initially ignored as well.
- Generates an initial mesh with low quality.
- Improves the quality by inserting points at the centers of large Delaunay circumcircles

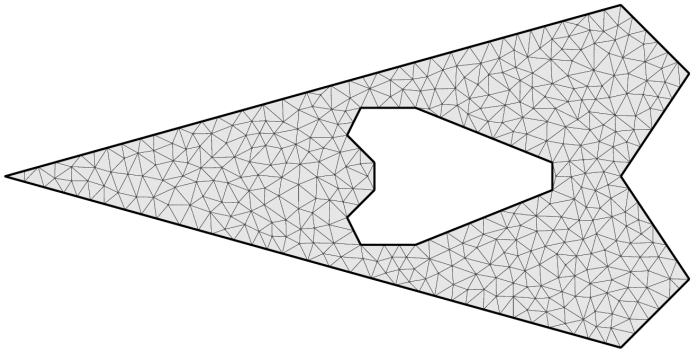


Relationship between the two approaches

Our Approach

- Points are generated to satisfy a given function $\mathcal{F}$.
- $\mathcal{F}$ is chosen based on (geometry and physics).
- The extra information enables "nicer" implementation
- Once the point cloud is generated no more point insertion or movement (termination guarantee).
- The quality measures would be decided according to $\mathcal{F}$.

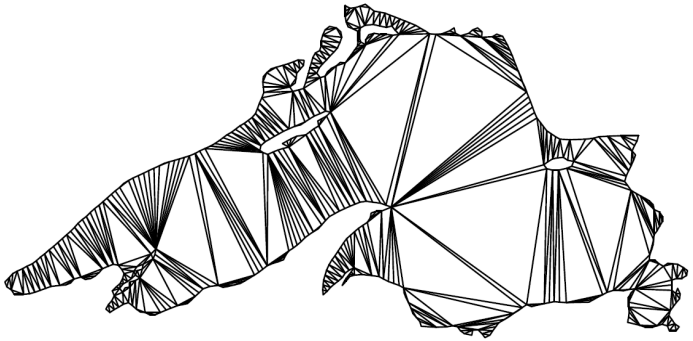


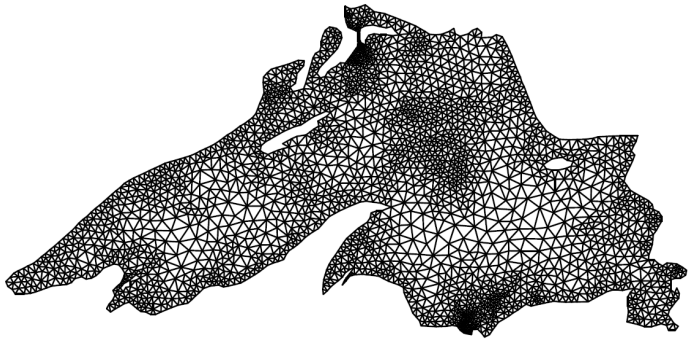


Relationship between the two approaches

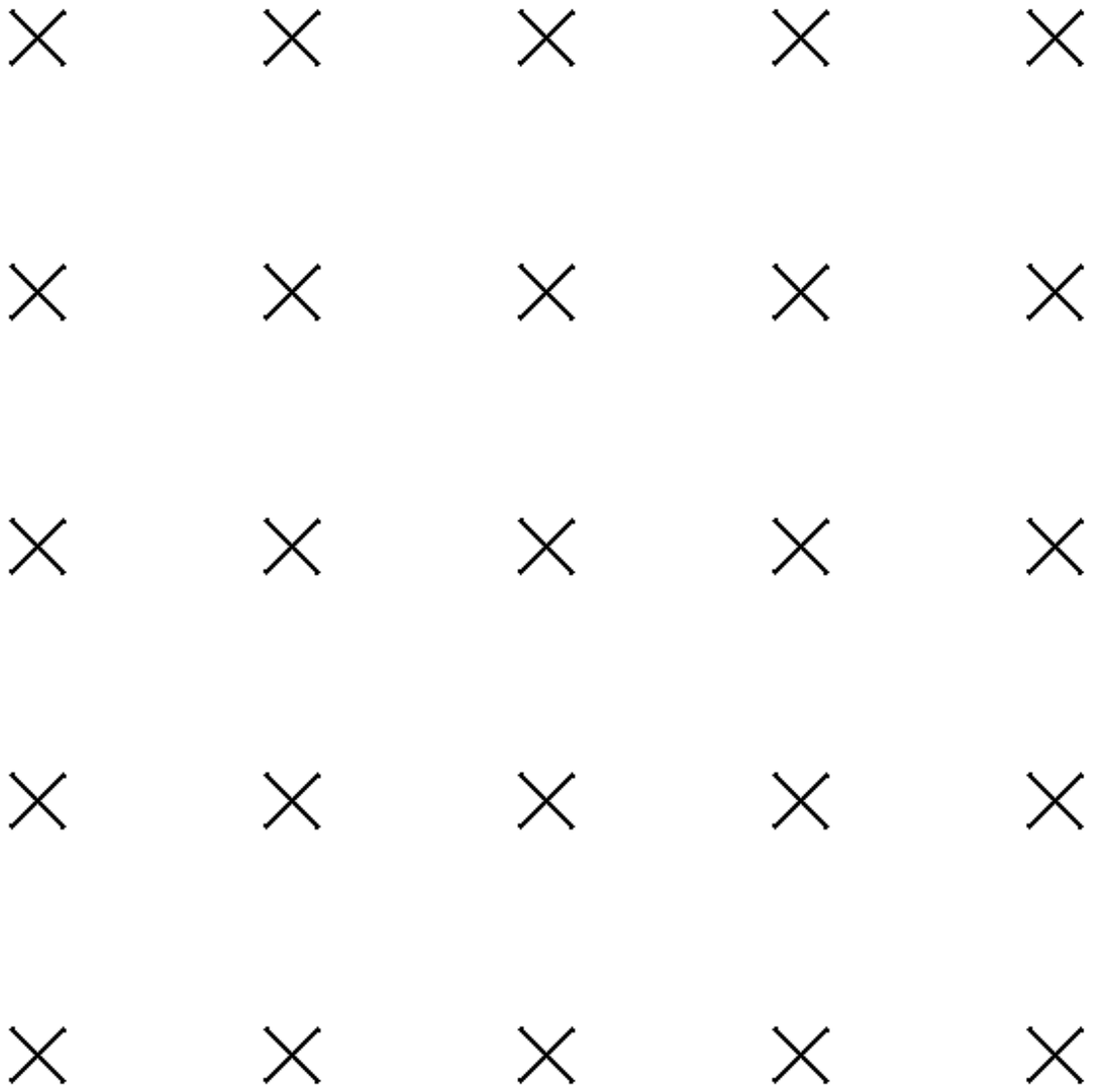
Point insertion methods

- The information associated with the point cloud is missing/discarded.
- A sequence of Delaunay meshes is generated trying to find a maximal distribution for an unknown implicit function, $\mathcal{F}$ associated with an input desired quality measure.
- Solution may not exist, and if it exists, it may not be desirable!
- Low quality meshes can cause robustness issues esp. in 3D.





A simple demonstration using an extremely nice point cloud



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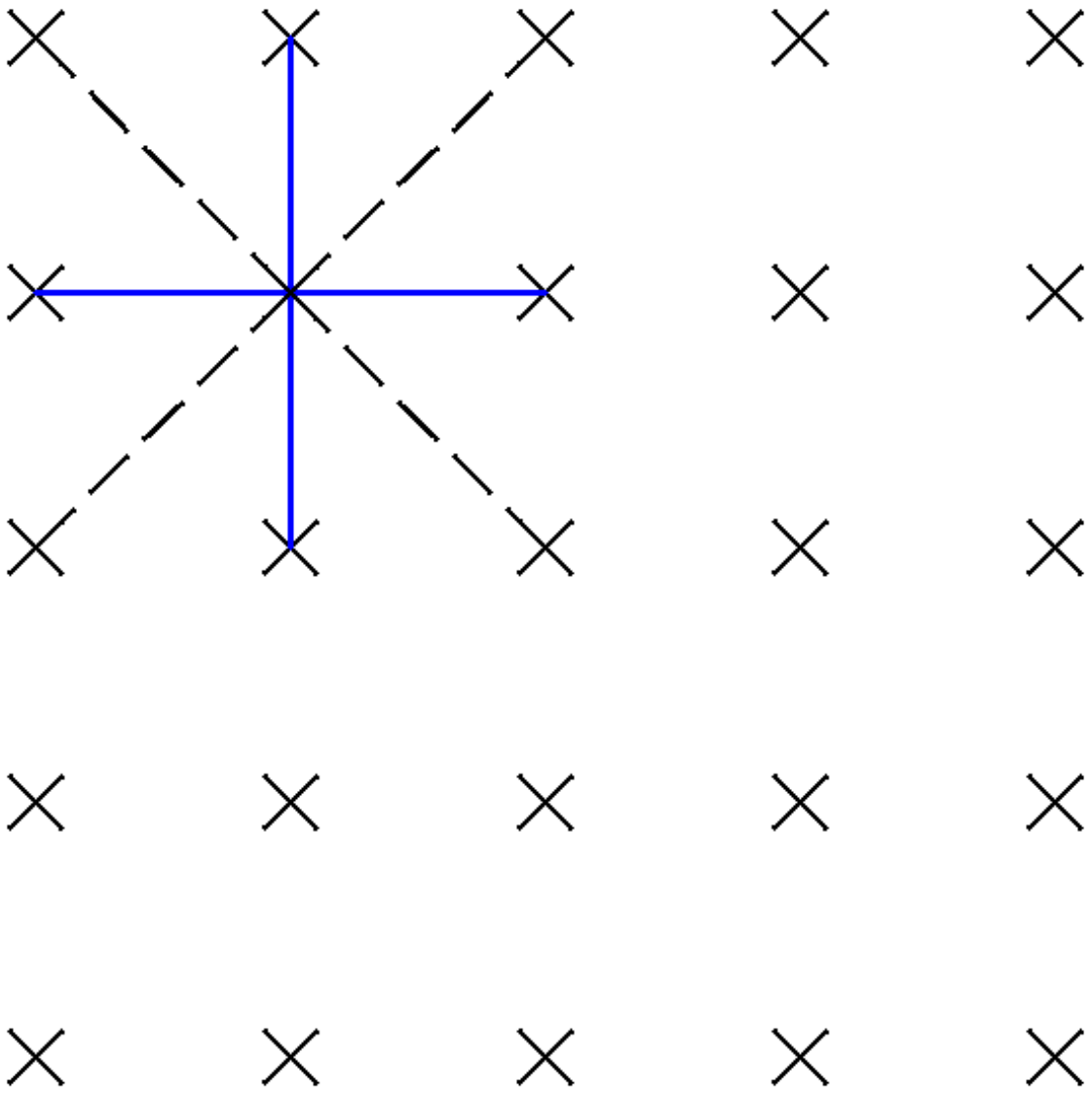

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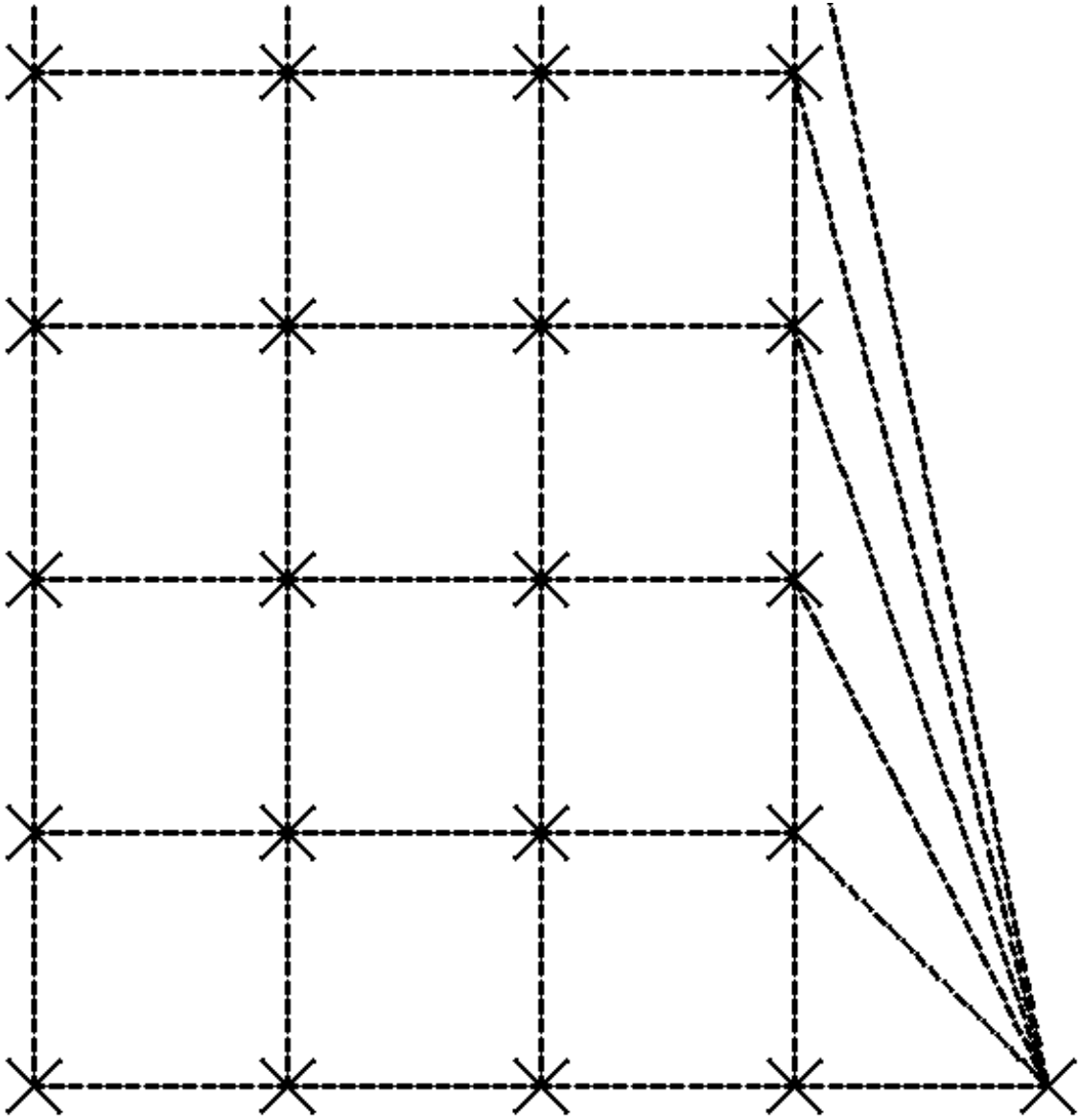
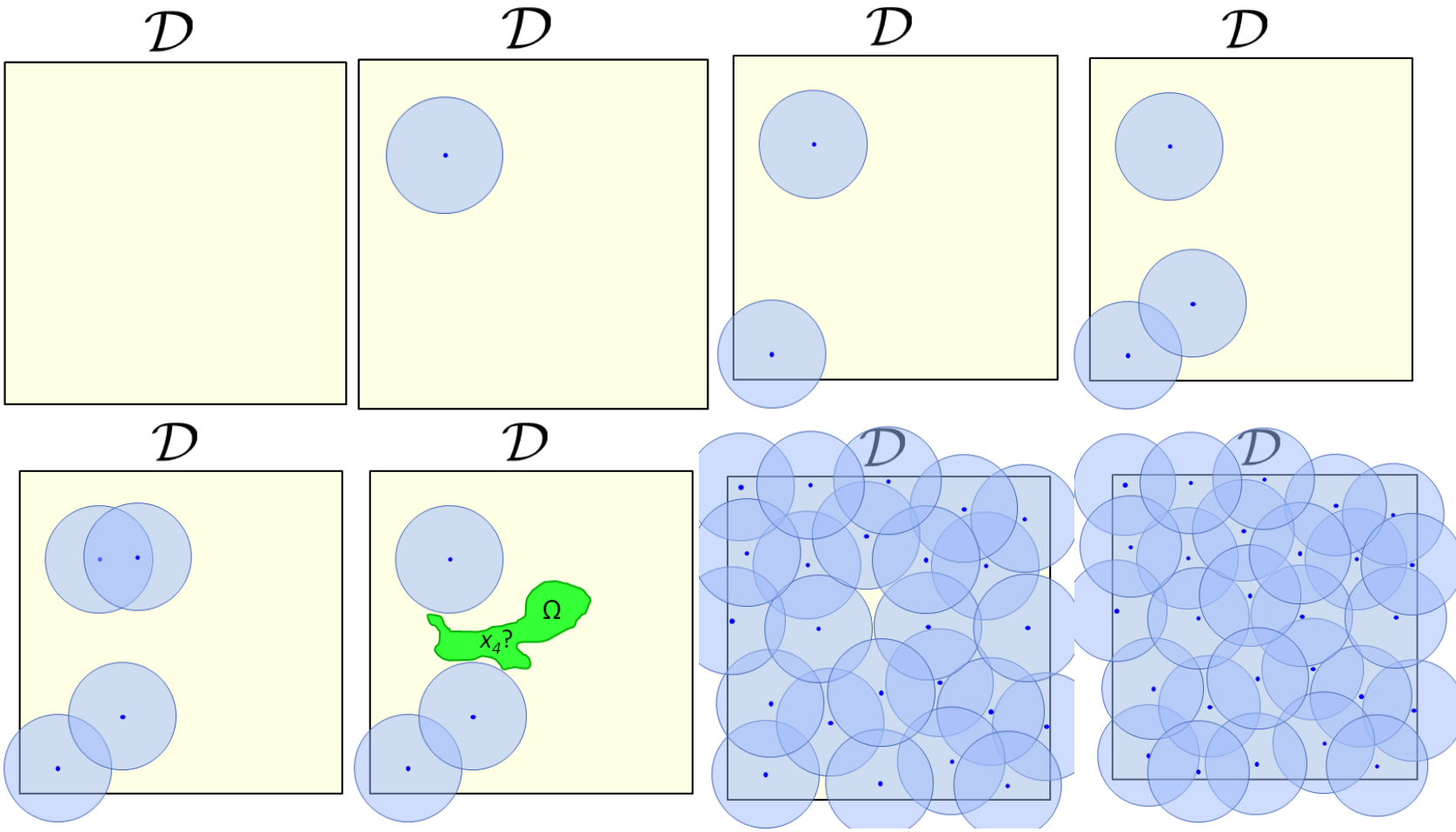


A simple demonstration using an extremely nice point cloud

Convinced? If so, let's see how we solved the uniform case :)

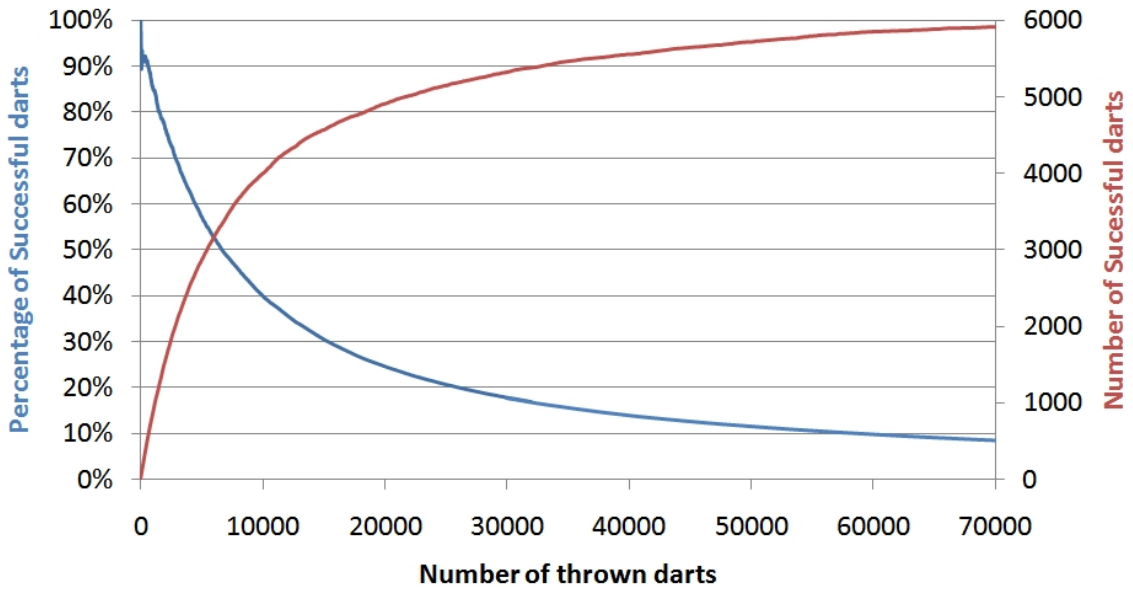
Classical dart throwing



The maximal Poisson-disk sampling

Challenges of classical dart throwing:

- An efficient method to retrieve conflicts.
- Filling the small gaps between the disks.
- Detection of the termination condition.



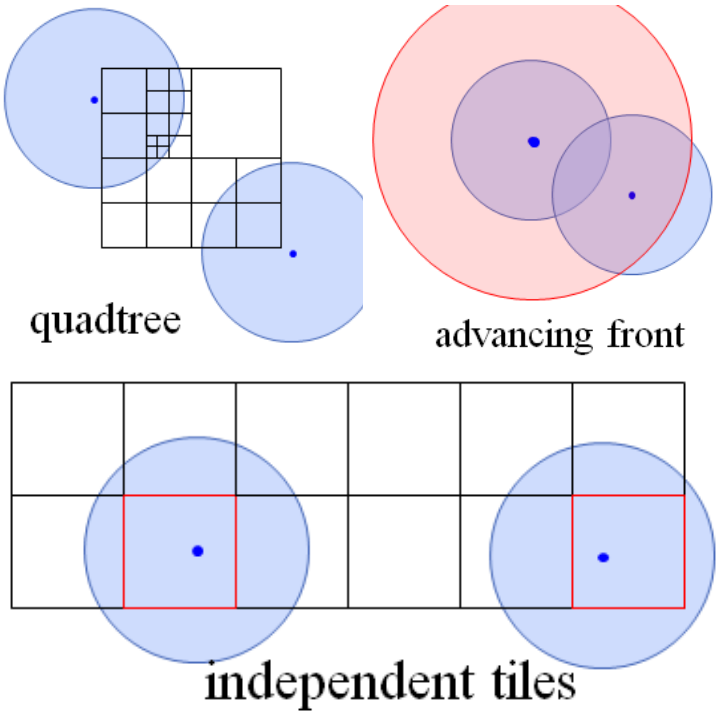
The maximal Poisson-disk sampling: Existing solutions

Approaches

- Quadtrees methods.
- Advancing front methods.
- Tiles to improve parallelism.

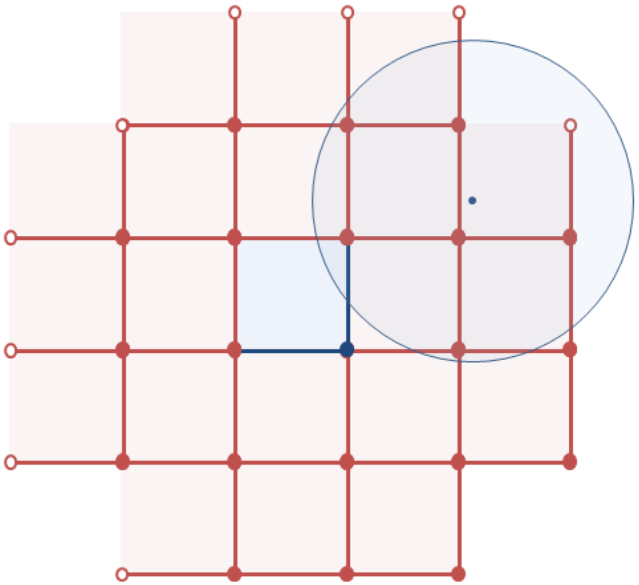
Common issues:

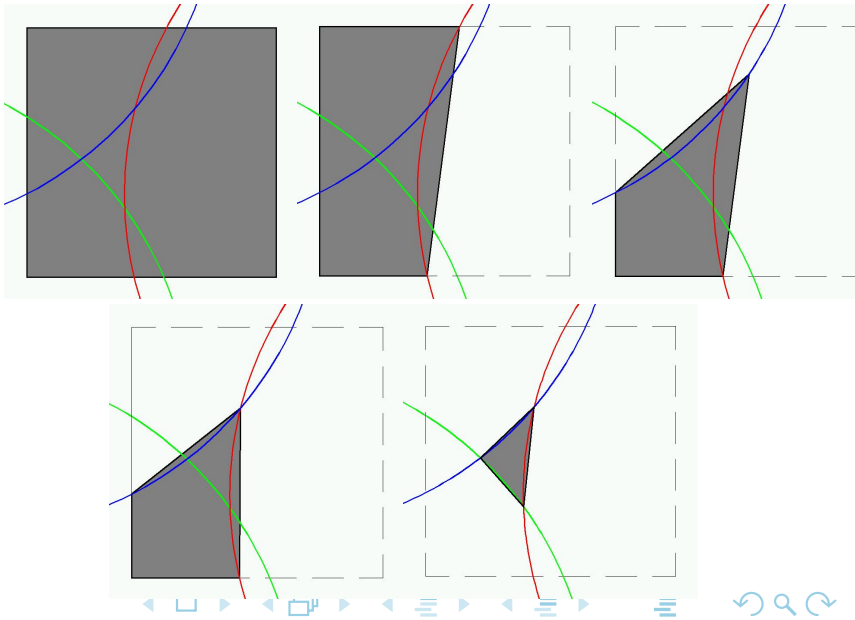
- Not strictly "unbiased".
- Not maximal: dependent on finite precision.
- Memory or run-time complexity.



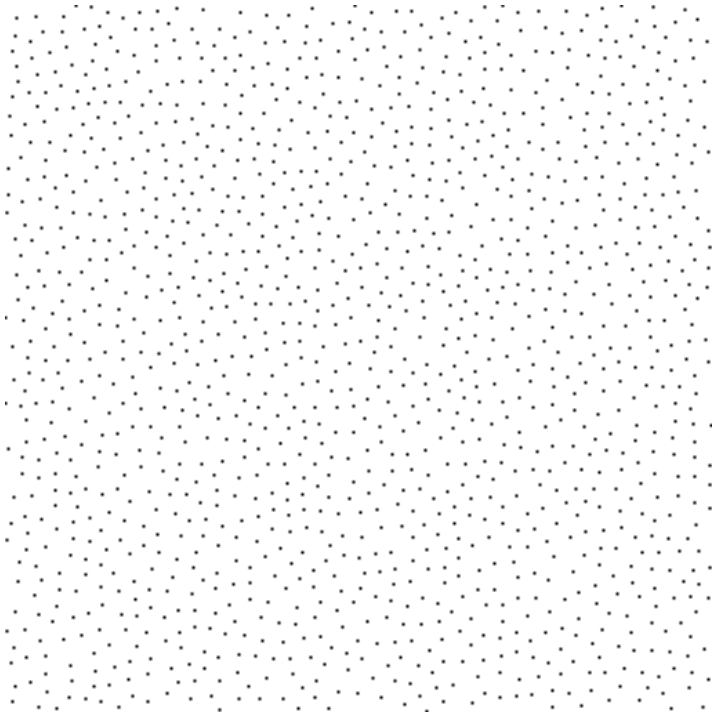
Our first solution: SIGGRAPH2011

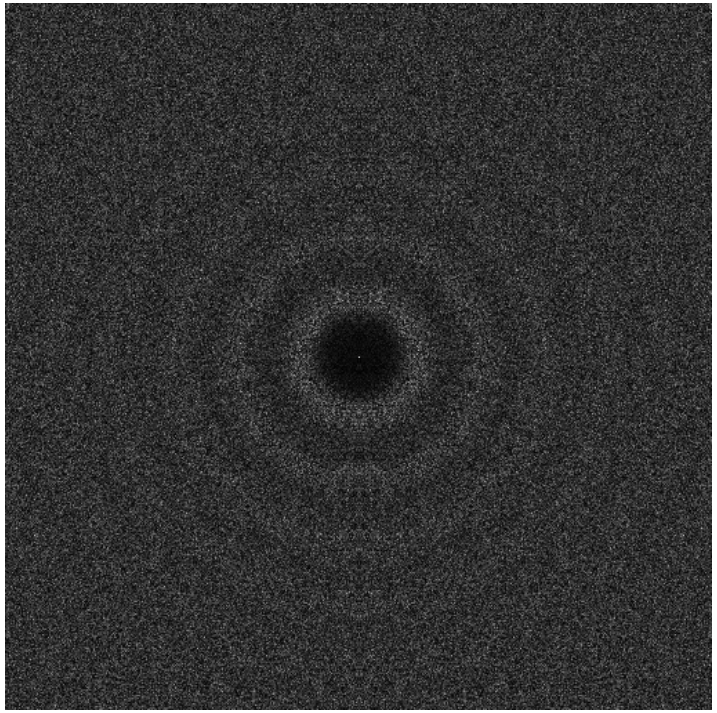
- The first provably bias-free, maximal, $E(n \log n)$ time $O(n)$ space.
- Utilizes a Cartesian background grid
- Dynamic linear representation of the voids in the domain.





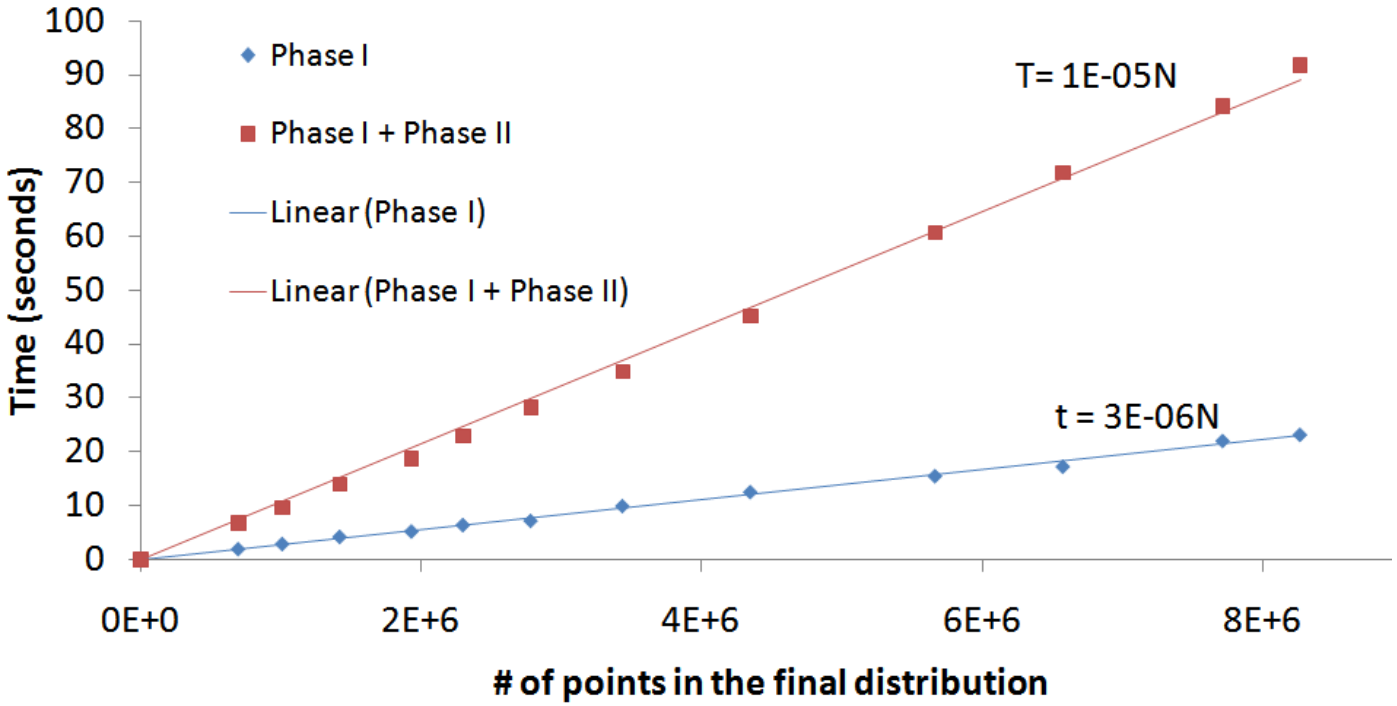
Our first MPS solution: Results





The generated point cloud have a desired blue noise property.

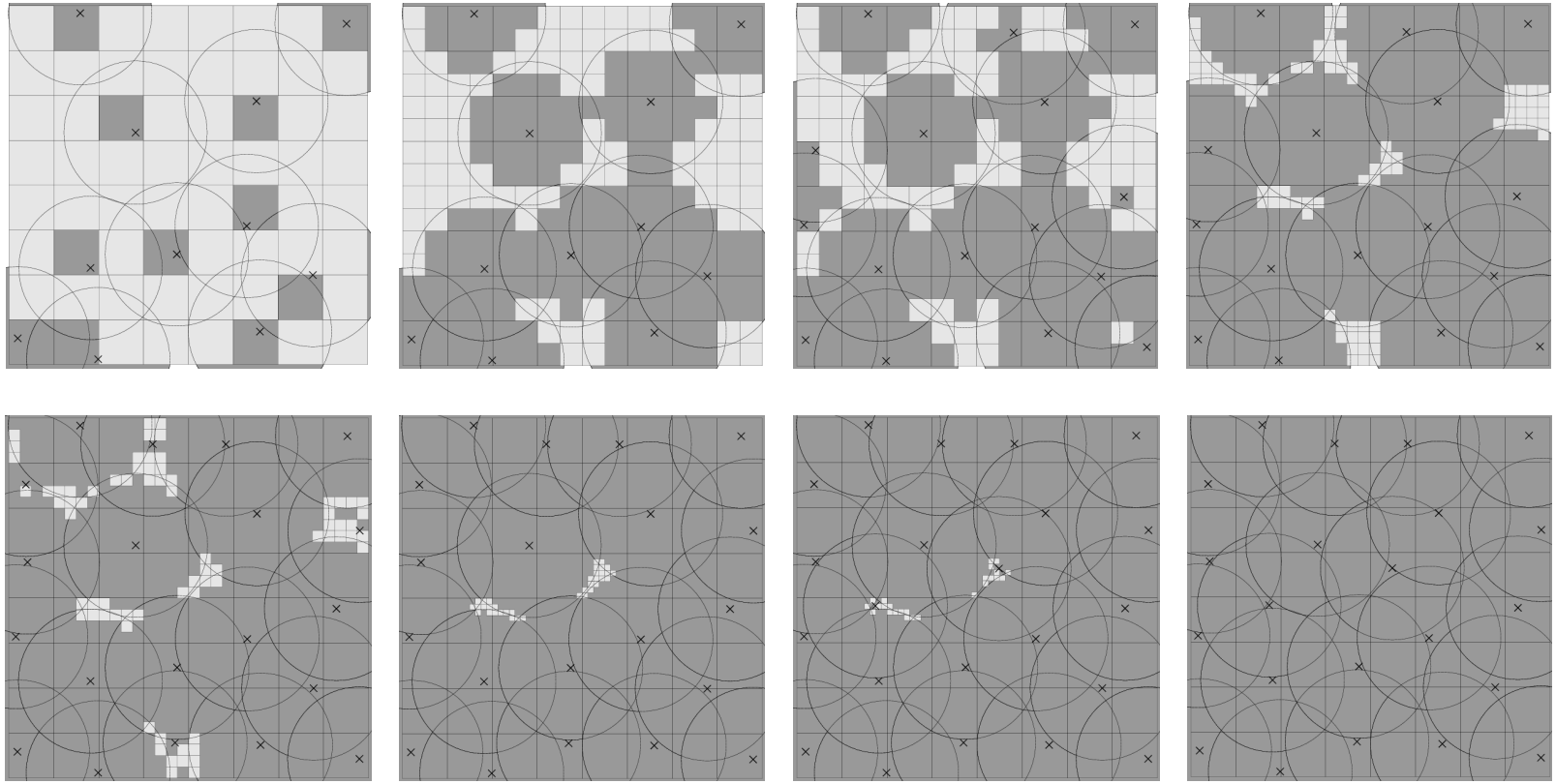
Our first MPS solution: Results



1M 2D points in less than 10 seconds using a modern laptop.

8 M 2D points using 2GB of memory

Our second MPS solution (Eurographics2012 - in preparation)

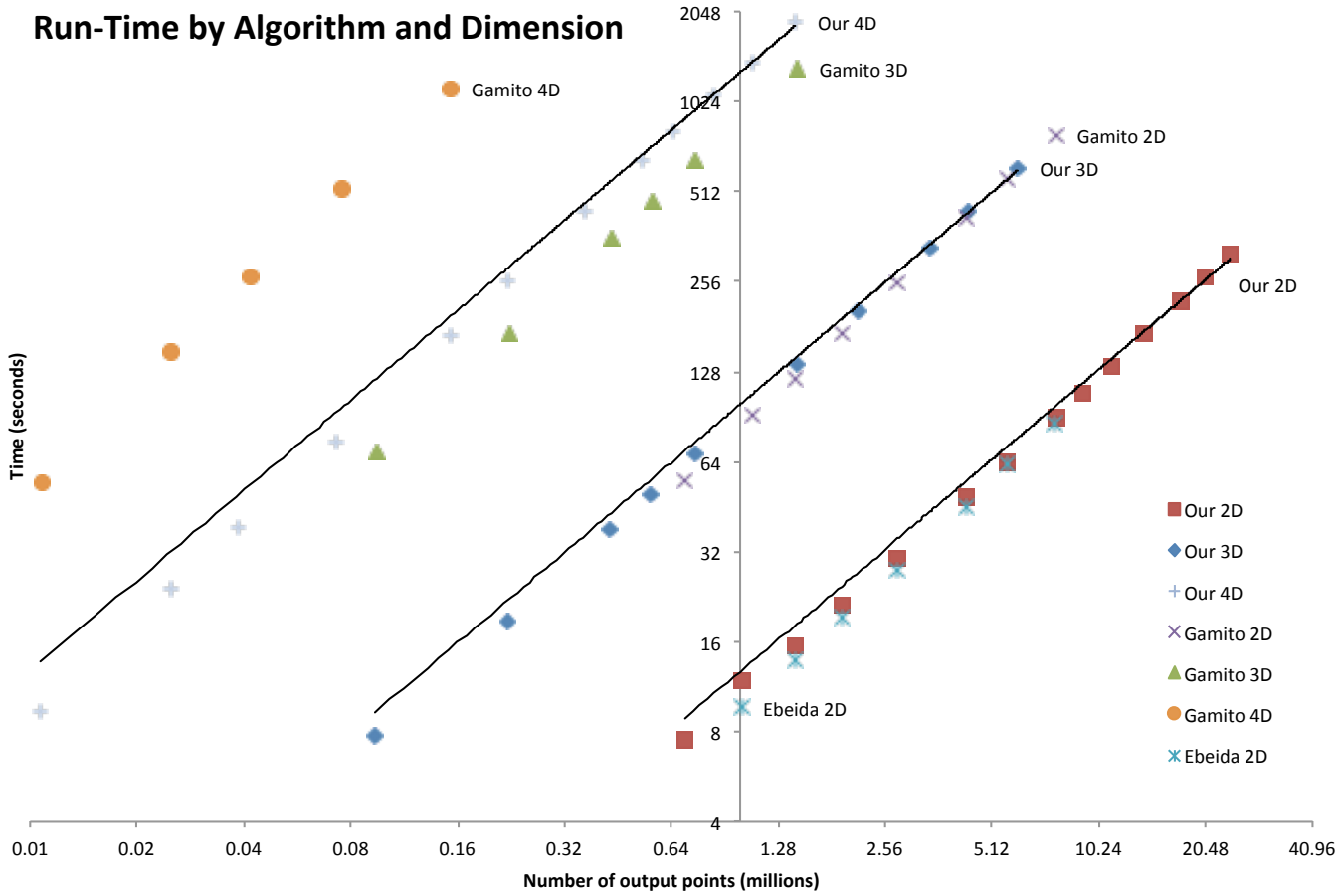


Almost same speed as Solution I (at least in 2D).

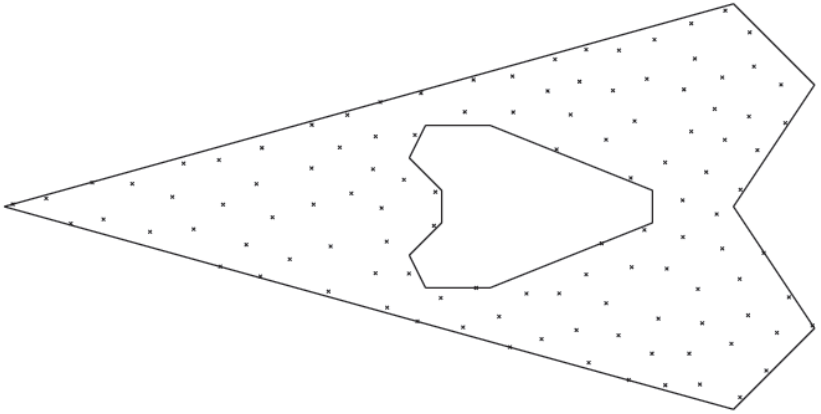
24/8 M 2D/3D points using 2GB of memory

Generic code in any d-dimensional space (if you can afford it!).

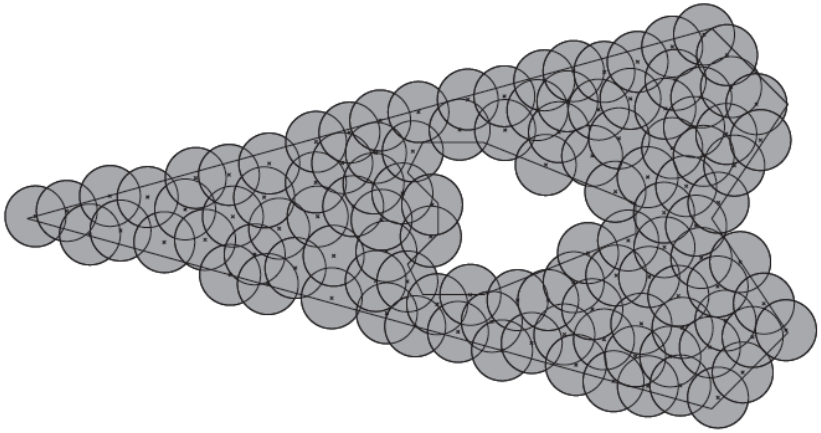
Our second MPS solution: Results



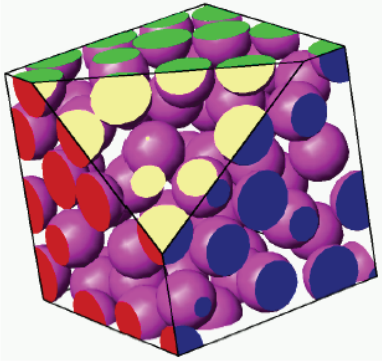
Our second MPS solution: Results



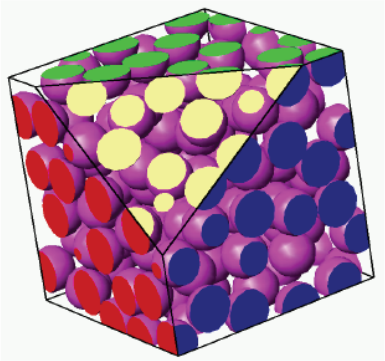
(a) Samples



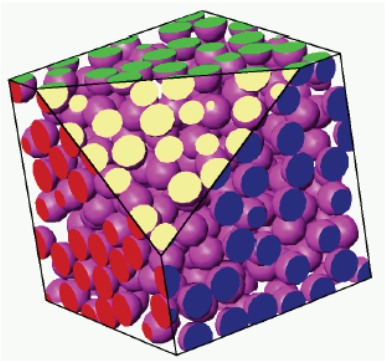
(b) Poisson-Disks cover the entire domain



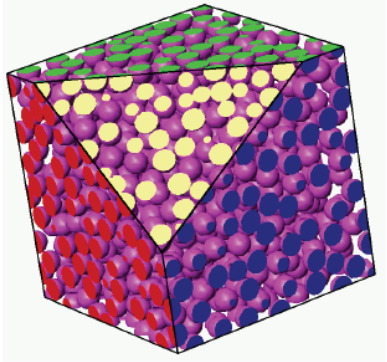
(c) $r = 0.25$



(d) $r = 0.20$



(e) $r = 0.15$



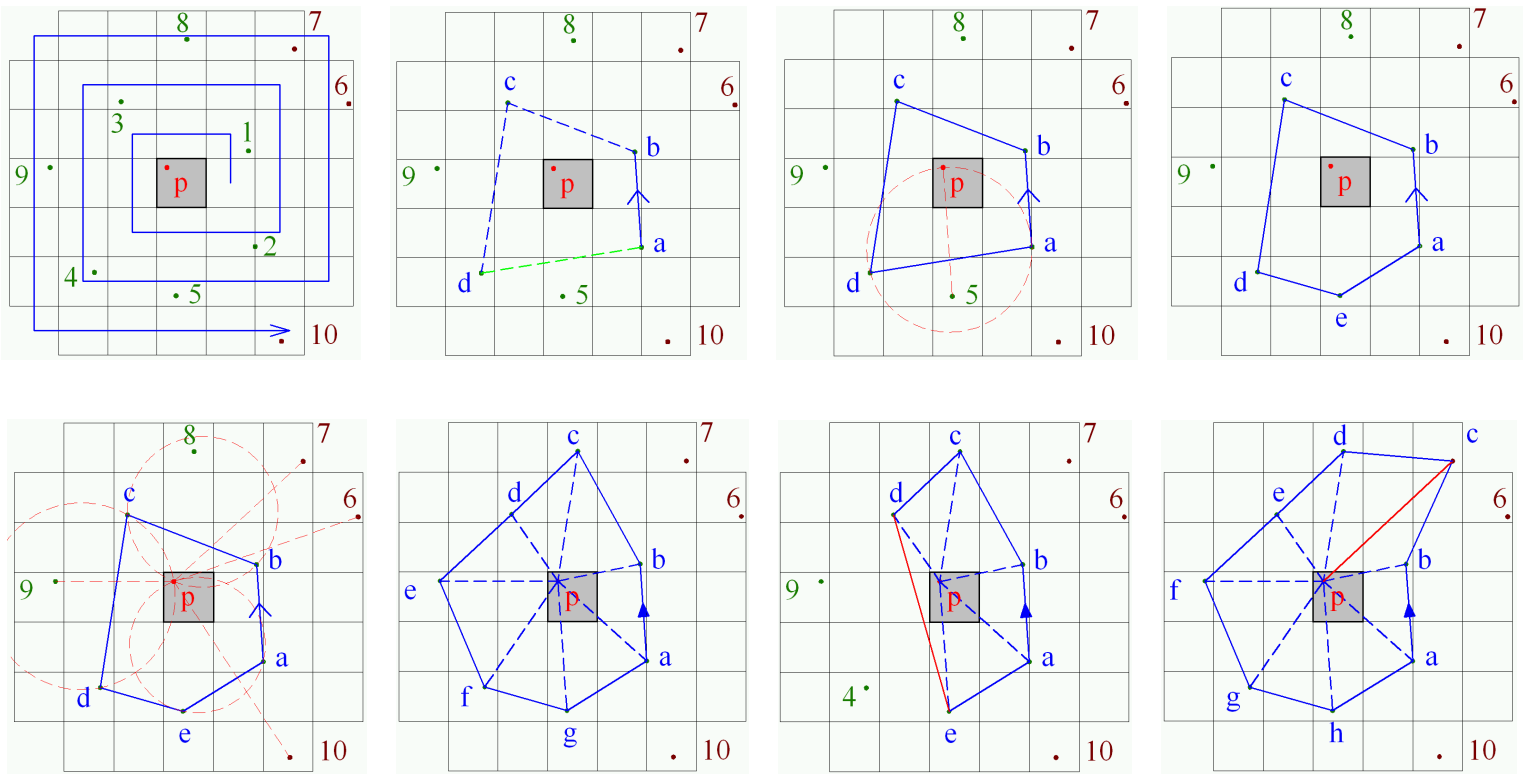
(f) $r = 0.10$

Question?

... So we now have a random point cloud

Can we use the properties associated with its distance function to improve the Delaunay/Voronoi meshing algorithms?

1. An Indirect method using a novel CDT algorithm (SIAM-GD 2011)

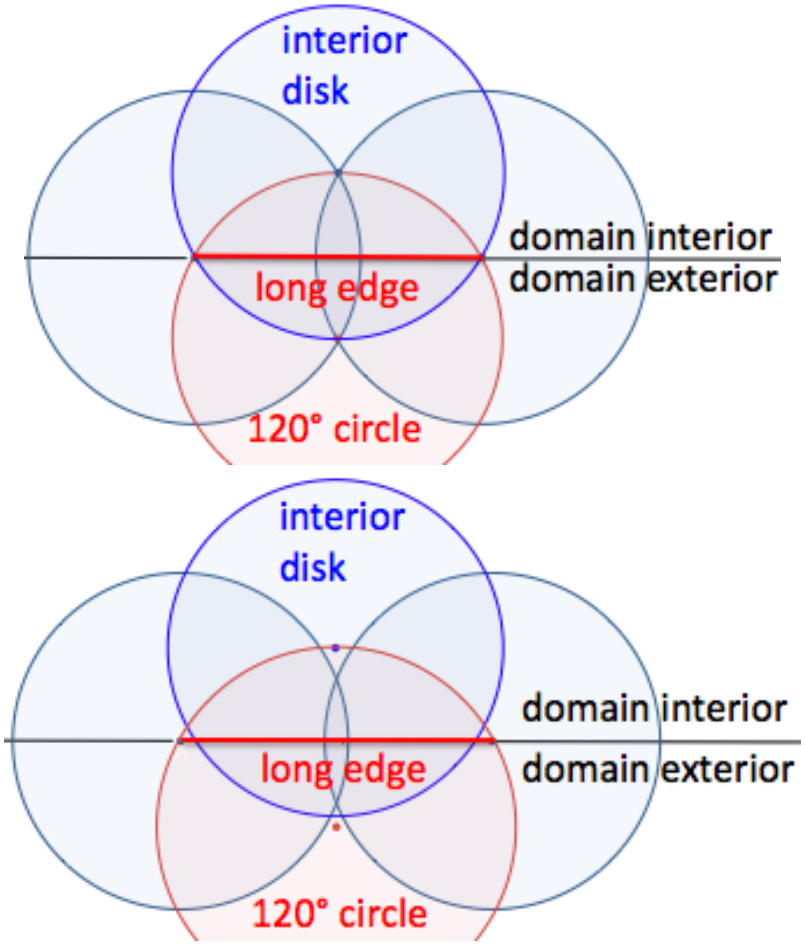


We were able to process 1 Million points in 2.7 seconds using a modern laptop.

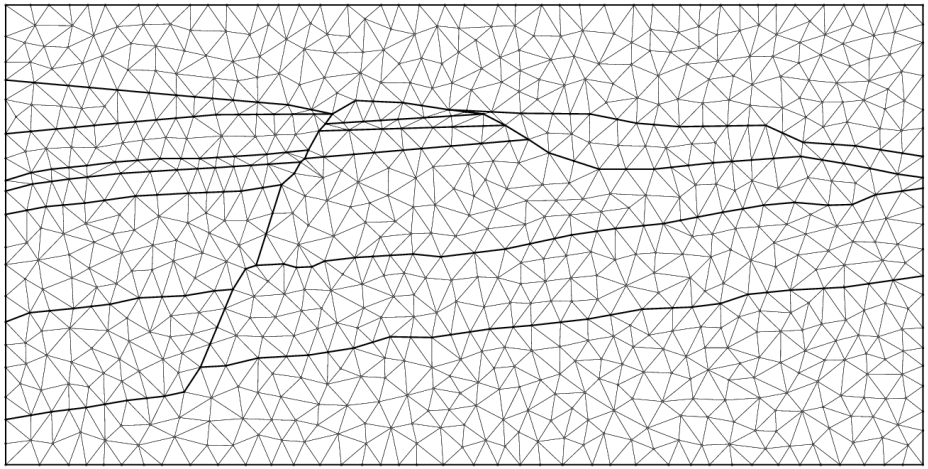
A linear CDT algorithm: Protecting the domain boundaries

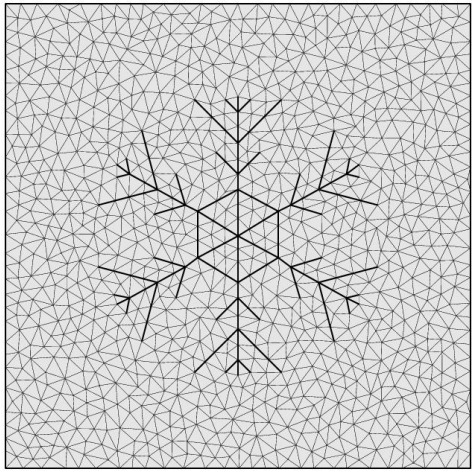
Two Approaches

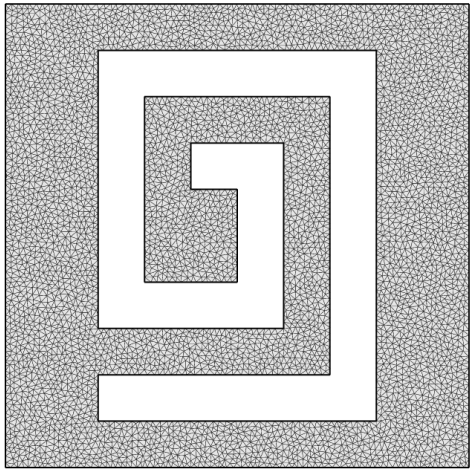
- Protecting with close disks: Sample the boundary edges using a smaller distribution radius.
- Protecting with interior disks: Sample the boundary edges with the same distribution radius and then introduce some interior points to avoid large boundary angles

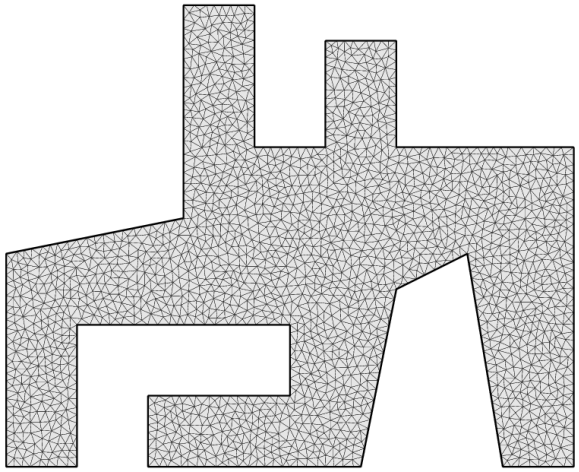


A linear CDT algorithm: Examples

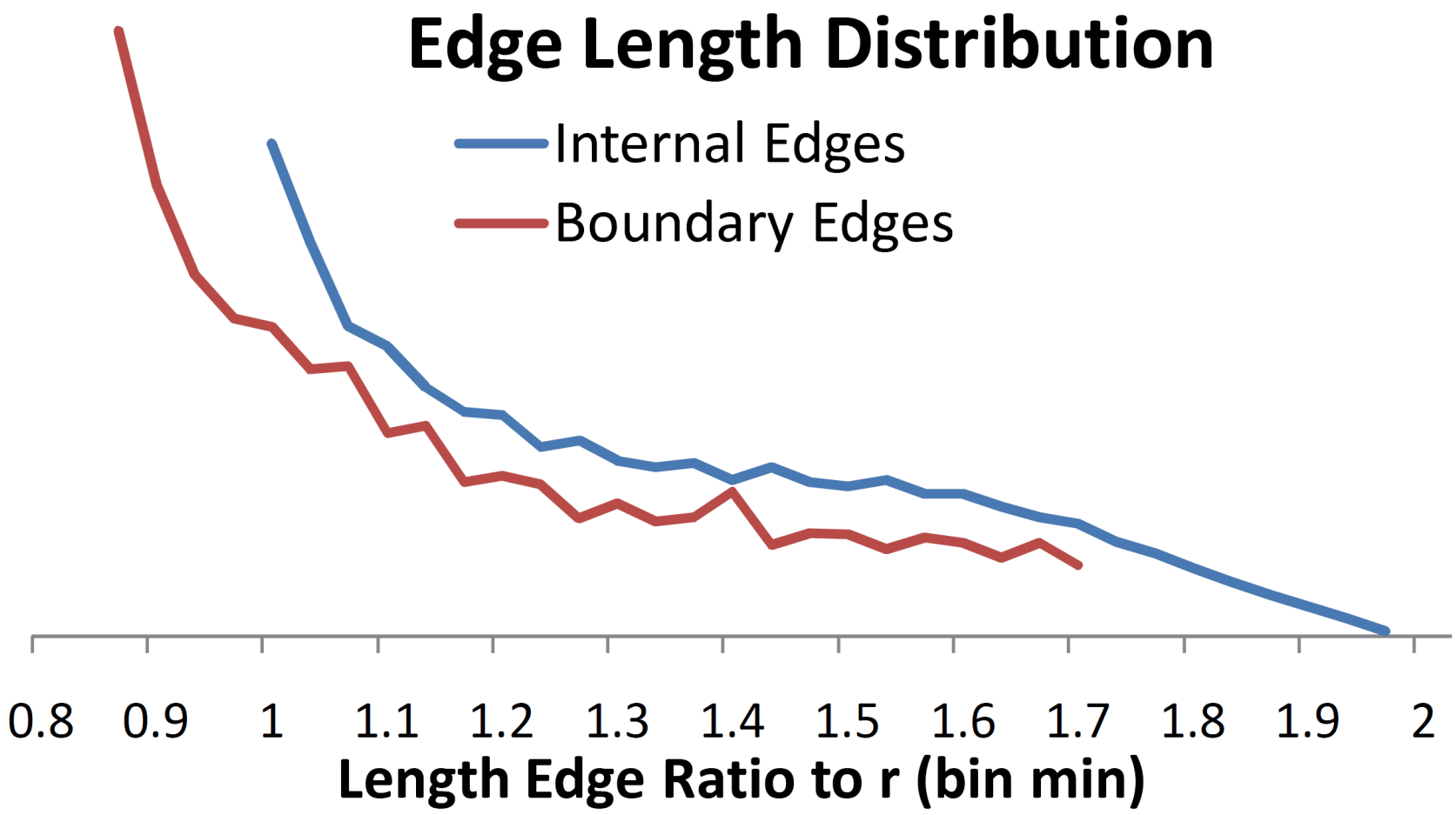




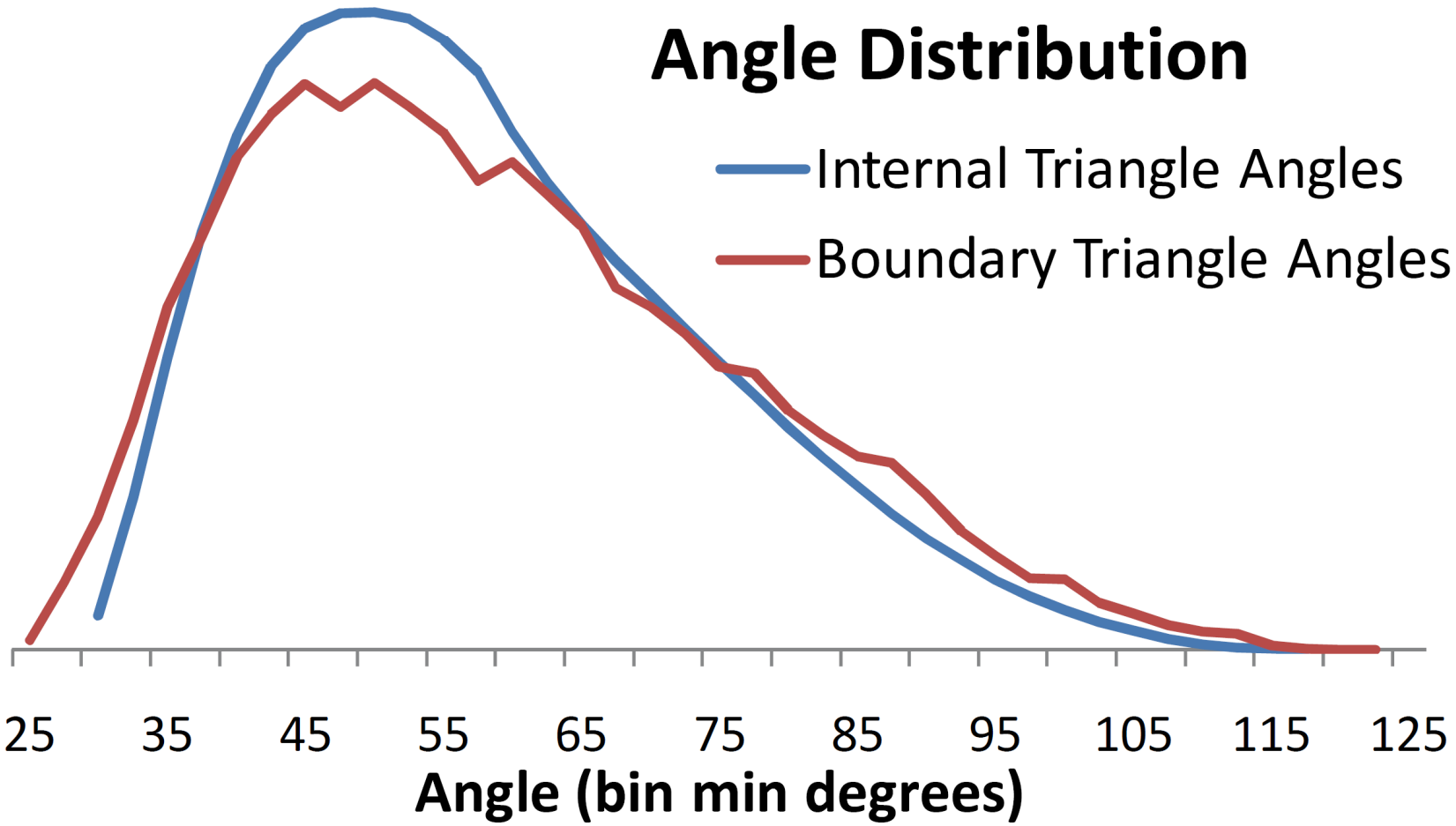




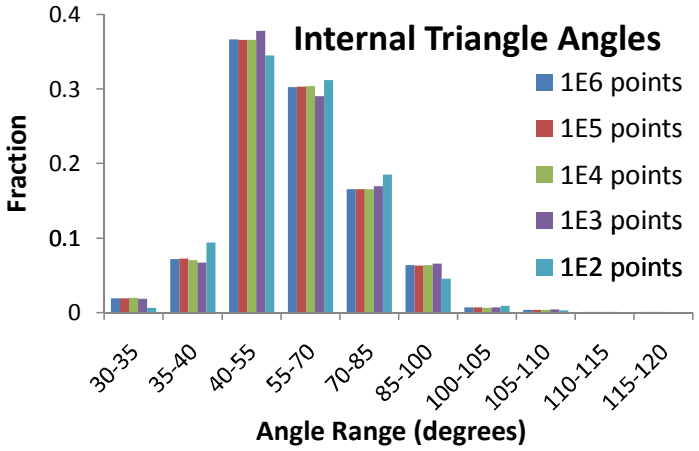
A linear CDT algorithm: Quality

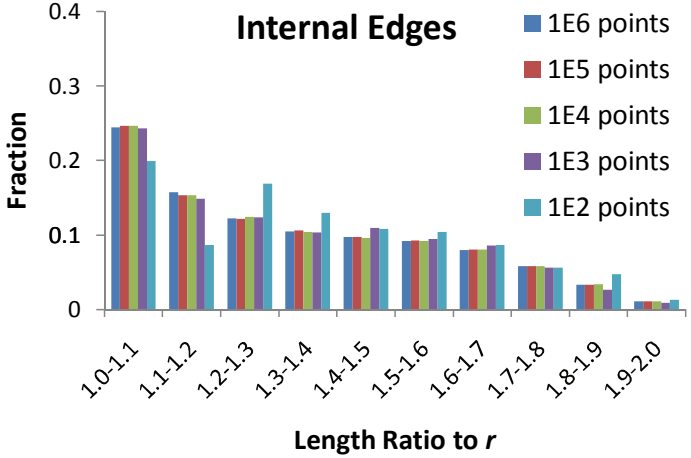


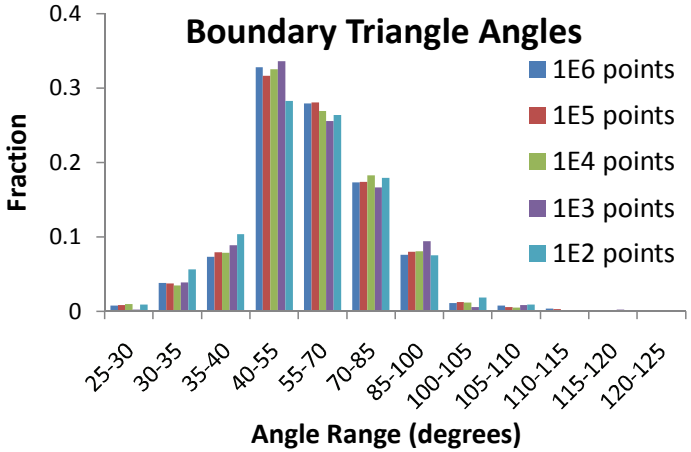
A linear CDT algorithm: Quality

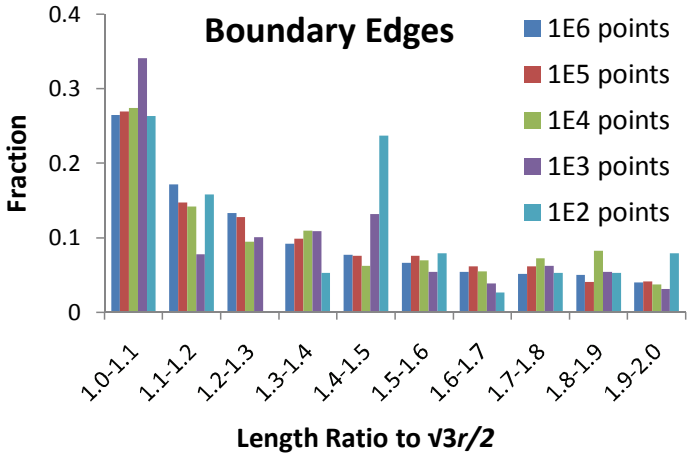


A linear CDT algorithm: Quality

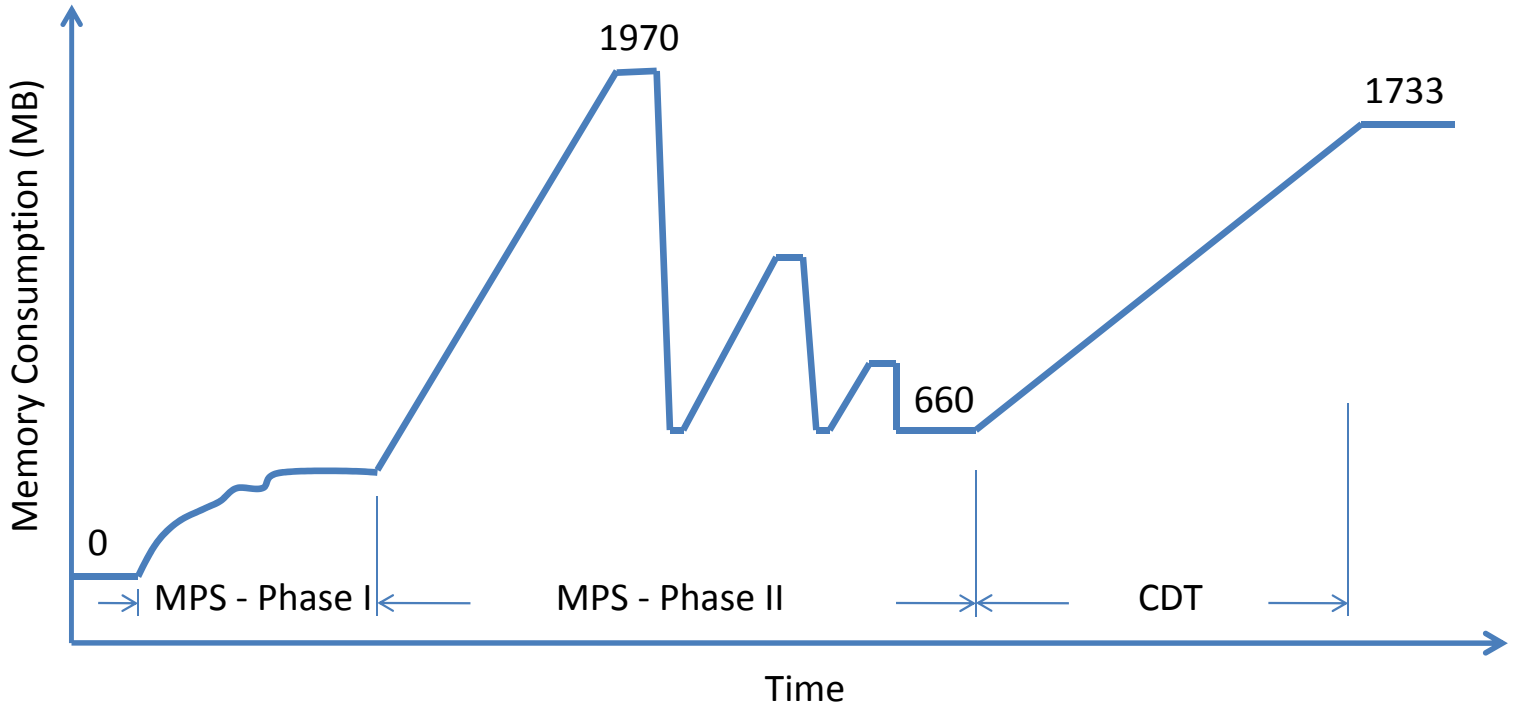




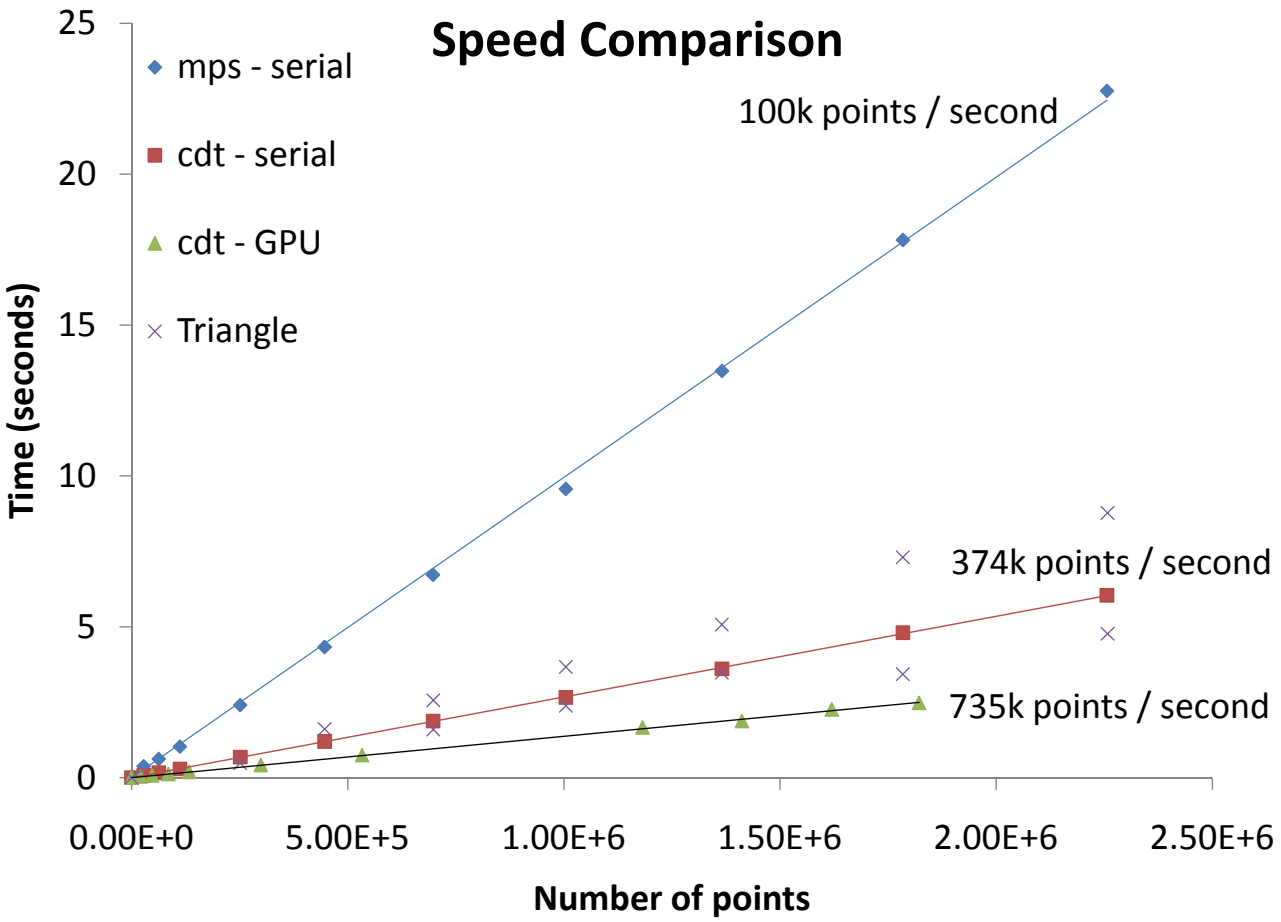




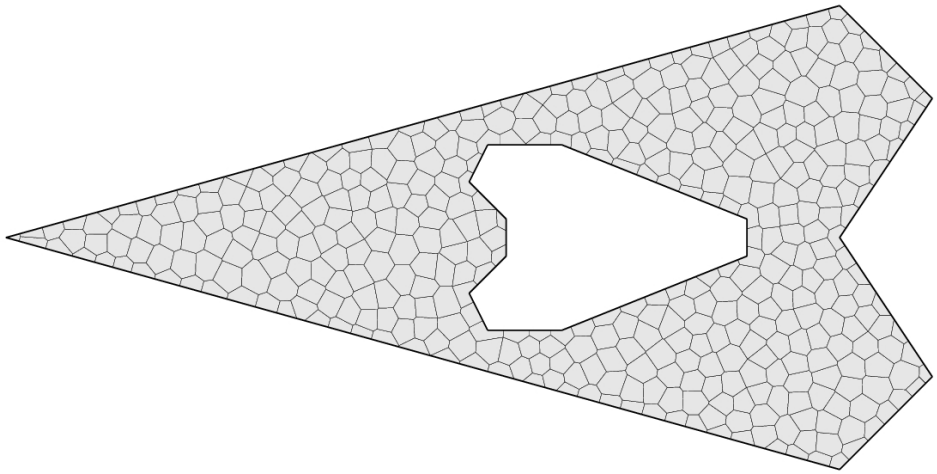
A linear CDT algorithm: Memory Consumption

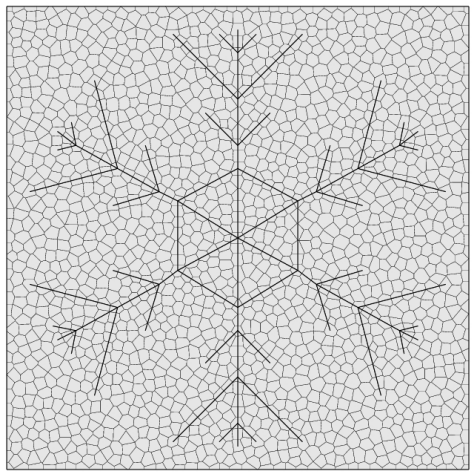


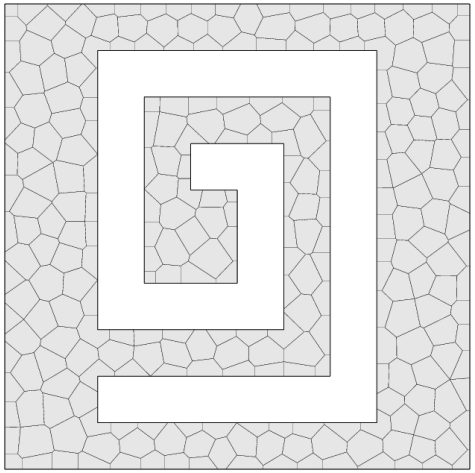
A linear CDT algorithm: Speed Comparison

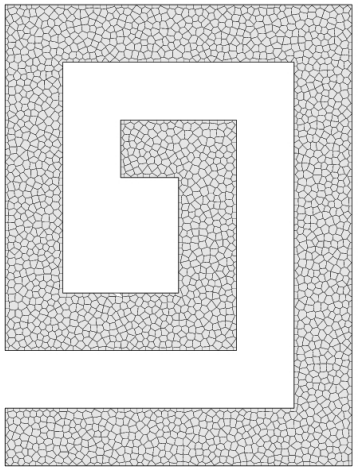


An indirect CVM algorithm: Examples

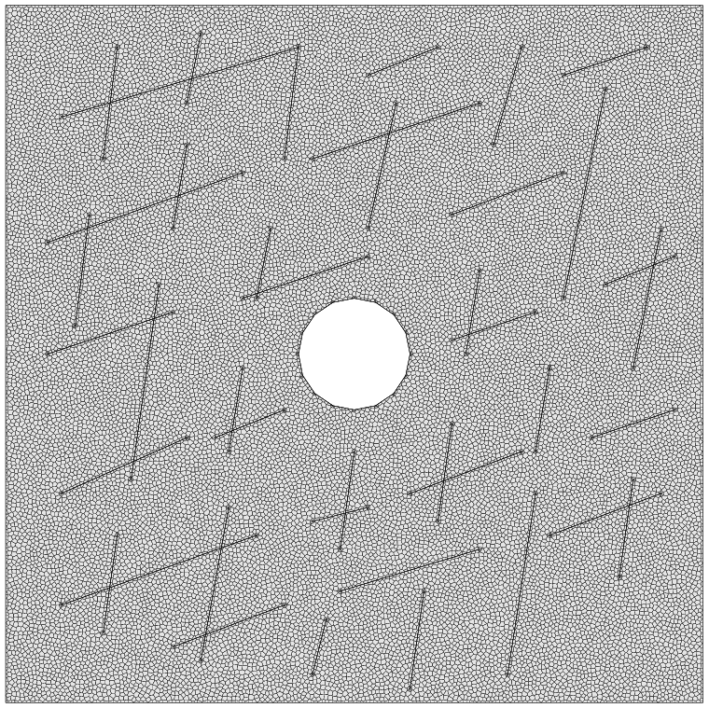


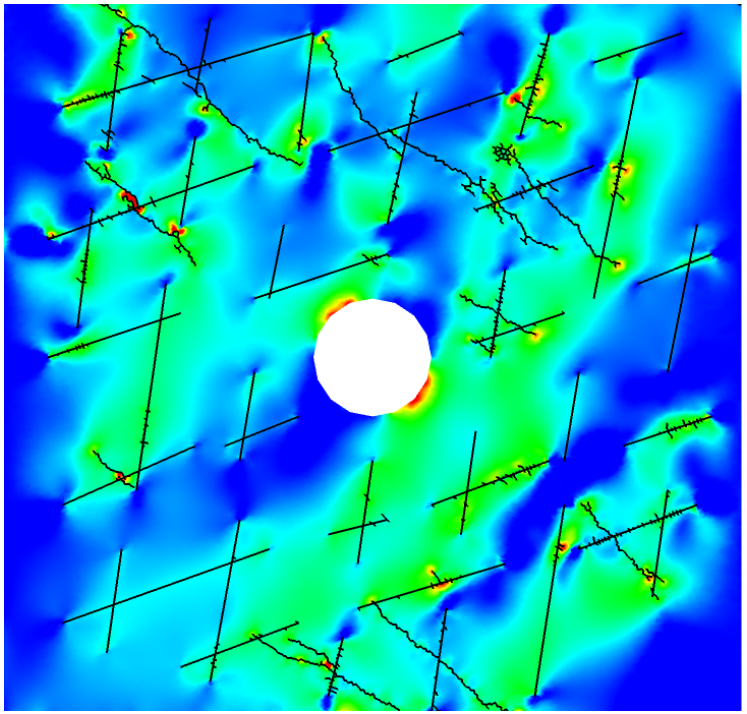




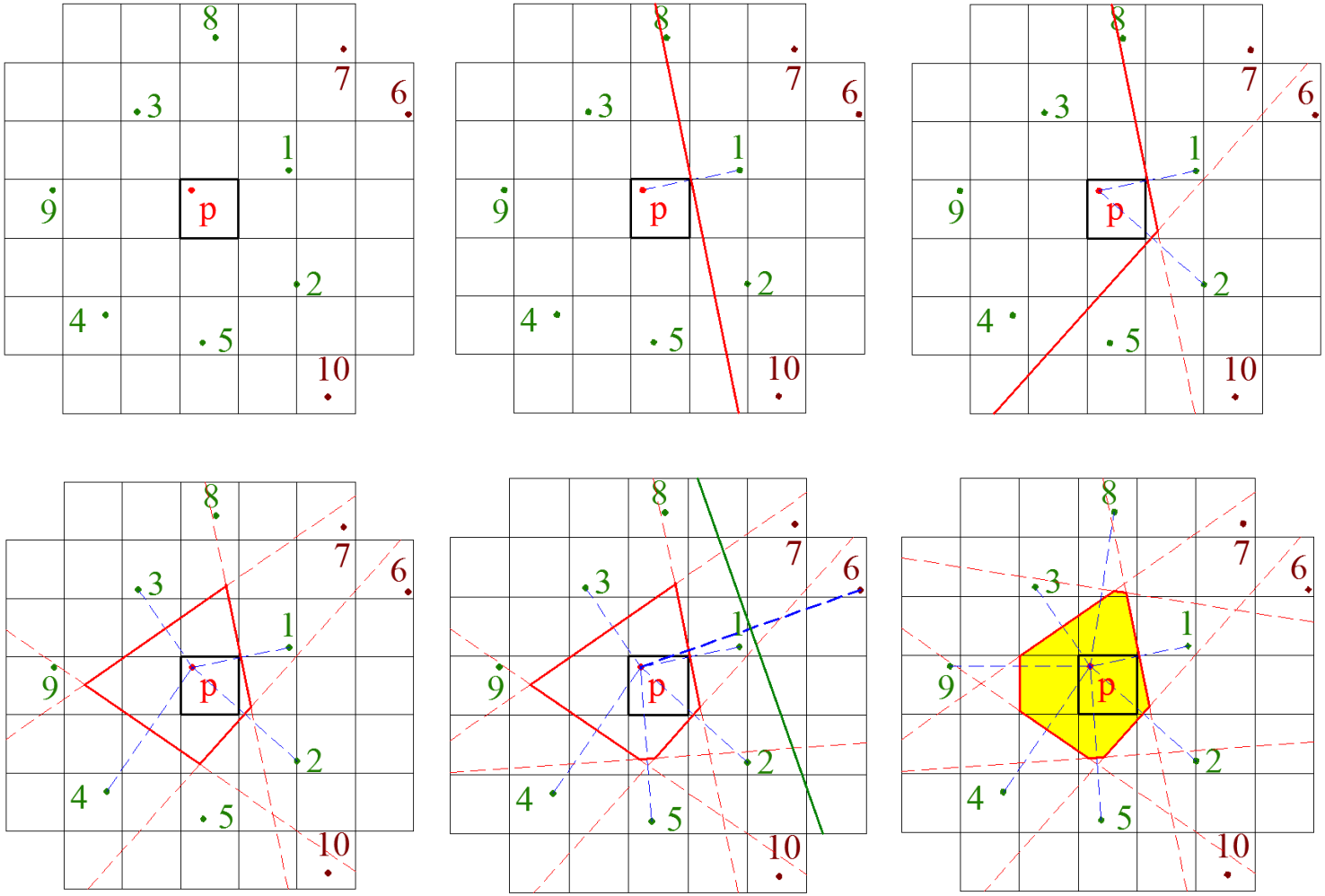


Constraint Voronoi Meshing: Example II
Joseph E. Bishop and Mario J. Martinez

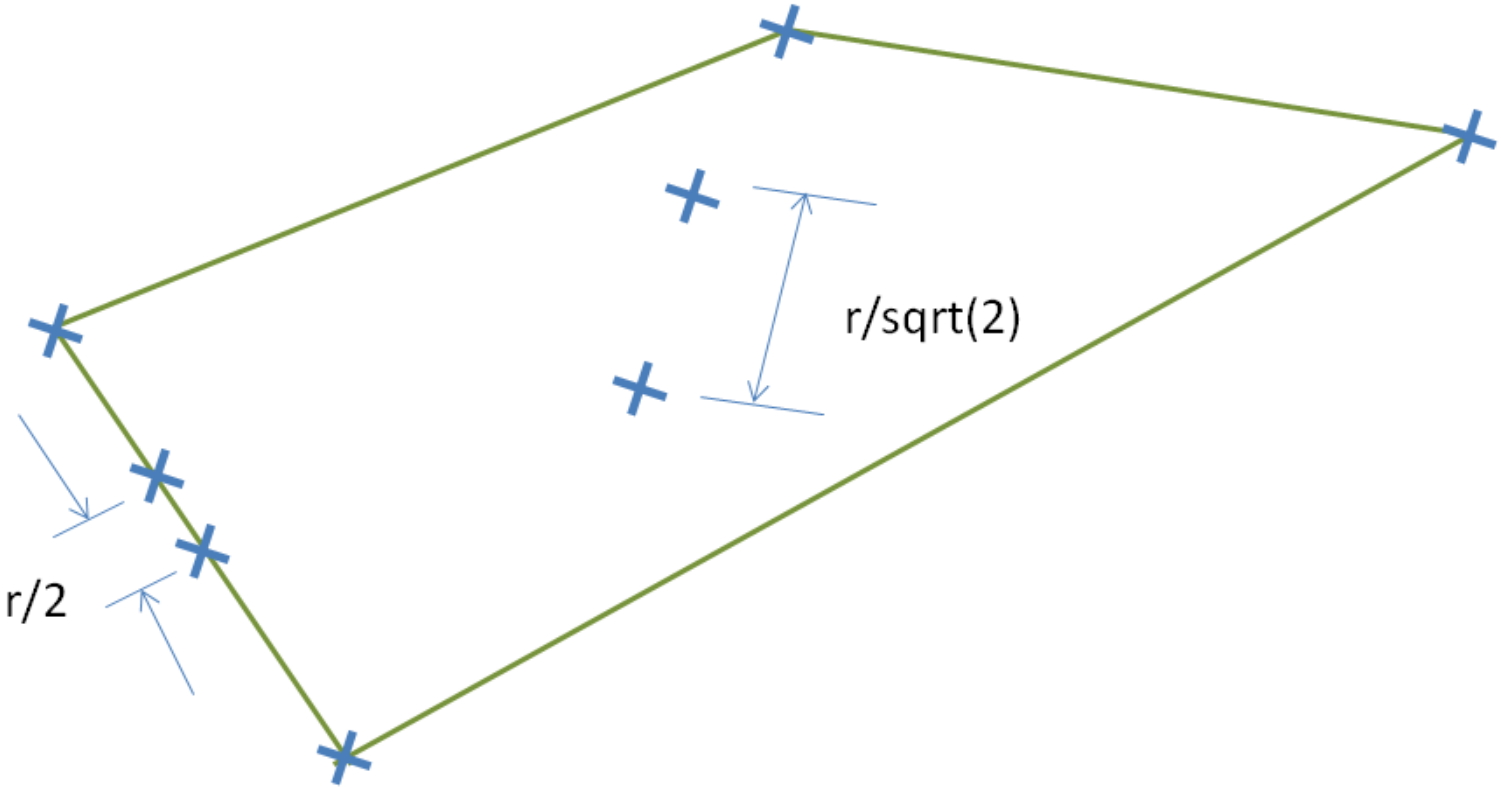




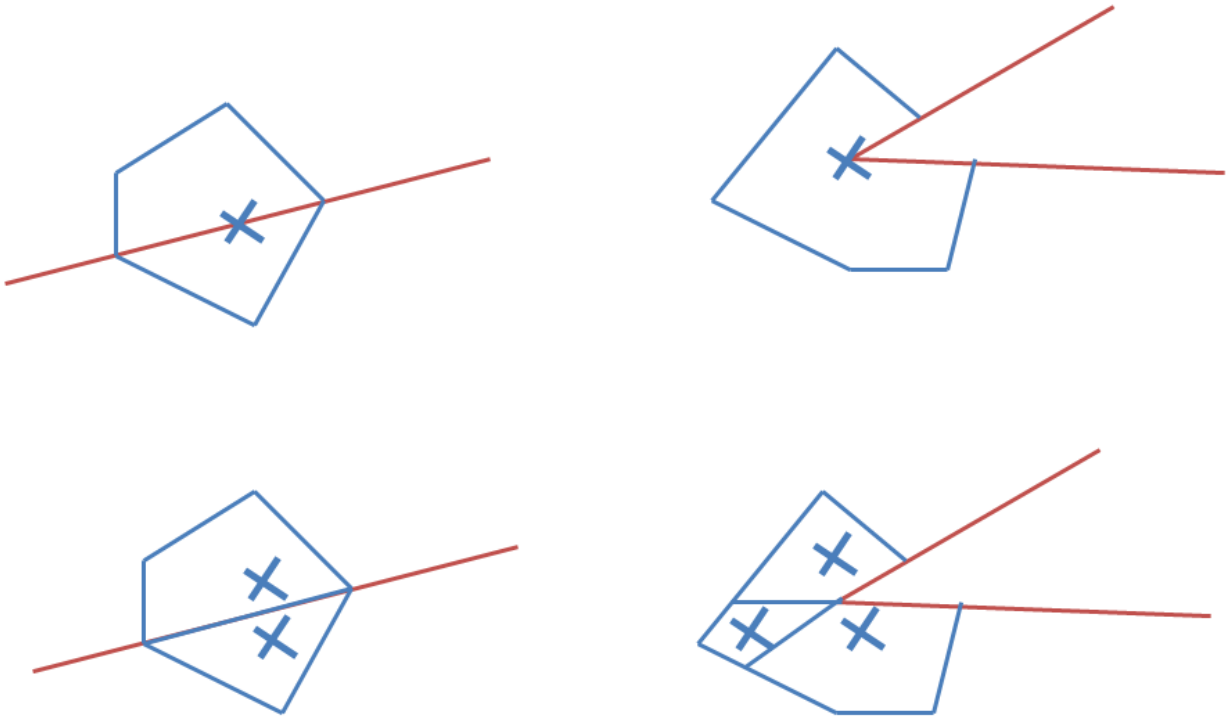
A direct method using Edge/Face trimming (IMR2011)



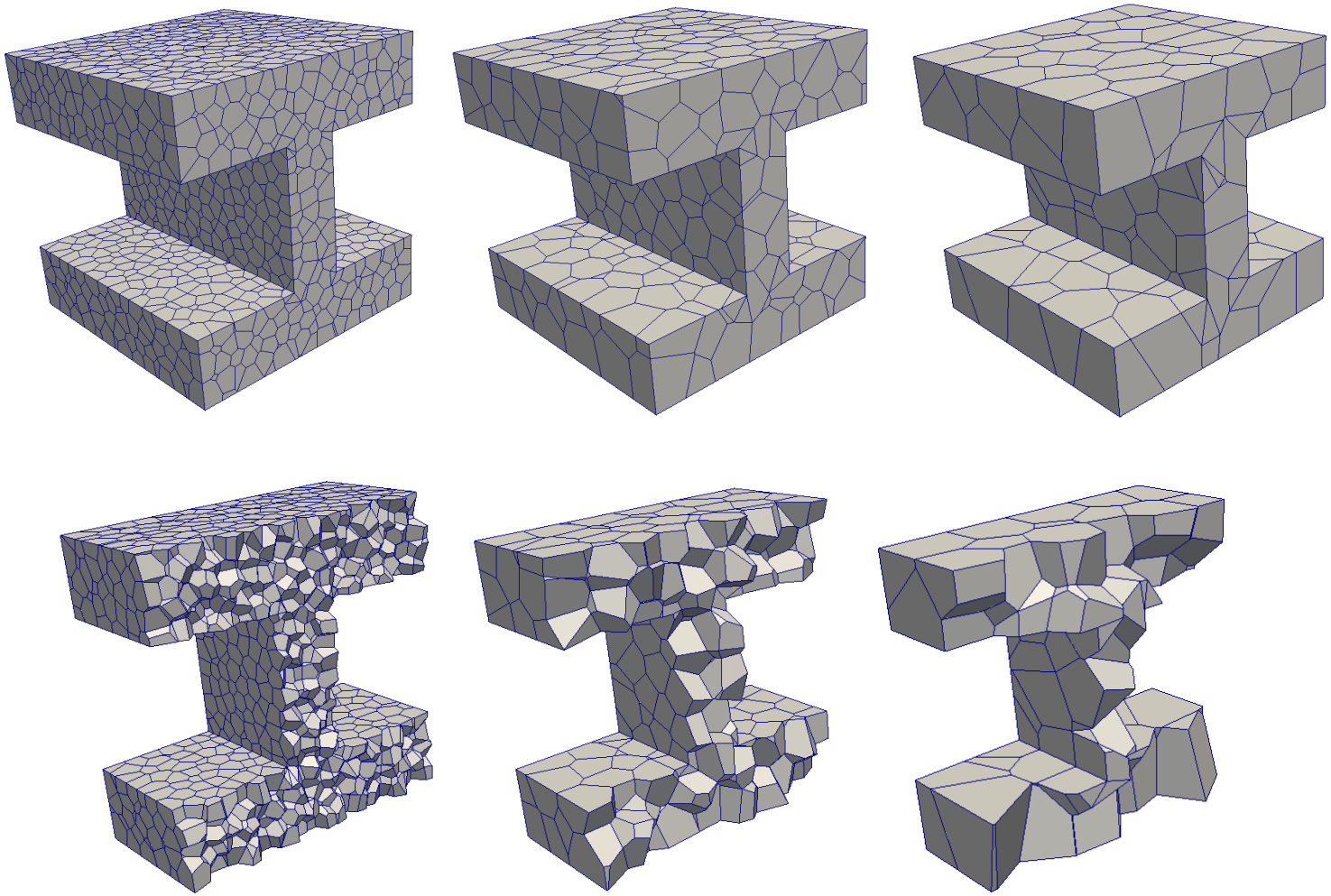
A direct method using Edge/Face trimming: handling internal interfaces



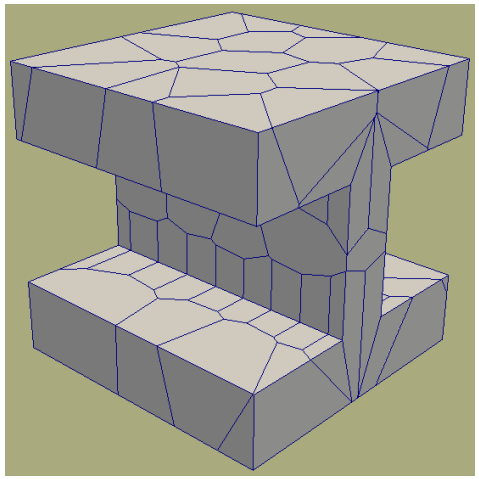
A direct method using Edge/Face trimming: handling non-convex cells

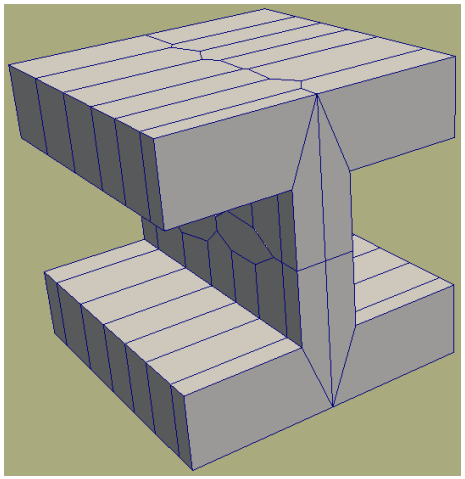


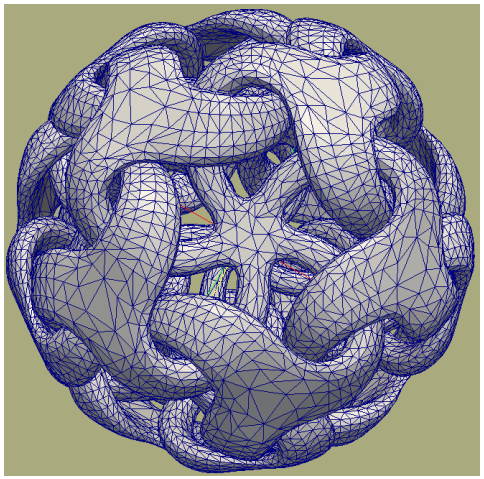
A direct method using Edge/Face trimming

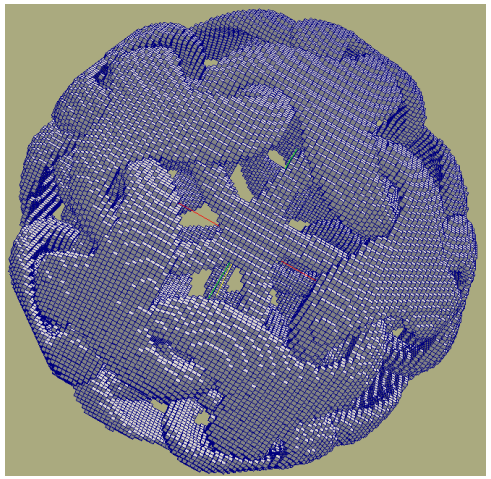


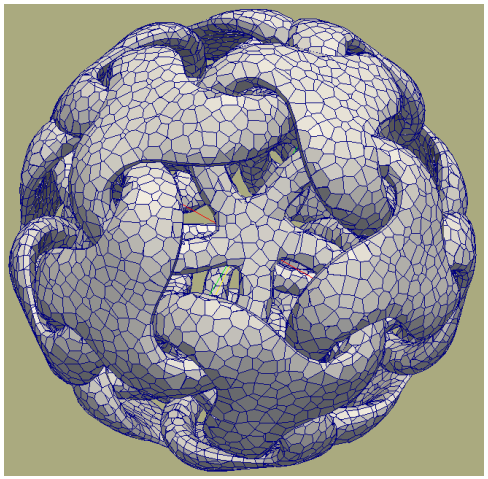
A direct method using Edge/Face trimming



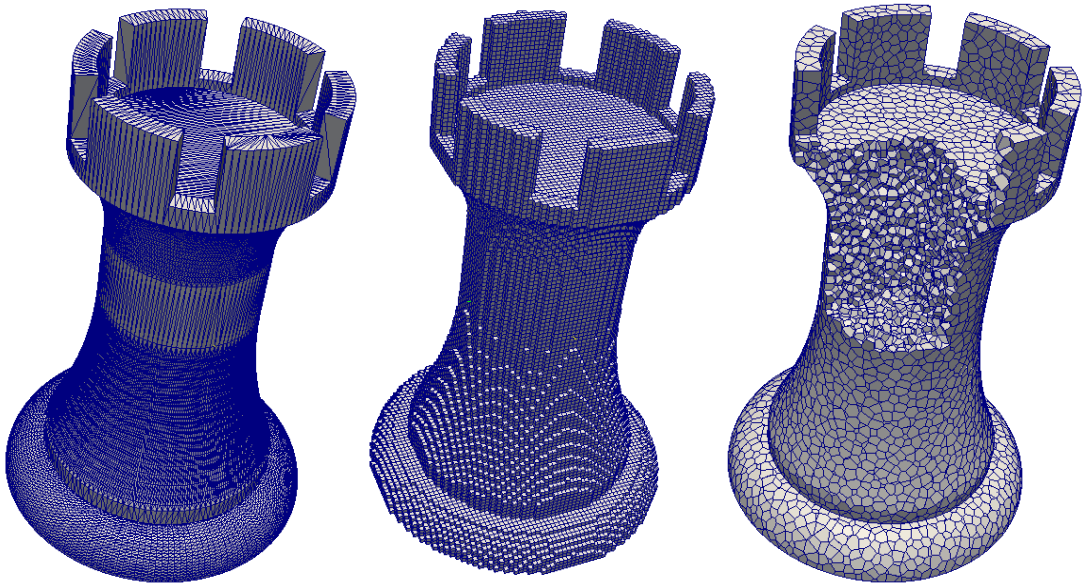


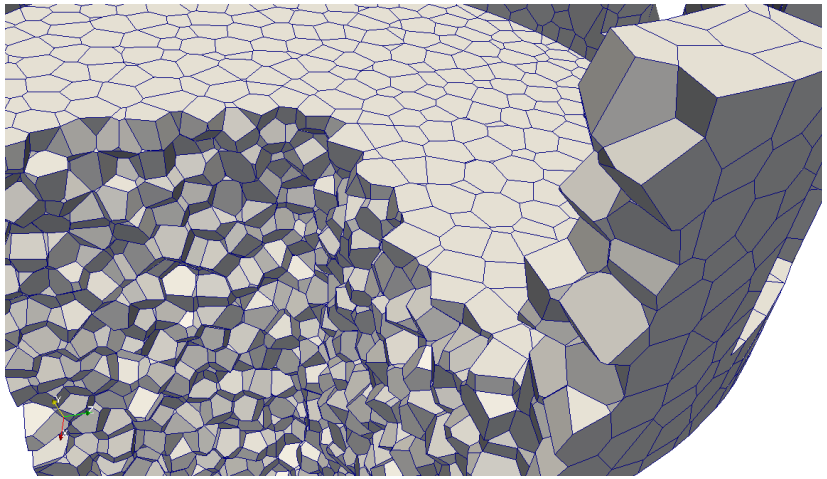


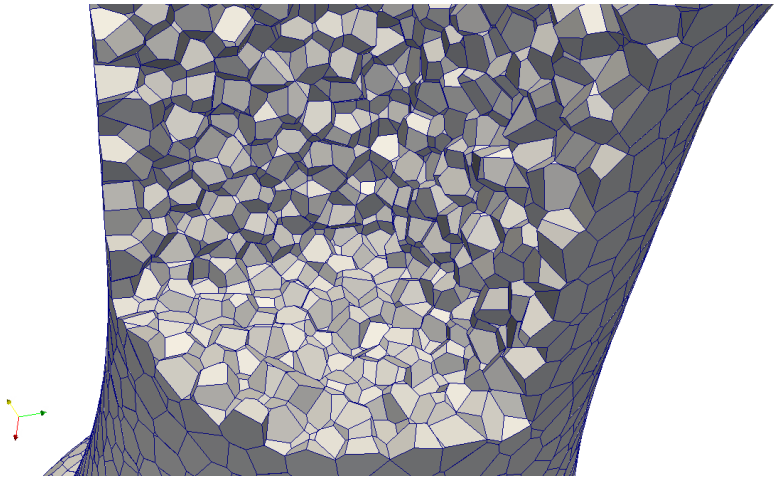




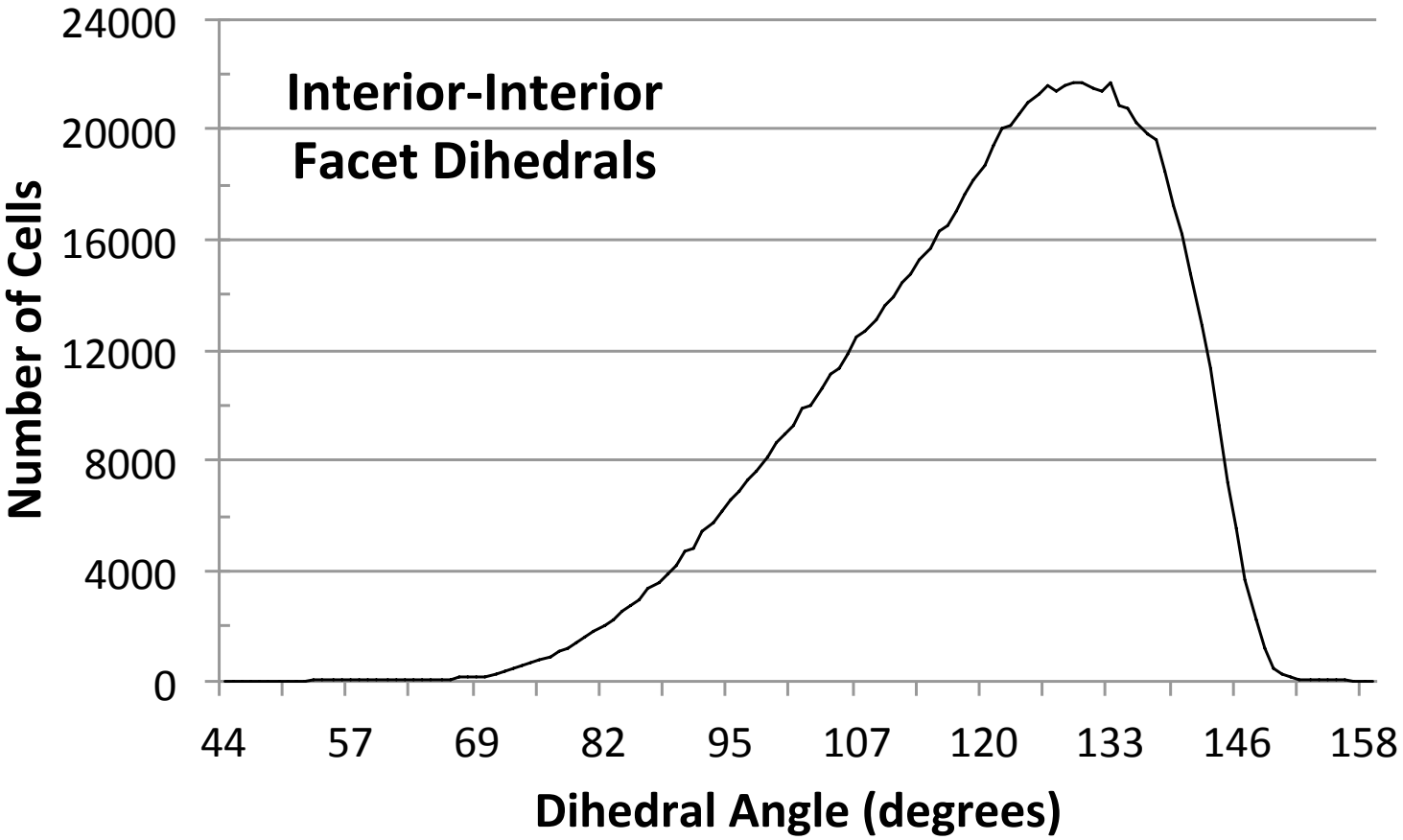
A direct method using Edge/Face trimming



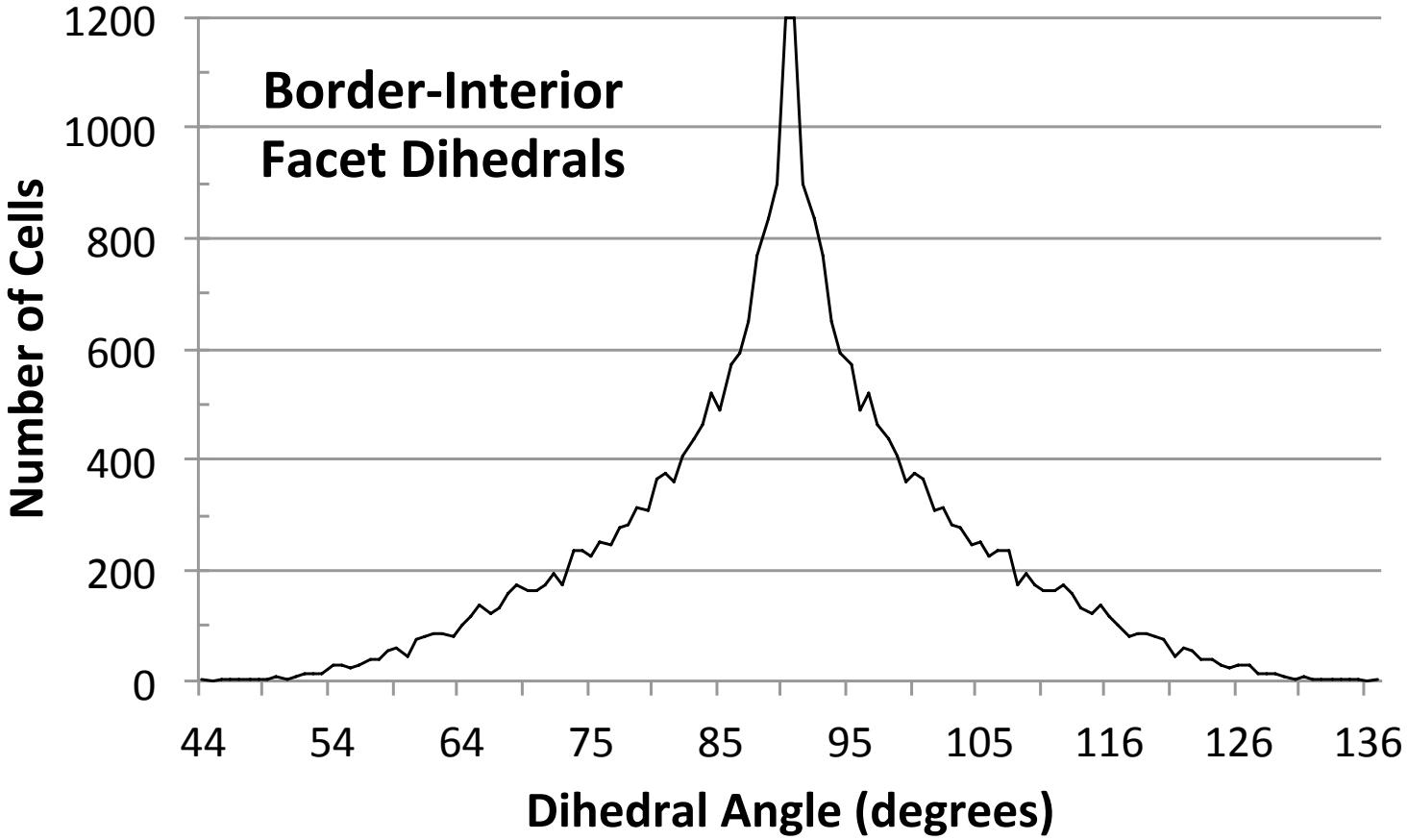




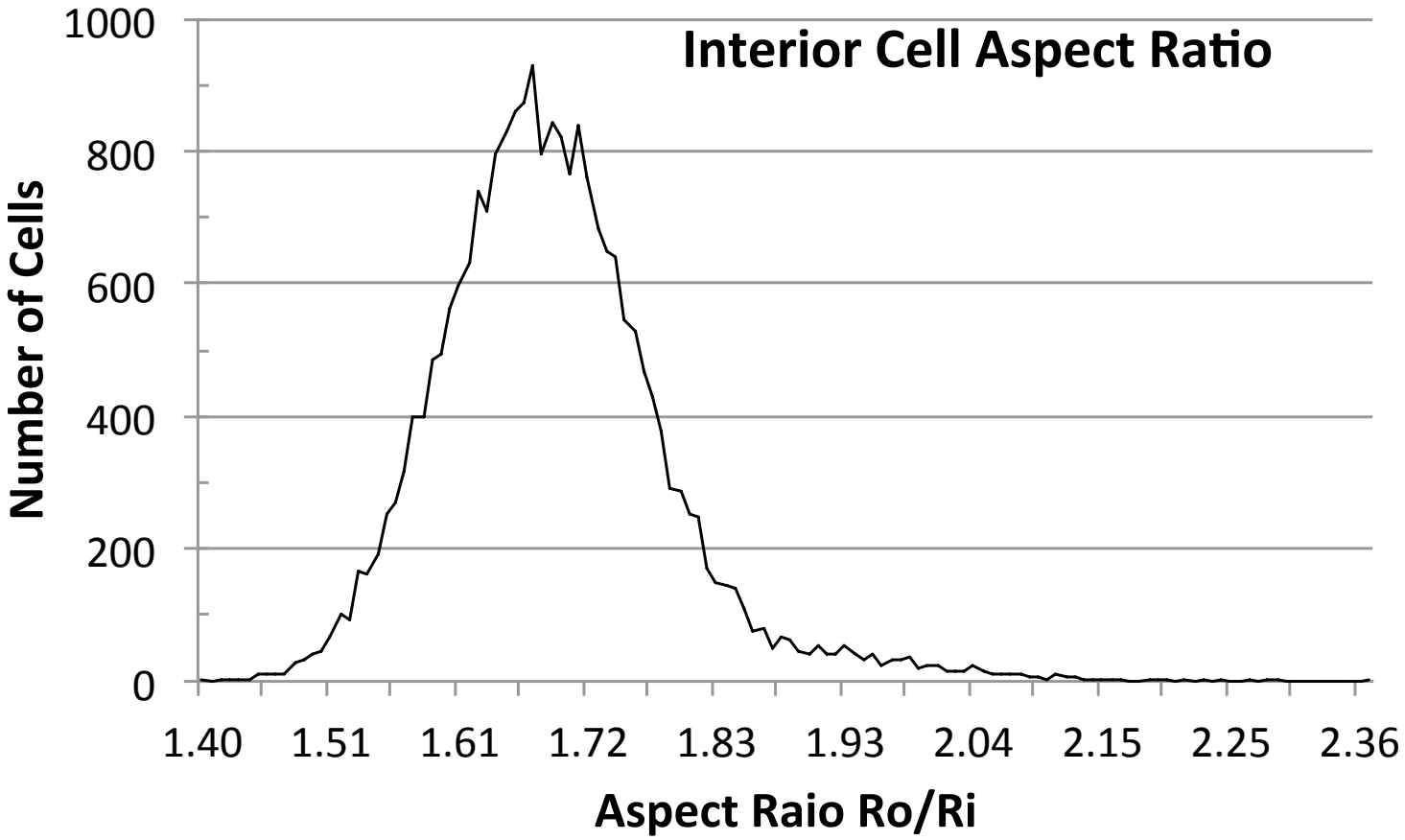
A direct method using Edge/Face trimming



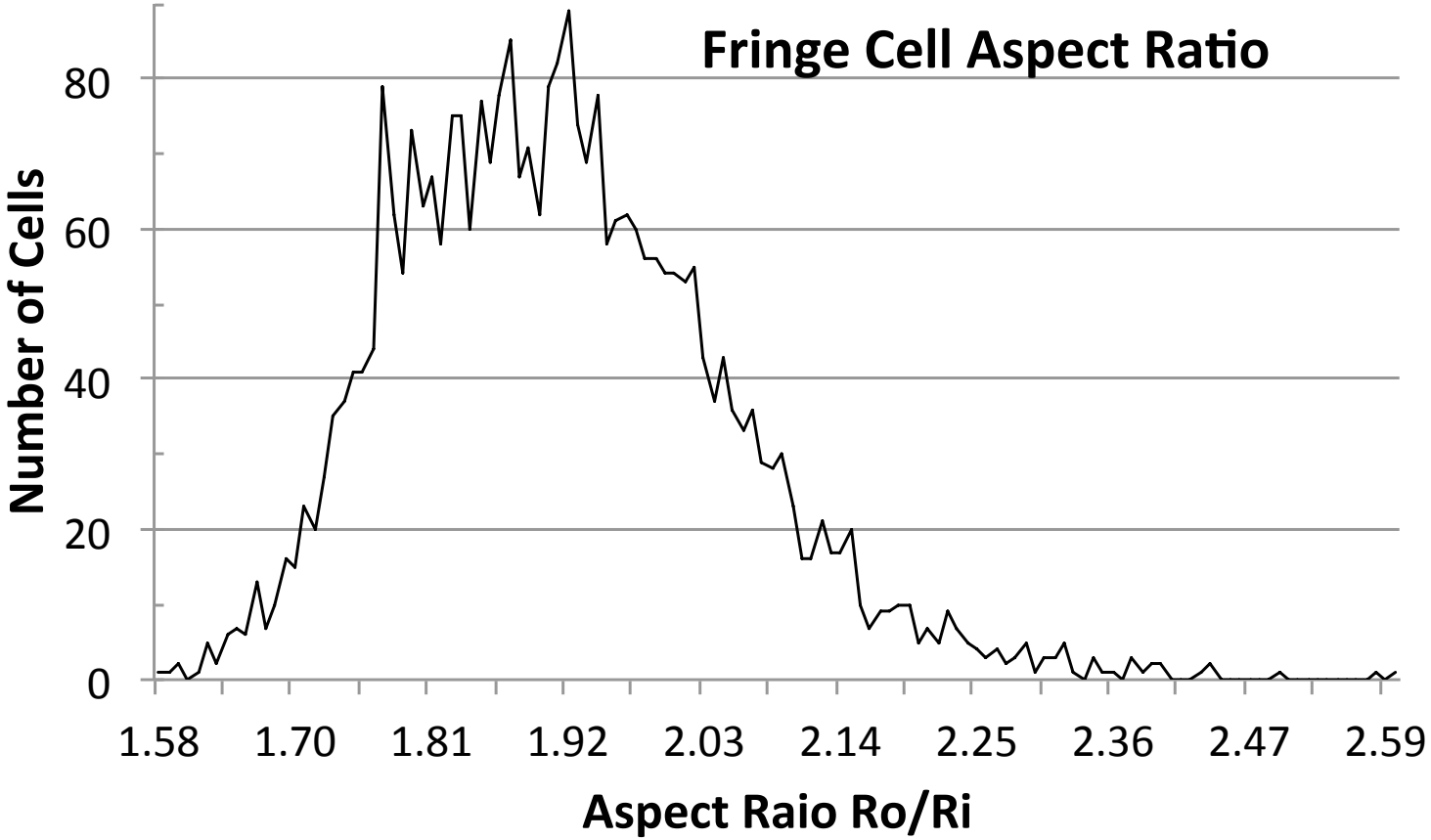
A direct method using Edge/Face trimming



A direct method using Edge/Face trimming



A direct method using Edge/Face trimming



Here is what we can do next:

1. Sampling (Meshing) of curved surfaces:

Existing Approaches

- Not strictly "unbiased".
- Not maximal: dependent on finite precision.
- Memory or run-time complexity.
- Geodesic approximations.
- The extra step (Delaunay / Voronoi meshing) is missing.

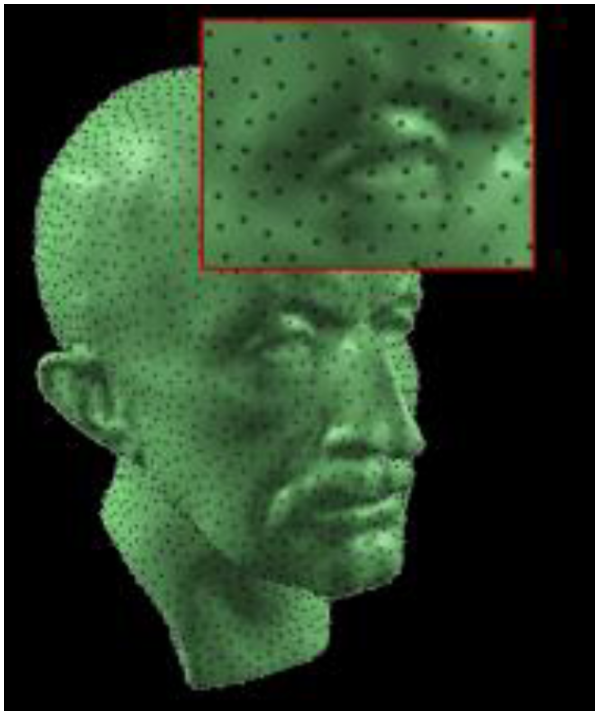


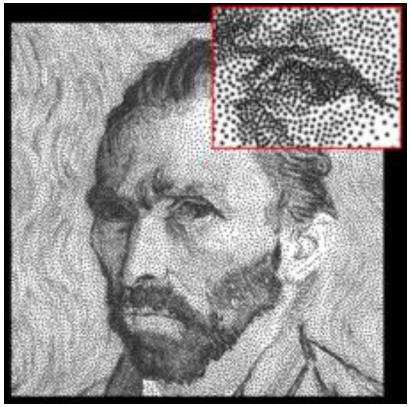
Image courtesy of Li-Yi Wei and Rui Wang (SIGGRAPH 2011)

Here is what we can do next:

2. Adaptive / Anisotrpoic Sampling (Meshing):

Existing Approaches

- The problem is not well defined.
- Ambiguous algorithms.
- Adaptive: r is not constant.
- Anisotropic: Ellipses are utilized instead of circles.
- Missing analysis of the associated Delaunay/Voronoi Meshes.
- Parallel implementations using MPI.



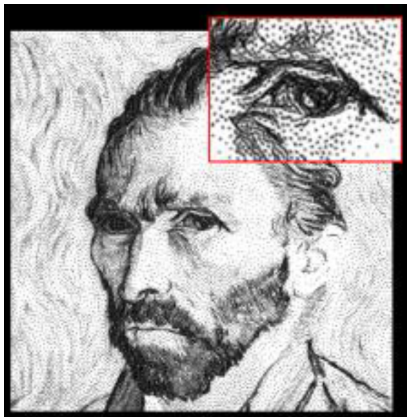
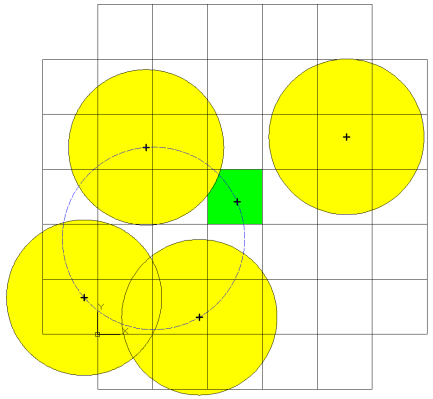


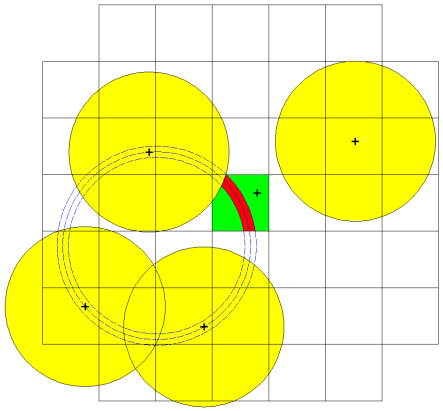
Image courtesy of Li-Yi Wei and Rui Wang (SIGGRAPH 2011)

Here is what we can do next:

3. Voronoi Meshing with no short edges (A unique Delaunay Mesh) :

- Constrained sampling to avoid more than d+1 points on the same circumcircle
- Short edges affect the selection of time steps in unsteady simulations.
- Edge collapse will result in non-planar faces
- Unique Delaunay meshes would require no communication



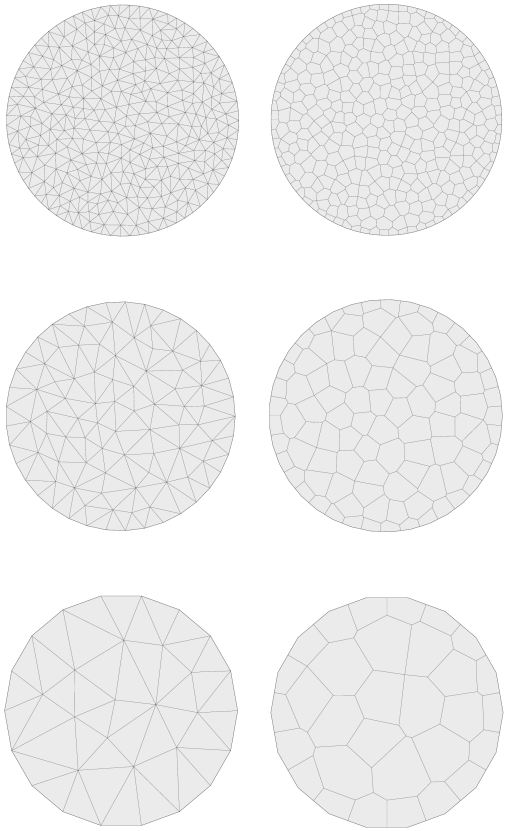


Here is what we can do next:

4. Unstructured Geometric Multigrid based on MPS:

Proposed Algorithm

- Construct a sequence of meshes by scaling $\mathcal{F}$.
- Each grid will have a guaranteed quality
- Discertized systems will maintain sparsity.
- Retrieve the prolongation/restriction operators using the Voronoi relations between successive levels

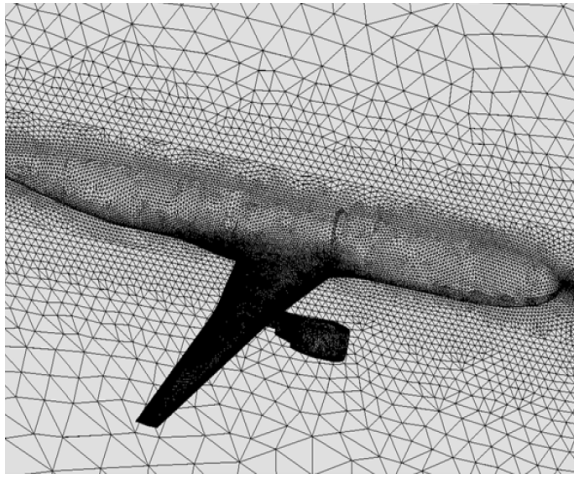


Here is what we can do next:

4. Unstructured Geometric Multigrid based on MPS:

Other Approaches

- Structured MG: Undesired distribution.
- Unstructured MG: Complicated Agglomeration techniques.
- Algebraic MG: Coarse levels loose sparsity.



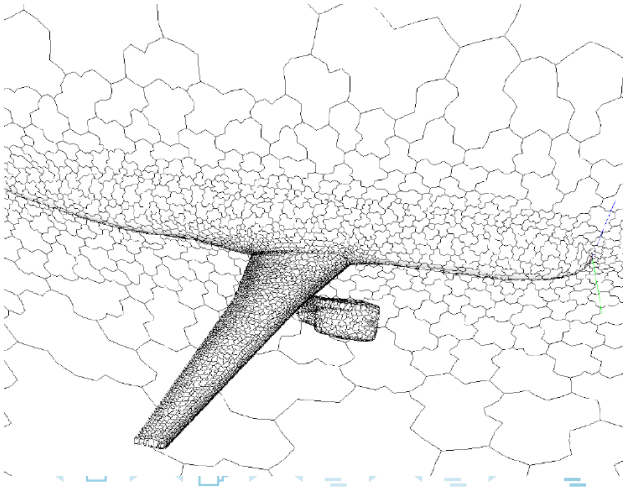


Image courtesy of Dimitri Mavriplis(Unstructured Grid Technology for CFD, 2008. Organized By: PET Program DoD)

Questions?!

Thank you!

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