Analysis of generalized square of opposition with intermediate quantifiers

Petra Murinová

Centre of Excellence IT4Innovations - Division University of Ostrava Institute for Research and Applications of Fuzzy Modeling
University of Ostrava
Czech Republic
petra.murinova@osu.cz

FSTA 2012, January 1, 2012
Outline

1 Motivation
2 Aristotle’s square and complete square of opposition
3 Łukasiewicz fuzzy type theory
4 Intermediate Generalized Quantifiers
5 Analysis of generalized square of opposition in Ł-FTT
Motivation

Motivation for this research

- Elaboration of theory of intermediate quantifiers from Peterson’s book *Intermediate Quantifiers* - analysis of complete square of opposition with the quantifiers *almost all, most, many*, etc.
- In the book of Peterson is no formal mathematical system.
- Application of Łukasiewicz fuzzy type theory.
Motivation for this research

- Elaboration of theory of intermediate quantifiers from Peterson’s book Intermediate Quantifiers - analysis of complete square of opposition with the quantifiers *almost all, most, many, etc.*
- In the book of Peterson is no formal mathematical system.
- Application of Łukasiewicz fuzzy type theory.
Motivation for this research

- Elaboration of theory of intermediate quantifiers from Peterson’s book Intermediate Quantifiers - analysis of complete square of opposition with the quantifiers *almost all, most, many*, etc.
- In the book of Peterson is no formal mathematical system.
- Application of Łukasiewicz fuzzy type theory.
# Contradictory, Contrary and Subcontrary

## Contradictory

- $x$ and $y$ are **contradictories** iff $x$ and $y$ cannot both be true; $x$ and $y$ cannot both be false.

## Contraries

- $x$ and $y$ are contraries iff $x$ and $y$ cannot both be true; $x$ and $y$ can both be false.

## Sub-contraries

- $x$ and $y$ are sub-contraries iff $x$ and $y$ cannot both be false; $x$ and $y$ can both be true.
### Contradictory

- **Contradictory**
  - \( x \) and \( y \) are contradictories iff \( x \) and \( y \) cannot both be true; \( x \) and \( y \) cannot both be false

### Contraries

- **Contraries**
  - \( x \) and \( y \) are **contraries** iff \( x \) and \( y \) cannot both be true; \( x \) and \( y \) can both be false

### Sub-contraries

- **Sub-contraries**
  - \( x \) and \( y \) are sub-contraries iff \( x \) and \( y \) cannot both be false; \( x \) and \( y \) can both be true
Contradictory, Contrary and Subcontrary

**Contradictory**
- $x$ and $y$ are contradictories iff $x$ and $y$ cannot both be true; $x$ and $y$ cannot both be false

**Contraries**
- $x$ and $y$ are contraries iff $x$ and $y$ cannot both be true; $x$ and $y$ can both be false

**Sub-contraries**
- $x$ and $y$ are sub-contraries iff $x$ and $y$ cannot both be false; $x$ and $y$ can both be true
Aristotle’s square

- A: All S is P
- E: No S is P
- I: Some S is P
- O: Some S is not P

Contraries: A ↔ O
Subalterns: A → I → E → O
Contradictories: A ↔ E
Subcontraries: I ↔ O
The first version of the complete square of opposition was introduced by P. Peterson in (1979) with “Almost-all” and “Many”.

Thompson extends the approach by the intermediate quantifier “Most” and introduced a complete square of opposition with contradictions, contraries and subalterns as follows:
The first version of the complete square of opposition was introduced by P. Peterson in (1979) with “Almost-all” and “Many”.

Thompson extends the approach by the intermediate quantifier “Most” and introduced a complete square of opposition with contradictions, contraries and subalterns as follows:
Complete square of opposition

A: All B are A  E: No B are A (universal)

P: Almost-all B are A  B: Few B are A (predominant)

T: Most B are A  D: Most B are not A (majority)

K: Many B are A  G: Many B are not A (common)

I: Some B are A  O: Some B are not A (particular)
Analysis of generalized square of opposition with intermediate quantifiers

Łukasiewicz fuzzy type theory

Structure of truth values-MVΔ-algebra

**MVΔ-algebra**

\[ \mathcal{L} = \langle L, \lor, \land, \otimes, \to, 0, 1, \Delta \rangle, \quad (1) \]

1. \( \langle L, \lor, \land, \otimes, \to, 0, 1, \rangle \) is an MV-algebra with involutive negation,

where

- \( \Delta a \lor \neg \Delta a = 1 \),
- \( \Delta(a \lor b) \leq \Delta a \lor \Delta b \),
- \( \Delta a \leq a \), \( \Delta a \leq \Delta \Delta a \),
- \( \Delta(a \to b) \leq \Delta a \to \Delta b \),
- \( \Delta 1 = 1 \).
Analysis of generalized square of opposition with intermediate quantifiers
Łukasiewicz fuzzy type theory

Example of MV\(_\Delta\)-algebra

**Standard Łukasiewicz algebra**

\[ \mathcal{L} = \langle [0, 1], \lor, \land, \otimes, \rightarrow, 0, 1, \Delta \rangle \quad (2) \]

1. \( \lor = \max \)
2. \( \land = \min \)
3. \( a \otimes b = \max(0, a + b - 1) \)
4. \( a \rightarrow b = 1 \land (1 - a + b) \)
5. \( \neg a = a \rightarrow 0 = 1 - a \)
Basic syntactical elements

The language of Ł-FTT denoted by $J$ consists of:

- variables $x_\alpha, \ldots$
- special constants $c_\alpha, \ldots (\alpha \in \text{Types})$
- $\lambda$ and brackets
- $E_{(\alpha\alpha)}\alpha$ for every $\alpha \in \text{Types}$ for fuzzy equality,
- $C_{(oo)}o$ for conjunction,
- $D_{(oo)}$ for delta operation.
The language of Ł-FTT denoted by $J$ consists of:

- variables $x_{\alpha}, \ldots$
- special constants $c_{\alpha}, \ldots$ ($\alpha \in Types$)
- $\lambda$ and brackets
- $E_{(o\alpha)\alpha}$ for every $\alpha \in Types$ for fuzzy equality,
- $C_{(oo)o}$ for conjunction,
- $D_{(oo)}$ for delta operation.
The language of Ł-FTT denoted by $J$ consists of:

- variables $x_\alpha, \ldots$
- special constants $c_\alpha, \ldots$ ($\alpha \in \text{Types}$)
- $\lambda$ and brackets
- $E_{(o\alpha)\alpha}$ for every $\alpha \in \text{Types}$ for fuzzy equality,
- $C_{(oo)o}$ for conjunction,
- $D_{(oo)}$ for delta operation.
The language of Ł-FTT denoted by $J$ consists of:

- variables $x_\alpha, \ldots$
- special constants $c_\alpha, \ldots$ ($\alpha \in Types$)
- $\lambda$ and brackets

- $E_{(o\alpha)\alpha}$ for every $\alpha \in Types$ for fuzzy equality,
- $C_{(oo)o}$ for conjunction,
- $D_{(oo)}$ for delta operation.
Analysis of generalized square of opposition with intermediate quantifiers
Łukasiewicz fuzzy type theory

Basic definitions

1. Equivalence: \( \equiv := \lambda x_\alpha \lambda y_\alpha (E_{(o\alpha)\alpha} y_\alpha)x_\alpha, \quad \alpha \in \text{Types}. \)

2. Conjunction: \( \land := \lambda x_o \lambda y_o (C_{(oo)o} y_0)x_o. \)

3. Delta connective: \( \Delta := \lambda x_o D_{oo} x_o. \)
Basic definitions

1. Equivalence: $\equiv := \lambda x_\alpha \lambda y_\alpha (E_{(\alpha \alpha)} y_\alpha) x_\alpha$, $\alpha \in \text{Types}$.

2. Conjunction: $\land := \lambda x_o \lambda y_o (C_{(oo)} y_o) x_o$.

3. Delta connective: $\Delta := \lambda x_o D_{oo} x_o$. 
Basic definitions

1. Equivalence: $\equiv := \lambda x_\alpha \lambda y_\alpha (E_{(o\alpha)\alpha} y_\alpha) x_\alpha, \quad \alpha \in \text{Types}.$
2. Conjunction: $\land := \lambda x_o \lambda y_o (C_{(oo)o} y_o) x_o.$
3. Delta connective: $\Delta := \lambda x_o D_{oo} x_o.$
Derived connectives

1. Representation of truth: $\top := \lambda x_0 x_0 \equiv \lambda x_0 x_0$.
2. Representation of falsity: $\bot := \lambda x_0 x_0 \equiv \lambda x_0 \top$.
3. Negation: $\neg := \lambda x_0 (x_0 \equiv \bot)$.
4. Implication: $\Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0$.
5. $\&$, $\bigtriangleup$, $\lor$ are defined as in Łukasiewicz logic.
6. General quantifier: $(\forall x_\alpha) A_\alpha := (\lambda x_\alpha A_\alpha \equiv \lambda x_\alpha \top)$.
7. Existential quantifier: $(\exists x_\alpha) A_\alpha := \neg (\forall x_\alpha) \neg A_\alpha$. 
Derived connectives

1. Representation of truth: $\top := \lambda x_0 x_0 \equiv \lambda x_0 x_0$.
2. Representation of falsity: $\bot := \lambda x_0 x_0 \equiv \lambda x_0 \top$.
3. Negation: $\neg := \lambda x_0 (x_0 \equiv \bot)$.
4. Implication: $\Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0$
5. $\&$, $\nabla$, $\lor$ are defined as in Łukasiewicz logic.
6. General quantifier: $(\forall x_\alpha)A_o := (\lambda x_\alpha A_o \equiv \lambda x_\alpha \top)$,
7. Existential quantifier: $(\exists x_\alpha)A_o := \neg (\forall x_\alpha)\neg A_o$. 
Analysis of generalized square of opposition with intermediate quantifiers

Łukasiewicz fuzzy type theory

Derived connectives

1. Representation of truth: $\top := \lambda x_0 x_0 \equiv \lambda x_0 x_0$.

2. Representation of falsity: $\bot := \lambda x_0 x_0 \equiv \lambda x_0 \top$.

3. Negation: $\neg := \lambda x_0 (x_0 \equiv \bot)$.

4. Implication: $\Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0$.

5. $\&$, $\nabla$, $\lor$ are defined as in Łukasiewicz logic.

6. General quantifier: $(\forall x_\alpha)A_o := (\lambda x_\alpha A_o \equiv \lambda x_\alpha \top)$.

7. Existential quantifier: $(\exists x_\alpha)A_o := \neg (\forall x_\alpha)\neg A_o$. 
Analysis of generalized square of opposition with intermediate quantifiers

Łukasiewicz fuzzy type theory

Derived connectives

1. Representation of truth: $\top := \lambda x_0 x_0 \equiv \lambda x_0 x_0$.
2. Representation of falsity: $\bot := \lambda x_0 x_0 \equiv \lambda x_0 \top$.
3. Negation: $\neg := \lambda x_0 (x_0 \equiv \bot)$.
4. Implication: $\Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0$
5. $\&$, $\nabla$, $\lor$ are defined as in Łukasiewicz logic.
6. General quantifier: $(\forall x_\alpha)A_o := (\lambda x_\alpha A_o \equiv \lambda x_\alpha \top)$,
7. Existential quantifier: $(\exists x_\alpha)A_o := \neg (\forall x_\alpha)\neg A_o$. 
Analysis of generalized square of opposition with intermediate quantifiers

- Łukasiewicz fuzzy type theory

Derived connectives

1. Representation of truth: \( \top := \lambda x_0 x_0 \equiv \lambda x_0 x_0 \).
2. Representation of falsity: \( \bot := \lambda x_0 x_0 \equiv \lambda x_0 \top \).
3. Negation: \( \neg := \lambda x_0 (x_0 \equiv \bot) \).
4. Implication: \( \Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0 \)

& , ∇ , ∨ are defined as in Łukasiewicz logic.

6. General quantifier: \( (\forall x_\alpha)A_o := (\lambda x_\alpha A_o \equiv \lambda x_\alpha \top) \),
7. Existential quantifier: \( (\exists x_\alpha)A_o := \neg (\forall x_\alpha)\neg A_o \).
Derived connectives

1. Representation of truth: $\top := \lambda x_0 x_0 \equiv \lambda x_0 x_0$.
2. Representation of falsity: $\bot := \lambda x_0 x_0 \equiv \lambda x_0 \top$.
3. Negation: $\neg := \lambda x_0 (x_0 \equiv \bot)$.
4. Implication: $\Rightarrow := \lambda x_0 \lambda y_0 (x_0 \land y_0) \equiv x_0$.
5. $\&$, $\lor$, $\lor$ are defined as in Łukasiewicz logic.
6. General quantifier: $(\forall x_\alpha) A_0 := (\lambda x_\alpha A_0 \equiv \lambda x_\alpha \top)$.
7. Existential quantifier: $(\exists x_\alpha) A_0 := \neg (\forall x_\alpha) \neg A_0$. 
Axioms and inference rules in Ł-FTT

- 17 axioms
- Two inference rules where the rules *modus ponens* and *generalization* are the rules derivative.
Semantics in Ł-FTT

A frame is a tuple

\[ \mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in \text{Types}}, \mathcal{L}_\Delta \rangle \]

1. \( (M_\alpha)_{\alpha \in \text{Types}} \) is a basic frame
2. \( \mathcal{L}_\Delta \) is MV-algebra with \( \Delta \)
3. \( =_\alpha \) is a fuzzy equality on \( M_\alpha \).

We say that a frame \( \mathcal{M} \) is a model of a theory \( T \) if all axioms are true in the degree 1 in \( \mathcal{M} \).
Trichotomous evaluative linguistic expressions

**TEE**

- are special expressions of natural language, e.g., *small*, *big*, *about fourteen*, *very short*, *more or less deep*, *not thick*.
- **Linguistic hedge** can be
  - *narrowing* — *extremely*, *significantly*, *very*
  - *widening* — *more or less*, *roughly*, *quite roughly*, *very roughly*
  - *empty hedge*

- We will work with expressions: *extremely big*, *very big*, *not small*.
- $T_{Ev}$ has 11 axioms.
Trichotomous evaluative linguistic expressions

**TEE**
- are special expressions of natural language, e.g., *small*, *big*, *about fourteen*, *very short*, *more or less deep*, *not thick*.
- Linguistic hedge can be
  - *narrowing* — *extremely*, *significantly*, *very*
  - *widening* — *more or less*, *roughly*, *quite roughly*, *very roughly*
  - *empty hedge*
- We will work with expressions: *extremely big*, *very big*, *not small*.
- $T^{Ev}$ has 11 axioms.
Theory of intermediate quantifiers \( T^{\text{IQ}} \)

1. is a special theory of Ł-FTT extending the theory \( T^{\text{Ev}} \) of evaluative linguistic expressions
2. we consider a special formula \( \mu \) of type \( o(o\alpha)(o\alpha) \) such that values of the measure are taken from the set of truth values
3. \( \mu \) has four axioms
Intermediate Generalized Quantifiers

Definition of intermediate generalized quantifiers

Definitions of intermediate generalized quantifiers of the form “Quantifier B’s are A”

(a) \( (Q_{Ev}^\forall x)(B, A) := (\exists z)(((\Delta(z \subseteq B) \& (\forall x)(z x \Rightarrow Ax)) \& Ev((\mu B)z)), \)

(b) \( (Q_{Ev}^\exists x)(B, A) := (\exists z)(((\Delta(z \subseteq B) \& (\exists x)(zx \& Ax)) \& Ev((\mu B)z)). \)
Definition of intermediate generalized quantifiers

**Explanation of definition of IGQ**

Each formula above consists of three parts:

\[
(\exists z)( (\Delta (z \subseteq B)) \quad \& \quad (\forall x)(z \Rightarrow Ax)) \quad \wedge \quad Ev((\mu B)z)) \tag{3}
\]

- \((\exists z)( (\Delta (z \subseteq B))\) “the greatest” part of \(B\)’s
- \((\forall x)(z \Rightarrow Ax))\) each \(z\)’s has \(A\)
- \(Ev((\mu B)z))\) size of \(z\) is evaluated by \(Ev\)
Definition of intermediate generalized quantifiers with presupposition

Interpretation of “Quantifier B’s are A” with presupposition

(a) \( (*Q^\forall_{Ev} x)(B, A) \equiv (\exists z)((\Delta(z \subseteq B) \& (\exists x)zx \& (\forall x)(z x \Rightarrow Ax)) \land Ev((\mu B)z)), \)

(b) \( (*Q^\exists_{Ev} x)(B, A) := (\exists z)((\Delta(z \subseteq B) \& (\exists x)zx \& (\exists x)(zx \land Ax)) \land Ev((\mu B)z)). \)

where only non-empty subsets of \( B \) are considered.
Analysis of generalized square of opposition with intermediate quantifiers

Intermediate Generalized Quantifiers

“All”, “No”, “Almost all”, “Few”, “Most”

A: All $B$ are $A$ := $Q_{Bi\Delta}^\forall (B, A) \equiv (\forall x)(Bx \Rightarrow Ax)$,

E: No $B$ are $A$ := $Q_{Bi\Delta}^\forall (B, \neg A) \equiv (\forall x)(Bx \Rightarrow \neg Ax)$,

P: Almost all $B$ are $A$ := $Q_{Bi\text{Ex}}^\forall (B, A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) & (\forall x)(zx \Rightarrow Ax)) \land (Bi\text{Ex})((\mu B)z)),$$

B: Few $B$ are $A$ (:= Almost all $B$ are not $A$) := $Q_{Bi\text{Ex}}^\forall (B, \neg A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) & (\forall x)(zx \Rightarrow \neg Ax)) \land (Bi\text{Ex})((\mu B)z)),$$

T: Most $B$ are $A$ := $Q_{Bi\text{Ve}}^\forall (B, A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) & (\forall x)(zx \Rightarrow Ax)) \land (Bi\text{Ve})((\mu B)z)),$$

D: Most $B$ are not $A$ := $Q_{Bi\text{Ve}}^\forall (B, \neg A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) & (\forall x)(zx \Rightarrow \neg Ax)) \land (Bi\text{Ve})((\mu B)z)),$$
“Many”, “Some”

**K:** Many \( B \) are \( A \) := \( Q^\forall_{-\bar{S}m\bar{v}}(B, A) \) \( \equiv \)
\[ (\exists z)((\Delta(z \subseteq B) \& (\forall x)(zx \Rightarrow Ax)) \land \neg(Sm\bar{v})(\mu Bz)), \]

**G:** Many \( B \) are not \( A \) := \( Q^\forall_{-\bar{S}m\bar{v}}(B, \neg A) \) \( \equiv \)
\[ (\exists z)((\Delta(z \subseteq B) \& (\forall x)(zx \Rightarrow \neg Ax)) \land \neg(Sm\bar{v})(\mu Bz)), \]

**I:** Some \( B \) are \( A \) := \( Q^\exists_{Bi\Delta}(B, A) \) \( \equiv (\exists x)(Bx \land Ax), \]

**O:** Some \( B \) are not \( A \) := \( Q^\exists_{Bi\Delta}(B, \neg A) \) \( \equiv (\exists x)(Bx \land \neg Ax). \]
Generalized definitions in Ł-FTT

**Contraries**

$P_1, P_2 \in Form_0$ are **contraries** in $T^{\text{IQ}}$ if in every model $\mathcal{M} \models T^{\text{IQ}}$ the following is true:

$$\mathcal{M}(P_1) \otimes \mathcal{M}(P_2) = \mathcal{M}(\bot).$$

We can alternatively say that $P_1$ and $P_2$ are contraries if $T^{\text{IQ}} \vdash P_1 \& P_2 \equiv \bot$. 
Generalized definitions in Ł-FTT

Sub-contraries

\( P_1, P_2 \in Form_0 \) are sub-contraries in \( T^{IQ} \) if in every model \( \mathcal{M} \models T^{IQ} \) the following is true:

\[ \mathcal{M}(P_1) \oplus \mathcal{M}(P_2) = \mathcal{M}(\top). \]

We can alternatively say that \( P_1 \) and \( P_2 \) are sub-contraries if \( T^{IQ} \vdash P_1 \sqcup P_2 \).
Generalized definitions in Ł-FTT

Sub-contraries

$P_1, P_2 \in \text{Form}_0$ are sub-contraries in $T^{IQ}$ if in every model $\mathcal{M} \models T^{IQ}$ the following is true:

$$\mathcal{M}(P_1) \oplus \mathcal{M}(P_2) = \mathcal{M}(\top).$$

We can alternatively say that $P_1$ and $P_2$ are sub-contraries if $T^{IQ} \vdash P_1 \nabla P_2$. 
Contradictories

$P_1, P_2 \in Form_o$ are contradictories in $T^{IQ}$ if in every model $\mathcal{M} \models T^{IQ}$ the following two equalities hold:

- $\mathcal{M}(\Delta P_1) \otimes \mathcal{M}(\Delta P_2) = \mathcal{M}(\bot)$,
- $\mathcal{M}(\Delta P_1) \oplus \mathcal{M}(\Delta P_2) = \mathcal{M}(\top)$.

Alternatively we can say that $P_1$ and $P_2$ are contradictories, if both $T^{IQ} \vdash \Delta P_1 \& \Delta P_2 \equiv \bot$ as well as $T^{IQ} \vdash \Delta P_1 \nabla \Delta P_2$. 

Generalized definitions in Ł-FTT
Generalized definitions in Ł-FTT

**Subaltern**

We say that $A$ is a subaltern of $S$ in $T^{IQ}$ if in every model $\mathcal{M} \models T^{IQ}$ the inequality

$$\mathcal{M}(A) \leq \mathcal{M}(S)$$

holds true. We will call $S$ as superaltern of $A$. Alternatively we can say that $A$ is a subaltern of $S$ if $T^{IQ} \vdash A \Rightarrow S$. 
The formulas $\ast A, E$ are contraries in $T^{IQ}$
($T^{IQ} \vdash \ast A \& E \equiv \bot$).

If $T^{IQ} \vdash (\exists x)Bx$ then the formulas $A, E$ are contraries in $T^{IQ}$.

The formulas $\ast O$ and $I$ are sub-contraries in $T^{IQ}$
($T^{IQ} \vdash \ast O \lor I$).

If $T^{IQ} \vdash (\exists x)Bx$, then the formulas $O$ and $I$ are
sub-contraries in $T^{IQ}$.

The formulas $A$ and $O$ are contradictories in $T^{IQ}$.

The formulas $E$ and $I$ are contradictories in $T^{IQ}$.
Aristotle’s square interpreted in $\mathcal{T}^{IQ}$

- **A**: All $S$ is $P$ — contraries — **E**: No $S$ is $P$
- **I**: Some $S$ is $P$ — subcontraries — **O**: Some $S$ is not $P$
- subalterns — contradictories — subalterns
Extension of the theory $T^{IQ}$

**Theory** $T[B, B']$

Let $B, B' \in Form_{\alpha}$. The theory $T[B, B']$ is a consistent extension of $T^{IQ}$ such that

(a) $T[B, B'] \vdash B \equiv B'$,

(b) $T[B, B'] \vdash (\exists x_\alpha) \Delta B x$ and $T[B, B'] \vdash (\exists x_\alpha) \Delta B' x$. 
Example of generalized Aristotelian square interpreted in $T[B, B']$

\[ M(A) = a = 0.2 \quad \text{contraries} \quad M(E) = e \leq 0.8 \]

\[ M(I) = i = 1 \quad \text{subcontraries} \quad M(O) = o = 1 \]
Properties of generalized quantifiers in $T[B, B']$

The main properties

- $T[B, B'] \vdash B \& P \equiv \bot$,
- $T[B, B'] \vdash D \& T \equiv \bot$,
- $T[B, B'] \vdash G \& K \equiv \bot$,
- $T[B, B'] \vdash G \& P \equiv \bot$,
- $T[B, B'] \vdash K \& B \equiv \bot$. 
## Properties of generalized quantifiers in $T[B, B']$

### Derived properties

- $T[B, B'] \vdash E \& K \equiv \bot$,  
- $T[B, B'] \vdash E \& T \equiv \bot$,  
- $T[B, B'] \vdash E \& P \equiv \bot$,  
- $T[B, B'] \vdash A \& G \equiv \bot$,  
- $T[B, B'] \vdash A \& D \equiv \bot$,  
- $T[B, B'] \vdash A \& B \equiv \bot$.  

Example of generalized complete square

A: \( a \leq p \)  
E: \( e \leq 1 - p \)

P: \( p = 0.4 \)  
B: \( e \leq b \leq 1 - p \)

T: \( p \leq t \)  
D: \( b \leq d \)  
(such that \( t \otimes d = 0 \))

K: \( t \leq k \)  
G: \( d \leq g \)  
(such that \( k \otimes g = 0 \))

I: \( i = 1 \)  
O: \( o = 1 \)
Analysis of generalized square of opposition with intermediate quantifiers

Analysis of generalized square of opposition in Ł-FTT

Generalized complete square of opposition

\[ A: a = 0.3 \quad E: e = 0.2 \]
\[ P: p = 0.4 \quad B: b = 0.3 \]
\[ T: t = 0.45 \quad D: d = 0.49 \]
\[ K: k = 0.5 \quad G: g = 0.5 \]
\[ I: i = 1 \quad O: o = 1 \]
Main results

Results

- I developed Ł-FTT.
- I proposed generalized definitions of properties which characterize relations among intermediate generalized quantifiers in the generalized square of opposition.
- I formally proved validity of these relations.
Thank you for your attention.