Observer-based sliding mode impedance control of bilateral teleoperation under constant unknown time delay

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Abstract

Sliding mode control has been used extensively in robotics to cope with parametric uncertainty and hard nonlinearities, in particular for time-delay teleoperators, which have gained gradual acceptance due to technological advancements. However, since the slave teleoperator is in contact with a rigid environment, the slave controller requires a free of chattering control strategy, thus making first order sliding mode teleoperation control unsuitable. As an alternative, chatter free, higher-order sliding mode teleoperator control is proposed in this paper to guarantee robust tracking under unknown constant time delay. Moreover, complete order observers are proposed to avoid measurement of velocity and acceleration, along with a formal closed-loop stability proof of the observer-based controller. Experimental results are presented and discussed, which reveals the effectiveness of the proposed teleoperation scheme.

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1. Introduction

Teleoperation systems are very useful and in widespread use over the Internet, despite evident stability problems, in particular because of the communication time-delay. Basically, a human operator interacts with an interface, called master teleoperator, and he/she drives it in order to govern the remote counterpart, on the opposite side, while another interface (slave teleoperator) is in charge of directly implementing commands received from the operator on the remote environment.

1.1. Background

Lots of teleoperation schemes that consider time-delays have been proposed in the last decades, most of them for linear systems. Kim and Hannaford [11] used force reflection and shared compliant control to improve teleoperation tasks under time-delays. The sensor force reading is filtered by a low-pass filter and added to the position error in order to decrease impact forces. However, Arcara and Melchiorri [3] demonstrated that this scheme is not stable for large time delays, and that stability is preserved if a tradeoff between controller gains and scale factors versus tracking performance is established. Anderson and Spong [2] proposed a scheme, based on scattering theory, to passify the communication block in order to attain stability of the whole teleoperation system. Further, Niemeyer and Slotine [12] extended this result by introducing the concept of wave variables, a more intuitive approach. These schemes guarantee stability for the whole teleoperation system under any, unknown and constant, time-delay. However, in practice, non-idealities of the mechanisms, actuators, and sensors may violate passivity, imposing practical limitations on these approaches [16]. Buttolo et al. [4] proposed model-based first order sliding mode controllers for both master and slave stations, achieving an improvement...
with respect to position tracking and parametric uncertainty. Unfortunately, with small time-delays the teleoperation system becomes unstable, furthermore, as the sliding mode control introduces high frequency signals, it makes it unsuitable to be implemented in a physical system, let alone for contact tasks. Park and Cho [14] proposed implementing an impedance control for the master and a modified first order sliding mode control for the slave, in which the gain that satisfies the sliding condition can be set independently of time-delay. Thus, this scheme attains robustness against constant time-delays, and for bounded varying time-delays. Nevertheless, it easily turns unstable when interacting with rigid environments. To deal with this problem, they proposed a sliding mode-based impedance control for the slave in order to reduce impact forces and achieve a sustained stable contact [6]. Also, in [5] the problem of parametric uncertainty is studied, and an algorithm for the parameters tuning is proposed. However, the signum function, used for the sliding mode, is replaced by a saturation function in order to reduce chattering, with consequent poor tracking performance, and even unstability of the system. On the other hand, García-Valdovinos et al. [7] proposed a modified scheme based on higher-order sliding modes so as to avoid introducing chattering to the system. In this case, simulations were presented wherein complete order sliding-observers for both master and slave robots are implemented to estimate velocity and acceleration.

Considering the latter, in this paper, a robust, though smooth control for a teleoperation system is proposed. The effective combination of second order sliding mode and impedance control, along with linear observers, allows for robustness against substantial unknown constant time delay, with a free of chattering control input and with only position and force measurements. A formal stability analysis of the proposed observer-based controller is presented, which reveals the exponential convergence of the system.

1.2. Motivation

First order sliding mode control has been proposed to deal with parametric uncertainties and hard nonlinearities present in any physical system; however, this approach cannot be implemented in real time because of the finite bandwidth of the teleoperators, and because it is unreasonable to allow high frequency. In particular, this is dangerous in the master teleoperator because the human is firmly handling it, and because the slave teleoperator is rigidly coupled to the environment, which eventually could destabilize the whole teleoperation system, even in the absence of time-delays. Despite the robustness of the first order sliding mode-based controller, this technique has remained elusive in bilateral teleoperation because of chattering phenomena. Therefore, it is interesting to study and synthesize a sliding mode-based controller that shows robustness and fast tracking without chattering.

On the other hand, to the best of our knowledge, no work has been done on using state observers for teleoperators. The avoidance of velocity and acceleration measurements is a plus in robotics because expensive and bulky tachometers are circumvented. Motivated by these facts, we propose a smooth teleoperator control based on second order sliding mode-impedance that solves the problem of chattering and avoids velocity measurements, offering robustness against unknown constant time-delays.

1.3. Contribution

A second order sliding mode-impedance control is proposed to solve the problem of chattering as well as fast tracking convergence. Simple linear observers are designed in order to avoid velocity and acceleration measurements. Altogether, this gives rise to a robust and smooth controller that achieves impedance tracking in the presence of substantial unknown time-delay. The proposed scheme is developed for linear systems and a formal analysis of stability is presented. Experimental results confirm the predicted behavior.

2. Dynamics of the teleoperator system

Next, the linear models of master and slave plants are presented.

2.1. Master and slave

The dynamics of the 1-dof master/slave system are modeled as a mass–damper system

\[ M_m \ddot{x}_m + B_m \dot{x}_m = F_h + F_{mc} \tag{1} \]
\[ M_s \ddot{x}_s + B_s \dot{x}_s = F_{sc} - F_e \tag{2} \]

where \( x_i \) denotes position, and \( \dot{x}_i, \ddot{x}_i \) velocity and acceleration, respectively; \( F_{ic} \) is torque control; \( M_i \) and \( B_i \) represent mass and viscous friction coefficient, respectively, with \( i = m, s \) denoting master and slave, respectively; \( F_h \) is the force applied at the master by the human operator, and \( F_e \) is the force exerted on the slave by the environment.

2.2. Delayed signals and scaling factors

Before designing the controllers, it is necessary to define the signals that are exchanged between each side. Signals sent from master to slave are the following

\[ x_{m}^{dy} = x_m(t - T), \quad \dot{x}_{m}^{dy} = \dot{x}_m(t - T) \]
\[ \ddot{x}_{m}^{dy} = \ddot{x}_m(t - T), \quad F_{h}^{dy} = F_h(t - T) \]

where superscript \( dy \) denotes delayed. The only one signal sent from slave to master is the measured force

\[ F_e^{dy} = F_e(t - T) \]

where \( T \) denotes the delay induced by the communication channel. It is assumed, for simplicity, the same constant time-delay in both directions. Position, velocity and acceleration of the master, and contact force from the slave can be scaled according to the application

\[ x_{sd} = k_p x_{m}^{dy}, \quad \dot{x}_{sd} = k_p \dot{x}_{m}^{dy} \]
\[ \ddot{x}_{sd} = k_p \ddot{x}_{m}^{dy}, \quad F_{hd} = k_f F_e^{dy} \]
where \( k_p, k_f > 0 \) are the position and force scaling factors, respectively. The subscript \( d \) denotes desired.

### 3. Controller design

#### 3.1. Master controller

Consider the following master control structure

\[
F_{mc} = -F_h + B_m \ddot{x}_m + \frac{M_m}{M_s} \times \left( F_h - k_f F_e^d - \ddot{B}_m \ddot{x}_m - \dddot{K}_m x_m \right)
\]  

(3)

where \( M_m, B_m, \dddot{K}_m > 0 \) are the desired inertia, damping, and stiffness, respectively, of a desired impedance. Substituting (3) into (1), the closed-loop impedance error equation arises

\[
\dddot{M}_m \dddot{x}_m + \dddot{B}_m \dddot{x}_m + \dddot{K}_m x_m = F_h - k_f F_e^d.
\]  

(4)

Notice that the master control imposes a desired impedance dynamics in the master teleoperator, between the speed of the master and the linear combination of the human force and the delayed contact force.

#### 3.2. Slave controller

Under a similar rationale as the master controller, consider the slave control design based on second order sliding mode approach to produce a desired impedance behavior modulated by the environmental contact forces, robust to unknown time-delay. Thus, we design the slave controller based on high order sliding mode control. To this end, consider the desired slave impedance

\[
\dddot{M}_s \dddot{x}_s + \dddot{B}_s \dddot{x}_s + \dddot{K}_s \dddot{x}_s = -F_e
\]  

(5)

where \( \dddot{M}_s, \dddot{B}_s, \dddot{K}_s > 0 \) are the desired inertia, damping, and stiffness, respectively, and \( \dddot{x}_s := \dddot{x}_s - k_p \dddot{x}_m \), \( \dddot{x}_s := \dddot{x}_s - k_p \dddot{x}_m \) are the slave tracking errors for acceleration, velocity, and position, respectively. Since we want to obtain (5) in closed loop, then naturally the (objective) sliding surface is

\[
I_c = \dddot{M}_s \dddot{x}_s + \dddot{B}_s \dddot{x}_s + \dddot{K}_s \dddot{x}_s + F_e = 0.
\]  

(6)

Now let us define the extended error variable as follows

\[
\Omega = \frac{1}{M_s} \int_0^t I_c(\tau) \, d\tau + K_1 \int_0^t \int_0^\sigma \text{sign}(I_c(\tau)) \, d\tau \, d\sigma
\]  

(7)

where \( K_1 > 0 \) is the sliding mode gain. Substituting (6) into (7), and integrating, we finally obtain

\[
\Omega = \dddot{x}_s + \dddot{B}_s \dddot{x}_s + \frac{1}{M_s} \int_0^t [\dddot{K}_s \dddot{x}_s + F_e] \, d\tau + \frac{K_1}{M_s} \int_0^t \int_0^\sigma \text{sign}(I_c(\tau)) \, d\tau \, d\sigma.
\]  

(8)

Consider the following slave controller \( F_{sc} \)

\[
F_{sc} = -\frac{M_s}{M_m} \left( \dddot{B}_s \dddot{x}_s + \dddot{K}_s \dddot{x}_s + F_e + K_I \int_0^t \text{sign}(I_c(\tau)) \, d\tau \right)
\]

\[
+ \frac{M_s}{M_m} k_p \left( F_e^{\text{dy}} - k_f F_e^{\text{dy}} - \dddot{B}_m \dddot{x}_m - \dddot{K}_m x_m \right)
\]  

\[
+ F_e + B_s \dddot{x}_s - K_g \dddot{\theta}
\]  

(9)

where \( F_e^{\text{dy}} = F_e(t - 2T) \), the superscript \( \text{dy} \) stands for the round trip delay \( 2T \), \( K_g > 0 \), and \( \text{sign}() \) is the discontinuous signum function. Note that chattering is avoided due to the integral of \( \text{sign}(I_c) \), unlike [5]. High frequency signals are inconceivable in electromechanical systems due to their finite bandwidth; that is why second order sliding modes are a better choice, rather than first order sliding modes. The term \( K_g \dddot{\theta} \) has been added to achieve stability as will be seen afterwards. Also, notice that (8) requires acceleration measurement because \( I_c \) depends on acceleration. To deal with this inconvenience, acceleration and velocity are estimated, at master and slave sides, by means of linear observers, introduced in the next section.

### 4. Observer design

For completeness the well-known complete order linear observer [10] is presented in a general form for both master and slave robots, where subscript \( i = m \) denotes the master and \( i = s \) denotes the slave. Thus, consider the observable system (1) and (2) described in state space

\[
\dot{x}_i = A_i x_i + b_i u_i
\]

\[
y_i = C_i x_i
\]  

(10)

where \( x_i \in \mathbb{R}^n \) denotes the state of the system, \( y_i \) is the measured output, \( u_i \) is the control input, \( A_i \in \mathbb{R}^{n \times n}, b_i \in \mathbb{R}^n \) contain system’s parameters and \( C_i \in \mathbb{R}^m \) is used to select the \( m \) outputs (with \( n = 2, m = 1 \)). The state \( x_i \) of (10) can be estimated by means of linear observers. Complete order linear observers are designed for both master and slave systems as follows

\[
\dot{\hat{x}}_i = A_i \hat{x}_i + b_i u_i + l_i (y_i - \hat{y}_i)
\]  

(11)

\[
\dot{\hat{y}}_i = C_i \hat{x}_i
\]  

(12)

where \( \hat{x}_i \in \mathbb{R}^n \), \( \hat{y}_i \) denote the estimated state and the estimated output of master \( (i = m) \) and slave \( (i = s) \), \( l_i \in \mathbb{R}^n \) is the observer gain vector that can be chosen such that the polynomial characteristic of \((A_i - l_i C_i)\) is Hurwitz. The dynamics of the estimation error, \( e_i = x_i - \hat{x}_i \), is given by

\[
\dot{e}_i = (A_i - l_i C_i) e_i \triangleq \hat{A}_i e_i.
\]  

(13)

To prove that the estimation error tends to zero asymptotically let us consider (13) together with the following Lyapunov equation

\[
P_i \hat{A}_i + \hat{A}_i^T P_i = -Q_i
\]  

where \( P_i \) and \( Q_i \) are positive definite symmetric matrices, with the Lyapunov candidate function \( V_{oi} = \frac{1}{2} e_i^T P_i e_i \) whose time derivative is
\[ \dot{V}_{el} = 2e^T_i P \dot{e}_i = 2e^T_i P \hat{A} \dot{e}_i + e^T_i \hat{A} P \dot{e}_i = -e^T_i Q \dot{e}_i \leq -\lambda_{\min}(Q) \| \dot{e}_i \|^2. \]  

(14)

From (14), it is evident that the estimation error tends asymptotically to zero. Considering the latter, in our case, \( x_m, \dot{x}_m \) are the measured outputs and, from (1) and (2), \( u_m = F_h + F_{mc}, \dot{u}_m = F_{sc} - F_e \) are the control inputs for the observer. \( \ddot{x}_m, \dot{x}_m, \dot{x}_s \) can now be replaced by their respective estimated variables. In this manner, with an appropriate choice of \( L_i \) and assuming a good knowledge of the system’s parameters, \( \hat{x}_i \rightarrow x_i \) and \( \hat{\dot{x}}_i \rightarrow \dot{x}_i \) as \( t \rightarrow \infty \).

5. Stability analysis of the controller–observer scheme

In this section, the formal stability analysis of the closed-loop system when the state is replaced by its estimate is addressed.

5.1. Master closed-loop dynamics

Replacing the estimated state \( \hat{x}_m \) into (3), the following arises

\[ F_{mc} = -F_h + B_m \ddot{x}_m + \frac{M_m}{M} x_m \]

× \( (F_h - k_f \dot{F}_e - \ddot{B}_m \ddot{x}_m - \dddot{K}_m x_m) \).  

(15)

Substituting (15) into (1) renders the following closed-loop dynamics

\[ \dddot{M}_m x_m + \dddot{B}_m \ddot{x}_m + \dddot{K}_m x_m = F_h - k_f \dot{F}_e + \left( \dddot{B}_m - \dddot{M}_m B_m \right) e_{m2} \]  

(16)

where \( e_{m} = [e_{m1} \ e_{m2}]^T \). Notice that \([\dddot{B}_m - \dddot{M}_m B_m / M] e_{m2}\) is the estimation error introduced by the observer. Note that if \( e_{m2} \rightarrow 0 \), then (16) becomes (4), providing the desired impedance by properly \( M_m, B_m \) and \( K_m \).

5.2. Slave closed-loop dynamics

Consider the sliding surface \( I_s \) replaced with its estimate \( \hat{X}_s \)

\[ \dot{I}_s = M_s \dddot{x}_s + \dddot{B}_s \dddot{x}_s + \dddot{K}_s x_s + F_e = 0 \]  

(17)

where \( \dddot{x}_s := \dddot{x}_s - k_p x_{\dddot{m}}, \dddot{x}_s := \dddot{x}_s - k_p \dot{x}_{\dddot{m}} \) are the observed errors of acceleration and velocity, respectively. So, the extended error variable \( \dddot{\Omega} \), a function of \( \dddot{I}_s \), is the following

\[ \dddot{\Omega} = \frac{1}{M_s} \left[ \int_0^t \dddot{I}_s(\tau) d\tau + K_s \int_0^t \int_0^{\sigma} \text{sign}(\dddot{I}_s(\tau)) d\tau \right] \]

\[ = \dddot{x}_s + \frac{\dddot{B}_s}{M_s} \dddot{x}_s + \frac{1}{M_s} \left[ \dddot{K}_s \dddot{x}_s + F_e \right] \]

\[ + \frac{1}{M_s} \int_0^t \int_0^{\sigma} \text{sign}(\dddot{I}_s(\tau)) d\tau d\sigma. \]  

(18)

Similar to the master controller, \( \hat{x}_s \) is substituted into the slave controller (9) instead of \( x_s \) as follows

\[ F_{sc} = -\frac{M_s}{M} \left( \dddot{B}_s \dddot{x}_s + \dddot{K}_s \dddot{x}_s + F_e + K_s \int_0^t \text{sign}(\dddot{I}_s(\tau)) d\tau \right) \]

\[ + \frac{M_s}{M} \kappa_p \left( \dot{r}_h - k_f \dot{F}_e - \dddot{B}_m x_{\dddot{m}} - \dddot{K}_m x_{\dddot{m}} \right) \]

\[ + F_e + \dddot{B}_s \dddot{x}_s - K_s \dddot{\Omega}. \]  

(19)

Now, substituting (19) into (2), adding and subtracting \( \pm(M_s k_p x_{\dddot{m}} + \dddot{B}_s \dddot{x}_s + \dddot{B}_s k_p x_{\dddot{m}} + \dddot{B}_m x_{\dddot{m}}) \) and considering (16) we obtain the closed-loop system expressed in terms of the estimation errors

\[ M_s \left( \dddot{x}_s - k_p x_{\dddot{m}} \right) + \frac{M_s}{M_s} \dddot{B}_s \left( \dddot{x}_s - k_p x_{\dddot{m}} \right) + \frac{M_s}{M_s} \dddot{K}_s \dddot{x}_s \]

\[ + \frac{M_s}{M_s} \left[ F_e + K_s \int_0^t \text{sign}(\dddot{I}_s(\tau)) d\tau \right] + \frac{M_s}{M_s} \dddot{\Omega} \]

\[ = \left[ \frac{M_s}{M_s} \dddot{B}_s - B_s \right] e_{s2} + \left[ \frac{M_s}{M_s} \dddot{B}_m k_p - \frac{M_s}{M_s} \dddot{B}_s \kappa_p \right] e_{m2} \]

\[ - \frac{M_s}{M_s} \kappa_p \left( \dddot{B}_m - \dddot{M}_m B_m \right) e_{m2}. \]  

(20)

Multiplying (20) by \( \left( \dddot{M}_s / M_s \right) \) and adding and subtracting \( \pm(M_s \dddot{X}_s + \dddot{M}_s k_p x_{\dddot{m}} + \dddot{B}_s \dddot{x}_s + \dddot{B}_s k_p x_{\dddot{m}}) \) gives rise to

\[ \dddot{M}_s \left( \dddot{x}_s - k_p x_{\dddot{m}} \right) + \dddot{B}_s \left( \dddot{x}_s - k_p x_{\dddot{m}} \right) + \dddot{K}_s x_s \]

\[ + F_e + K_s \int_0^t \text{sign}(\dddot{I}_s(\tau)) d\tau \]

\[ = \dddot{M}_s \left( e_{obs1} - e_{obs2} \right) - \dddot{M}_s \dot{e}_{s2} + \dddot{M}_s k_p e_{m2} \]

\[ - \dddot{B}_s e_{s2} + \dddot{B}_s k_p e_{m2}. \]  

(21)

where \( e_i = [e_i \ e_{i2}]^T \) with \( e_{i2} = \dddot{x}_i - \dddot{x}_i, e_{m} = [e_{m1} \ e_{m2}]^T \) with \( e_{m2} = \dddot{x}_m - \dddot{x}_m, e_{i} = [\dot{e}_1 \ \dot{e}_{i2}]^T \) with \( \dot{e}_{i2} = \dddot{x}_i - \dddot{x}_i, e_{m} = [e_{m1} \ e_{m2}]^T \) with \( e_{m2} = \dddot{x}_m - \dddot{x}_m \) (i = m, s). Rewriting (21), we have

\[ \dddot{M}_s \dddot{x}_s + \dddot{B}_s \dddot{x}_s + \dddot{K}_s \dddot{x}_s + F_e + K_s \int_0^t \text{sign}(\dddot{I}_s(\tau)) d\tau \]

\[ = \dddot{M}_s \left( e_{obs1} - e_{obs2} \right) - \dddot{M}_s \dot{e}_{s2} + \dddot{M}_s k_p e_{m2} - \dddot{B}_s e_{s2} \]

\[ + \dddot{B}_s k_p e_{m2}. \]  

(22)

After multiplying (22) by \( 1/\dddot{M}_s \), this can be expressed in terms of \( \dddot{\Omega} \) as follows

\[ \dddot{\Omega} + \frac{K_s}{M_s} \dddot{\Omega} = v_1 e_1 + v_2 e_{m1} + v_3 \dddot{\Omega} + v_4 \dddot{x}_s + v_5 \dddot{x}_m \]  

(23)
where
\[
\begin{align*}
v_1 &= \begin{bmatrix} 0 & -B_s/M_s \end{bmatrix} \\
v_2 &= \begin{bmatrix} 0 & \tilde{B}_m k_p - \tilde{B}_s k_p/M_m \\
v_3 &= \begin{bmatrix} 0 & \tilde{B}_s k_p - k_p/M_m (\tilde{M}_m - \tilde{M}_m B_m) \\
v_4 &= [0 & -1] \\
v_5 &= [0 & k_p].
\end{align*}
\]

Then, we have
\[
\dot{\Omega} + \alpha \dot{\Omega} = a_1 e_s + a_2 e_m + a_3 e_m^{dy}
\] (24)
where \( \alpha = K_s/M_s, a_1 = v_1 + v_3 \tilde{A}_s, a_2 = v_2 \) and \( a_3 = v_3 + v_5 \tilde{A}_m \). Notice that if \( e_s, e_m \) and \( e_m^{dy} \) are zero, then \( \dot{\Omega} \rightarrow 0 \) and \( \dot{I}_z \rightarrow 0 \). Since the system (13) is time invariant, we also have
\[
e_m^{dy} = \tilde{A}_m e_m^{dy}.
\] (25)
Furthermore
\[
e_s = \tilde{A}_s e_s.
\]

Next, the following theorem is given.

**Theorem.** Consider the system (1) and (2) in closed-loop with the control laws (15) and (19) and the observer (11) and (12), then the different error dynamics of the system given by (13), (24) and (25), are all asymptotically stable if positive definite feedback gains \( K_s, K_l \) and \( Q_m, Q_s \) are properly chosen.

**Proof.** Consider the following Lyapunov candidate function
\[
V = \frac{1}{2} \dot{\Omega}^2 + e_s^T P_s e_s + e_m^T P_m e_m + e_m^{dyT} P_m e_m^{dy}
\]
whose time derivative along the solution of (13), (24) and (25) is
\[
\dot{V} = \dot{\Omega} \dot{\Omega} + 2 e_s^T P_s \dot{e}_s + 2 e_m^T P_m \dot{e}_m + 2 e_m^{dyT} P_m \dot{e}_m^{dy}
\]
\[
= -\alpha \dot{\Omega}^2 + \dot{\Omega} a_1 e_s + \dot{\Omega} a_2 e_m + \dot{\Omega} a_3 e_m^{dy}
\]
\[
- e_s^T Q_s e_s - e_m^T Q_m e_m - e_m^{dyT} Q_m e_m^{dy}
\]
\[
\leq -\alpha \dot{\Omega}^2 + \tilde{a}_1 |\dot{\Omega}| \| e_s \| + \tilde{a}_2 |\dot{\Omega}| \| e_m \| + \tilde{a}_3 |\dot{\Omega}| \| e_m^{dy} \|
\]
\[
- 2 \lambda_s \| e_s \|^2 - 2 \lambda_m \| e_m \|^2 - 2 \lambda_m \| e_m^{dy} \|^2
\] (26)
where \( \lambda_s = \frac{1}{2} \lambda \min (Q_s), \lambda_m = \frac{1}{2} \lambda \min (Q_m) \) and \( \tilde{a}_1 = \| a_1 \|, \tilde{a}_2 = \| a_2 \|, \tilde{a}_3 = \| a_3 \|. \) Since \( \alpha = K_s/M_s \) and \( K_s > 0 \)
can be chosen arbitrarily, consider
\[
\alpha = \delta + \frac{\tilde{a}_1^2}{4 \lambda_s} + \frac{\tilde{a}_2^2}{4 \lambda_m} + \frac{\tilde{a}_3^2}{4 \lambda_m}
\]
to obtain
\[
\dot{V} \leq -\delta \dot{\Omega}^2 - \lambda_s \| e_s \|^2 - \lambda_m \| e_m \|^2 - \lambda_m \| e_m^{dy} \|^2
\]
\[
- \frac{\tilde{a}_1^2}{4 \lambda_s} \dot{\Omega}^2 + \tilde{a}_1 |\dot{\Omega}| \| e_s \| - \lambda_s \| e_s \|^2
\]
\[
- \frac{\tilde{a}_2^2}{4 \lambda_m} \dot{\Omega}^2 + \tilde{a}_2 \dot{\Omega} \| e_m \| - \lambda_m \| e_m \|^2
\]
\[
- \frac{\tilde{a}_3^2}{4 \lambda_m} \dot{\Omega}^2 + \tilde{a}_3 \dot{\Omega} \| e_m^{dy} \| - \lambda_m \| e_m^{dy} \|^2.
\] (27)

Notice that
\[
- \frac{\tilde{a}_j}{4 \lambda_j} \dot{\Omega}^2 + \tilde{a}_j |\dot{\Omega}| \| e_i \| \lambda_i \| e_i \|^2
\]
\[
= - \left( \frac{\tilde{a}_j}{2 \sqrt{\lambda_j}} \dot{\Omega}^2 - \sqrt{\lambda_j} \| e_i \|^2 \right)^2
\]
with \( j = 1, 2, 3 \) and \( i = m, s \). Finally
\[
\dot{V} \leq -\delta \dot{\Omega}^2 - \lambda_s \| e_s \|^2 - \lambda_m \| e_m \|^2 - \lambda_m \| e_m^{dy} \|^2.
\]

So far, one can conclude that the system is globally asymptotically stable. Moreover, exponential stability of the system can be proved by making
\[
V = z^T \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & P_m & 0 & 0 \\
0 & 0 & 0 & P_m & 0 \\
0 & 0 & 0 & 0 & P_m \end{bmatrix} z
\]
where \( z = \begin{bmatrix} \dot{\Omega} & e_s^T & e_m^T & e_m^{dyT} \end{bmatrix} \). Let \( \lambda_1, \lambda_2 \) be positive constants such that
\[
\lambda_1 \| z \|^2 \leq V \leq \lambda_2 \| z \|^2 \Rightarrow \left\{ \begin{array}{l}
- \| z \|^2 \geq - \frac{V}{\lambda_1} \\
- \| z \|^2 \leq - \frac{V}{\lambda_2}.
\end{array} \right.
\]

If \( \dot{V} \leq 0 \), we have
\[
\lambda_1 \| z \|^2 \leq V \leq V(0) \leq \lambda_2 \| z(0) \|^2
\]
\[
\Rightarrow \| z \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} \| z(0) \|.
\]

But
\[
\dot{V} \leq -\varepsilon \| z \|^2, \quad \varepsilon = \min(\delta, \lambda_s, \lambda_m).
\]

Then
\[
\dot{V} \leq -\frac{\varepsilon}{\lambda_2} V \Rightarrow V(t) \leq V(0)e^{-\frac{\varepsilon}{\lambda_2} t}.
\]

Therefore
\[
\lambda_1 \| z \|^2 \leq \lambda_2 \| z(0) \|^2 e^{-\frac{\varepsilon}{\lambda_2} t}.
\] (28)

Solving for \( \| z \| \) in (28) we get
\[
\| z \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} \| z(0) \| e^{-\frac{\varepsilon}{\lambda_2} t},
\]
which proves that the system is exponentially stable. Considering the latter, from (23), \( \dot{\Omega} \) is bounded. By computing the derivative of (23), one concludes that \( |\dot{\Omega}| < \epsilon_1 \) for a positive constant \( \epsilon_1 \), because \( \dot{e}_s, \dot{e}_m \) and \( \dot{e}_m^{dy} \) are bounded.
Sliding mode condition. Now, to prove the existence of the sliding mode it is necessary to verify the sliding mode condition
\[ \dot{\hat{e}}_c \leq -\mu |\hat{e}_c| \]. Differentiating twice (18), we obtain
\[ \ddot{\hat{e}}_c + \kappa_1 \text{sign}(\hat{e}_c) = 0 \].

Multiplying the above equation by \( \hat{e}_c \) and applying norms, gives rise to
\[ \hat{e}_c \leq -\kappa_1 |\hat{e}_c| + \hat{e}_c \Omega |\hat{e}_c| \]
\[ \leq -(\kappa_1 - \epsilon_1)|\hat{e}_c| \]
\[ \leq -\mu |\hat{e}_c| \]
(29)
with \( \epsilon_1 = \epsilon_1 \hat{M}_s \). Thus, if \( \kappa_1 > \epsilon_1 \), then \( \mu > 0 \), and the sliding mode condition is established at \( \hat{e}_c = 0 \) at time \( t_i \leq \hat{L}_{(h)}/\mu \) [17].

Therefore, Eq. (29) leads to (17), that is exponential impedance tracking \( \hat{e}_c = 0 \) implies \( \hat{M}_s \hat{x}_s + \hat{B}_s \hat{x}_s + \hat{K}_s \hat{x}_s = -F_e \). \( \square \)

6. Discussion

6.1. Smooth controller

High order sliding modes have emerged to solve the problem of chattering, which is induced by first order sliding modes. Along with keeping the main advantages of the original approach, at the same time they totally remove the chattering effect and provide for even higher accuracy in realization [15].

6.2. Communication channel stability

As human operator and environment are passive (by assumption), and controllers at master and slave render stable systems, let us analyze the communication block. A communication block can be analyzed as a two-port network, which relates flows and efforts from one side to another and vice versa. The relationship between network flows \((\hat{x}_m, \hat{x}_s)\) and efforts \((F_h, F_e)\) can be described by means of a hybrid matrix [8]. This matrix is obtained using the impedance models of master and slave, (4) and (5), respectively

\[
\begin{bmatrix}
F_h(s) \\
\hat{x}_s(s)
\end{bmatrix} =
\begin{bmatrix}
h_{11}(s) & h_{12}(s) \\
h_{21}(s) & h_{22}(s)
\end{bmatrix}
\begin{bmatrix}
\hat{x}_m(s) \\
-F_e(s)
\end{bmatrix}
\]

where
\[
h_{11} = \frac{F_h(s)}{\hat{x}_m(s)}|_{F_e=0} = \hat{M}_m s + \hat{B}_m + \frac{\hat{K}_m}{s}
\]
\[
h_{12} = \frac{F_h(s)}{F_e(s)}|_{\hat{x}_m=0} = -k_f e^{-sT}
\]
\[
h_{21} = \frac{\hat{x}_s(s)}{\hat{x}_m(s)}|_{F_e=0} = k_p e^{-sT}
\]
\[
h_{22} = \frac{\hat{x}_s(s)}{F_e(s)}|_{\hat{x}_m=0} = \frac{s}{\hat{M}_s s + \hat{B}_s + \hat{K}_s}.
\]

Llewellyn’s stability criteria [1] provides necessary and sufficient conditions for absolute stability. Absolute stability is a less conservative tool than passivity. Therefore, absolute stability does not imply passivity, but on the contrary passivity implies absolute stability. Then a two-port network is absolutely stable if and only if
(a) \( h_{11} \) and \( h_{22} \) has no poles in the right half plane,
(b) Any poles of \( h_{11} \) and \( h_{22} \) on the imaginary axis are simple with real and positive residues,
(c) For all real values of \( \omega \),
\[
\text{Re} h_{11} \geq 0, \quad \text{Re} h_{22} \geq 0,
\]
\[
2 \text{Re} h_{11} \text{Re} h_{22} - \text{Re}(h_{22} h_{11}) - |h_{12} h_{21}| \geq 0
\]

For the given two-port network, conditions (a) and (b), together with the first and second conditions in (c) are satisfied with positive impedance parameters. The last of condition (c) can be expressed by
\[
[\cos(2\omega T) - 1] k_p k_f + \frac{2 \hat{B}_m \hat{B}_s \omega^2}{(\hat{K}_s - \hat{M}_s \omega^2)^2 + (\hat{B}_s \omega)^2} \geq 0.
\]

If the design parameters satisfy (30), the teleoperation system will be stable for any set of passive human operators and environments [9].

6.3. How to tune the master controller

Master impedance parameters are tuned such that a comfortable operation can be achieved. A higher impedance yields a heavier and more sluggish performance, while a lower impedance makes the master teleoperator weightless and more difficult to drive. Then, tuning of the controller is made heuristically provided that the choosing of \( \hat{B}_m \) contributes to the slave’s stability, according to (30).

6.4. How to tune the slave controller

First of all, impedance parameters must be chosen such that (30) is fulfilled. Similar to the master controller tuning, slave impedance tuning is made heuristically provided that a wide range of frequencies are allowed when evaluating (30). Additional stability and improvement of the performance, as the stability proof suggests, is to set an arbitrary \( K_{\delta} > 0 \) such that all error dynamics remain bounded. Increase \( K_{\delta} \) until acceptable boundedness of \( \hat{\Omega} \) is achieved. Then, increase \( K_1 \) until the sliding mode arises \( (\dot{\hat{e}}_c \approx 0) \). Notice that \( K_1 \) is not a high gain result since a larger \( K_1 \) does not mean a larger domain of stability. Finally, a faster and better tracking position can be accomplished by increasing \( K_{\delta} \) and \( K_{\delta} \), respectively.

7. Experimental results

Performance of the proposed controller is verified through experiments with a 1-dof bilateral teleoperation system, shown in Fig. 1. The system is at the Laboratory of Robotics and Manufacturing of the Mechatronics Division.

7.1. Experimental setup

The experimental system consists of a one-axis master handle and a one-axis slave link driven by direct-drive Yaskawa AC servomotors SGM-04U3B4L and SGM-08A314,
respectively, with 2048 pulse integrated encoders. Each AC servomotor is controlled by its own servopack, which can be programmed by means of a Digital Operator. In this case, the servopacks are programmed in torque mode. To measure the master force, a load cell is attached at the end effector, while a JR3 force sensor 67M25A-I40-200N is mounted at the end effector of the slave robot. A custom-made conditioning circuit is used to amplify and transfer the load cell signal to the PC. A DSP for PCI Bus is in charge of reading and processing the signals from the JR3 force sensor. A Sensoray 626 I/O card is used to read encoder signals (with quadrature included), and the conditioned load cell signal via the A/D channel, while torque commands are transferred to the servopacks through the D/A channels. The control system is running in real-time with a sampling rate of 1 kHz on a PC over Linux-RTAI operating system.

7.1.1. System parameters
Master and slave are different in length and weight, so the identified parameters are shown in Table 1.

7.1.2. Impedance parameters
The desired values for the impedance models are shown in Table 2. Impedance parameters for master and slave are chosen according to Discussion 6.3 and Discussion 6.4, respectively.

7.1.3. Controller gains and scaling factors
The slave controller gains are set to $K_g = 25$, $K_i = 0.1$, and the scaling factors are set to $k_p = 1$, $k_f = 0.9$.

7.1.4. Linear observers
The observers have been designed such that the characteristic polynomial of $(A_i - I/C_i)$, for $i = m, s$, have eigenvalues at $[-350, -450]$ for the master, and $[-250, -350]$ for the slave. Thus, $I_m = [800 157 470]^T$ and $I_s = [600 87 346]^T$, according to Ackermann’s formula [13].

7.1.5. Desired task
The human operator pulls and pushes the master lever so that the slave makes intermittent contact with the environment, which is a sponge with a steel support. The communication channel is emulated with a FIFO memory buffer that introduces bilateral time delays. The data to be exchanged between the master and the slave are stored in the buffer instead of sending them immediately to their destination. Data are released only at the moment when the buffer is full. Thus, the size of the time delay depends on the size of the buffer.

7.1.6. Results
Two experiments are performed, the first one with a time delay $T = 1.0$ s and the second one with a time delay $T = 0.6$ s, which means a round trip time (RTT) of 2 s and 1.2 s, respectively. Fig. 2 shows the position tracking error at the slave side. As expected, $\dot{F} = 0$ holds. In free motion the scaled position, velocity and acceleration tend to the desired ones, instead, in constrained motion a trade-off between force and position tracking is established. Evidently, the proposed

---

Table 1
<table>
<thead>
<tr>
<th>Par</th>
<th>Mass</th>
<th>Length</th>
<th>Damping</th>
<th>Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slave</td>
<td>7.1956</td>
<td>0.4</td>
<td>0.0743</td>
<td>0.2879</td>
</tr>
<tr>
<td>Master</td>
<td>1.8941</td>
<td>0.27</td>
<td>0.001367</td>
<td>0.0363</td>
</tr>
<tr>
<td>Units</td>
<td>kg</td>
<td>m</td>
<td>N m s</td>
<td>kg m²</td>
</tr>
</tbody>
</table>

Table 2
<table>
<thead>
<tr>
<th>Par (master)</th>
<th>Value</th>
<th>Par (slave)</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{M}_m$</td>
<td>1.8641</td>
<td>$\bar{M}_s$</td>
<td>0.3</td>
<td>kg m²</td>
</tr>
<tr>
<td>$\bar{B}_m$</td>
<td>1.5</td>
<td>$\bar{B}_s$</td>
<td>0.5</td>
<td>N m s</td>
</tr>
<tr>
<td>$\bar{K}_m$</td>
<td>0.0</td>
<td>$\bar{K}_s$</td>
<td>15</td>
<td>N m</td>
</tr>
</tbody>
</table>

---

Fig. 1. Teleoperation experimental set up.
Fig. 2. Stable intermittent contact with a time delay of $T = 1 \text{s}$.

Fig. 3. Human operator and environment torques with a time delay of $T = 1 \text{s}$. Although the proposed scheme does not guarantee perfect transparency, even so the $F_e \approx F_{dh}$.

Although the proposed scheme does not achieve perfect transparency, but gives a good idea about the remote environment, even in the presence of substantial time delay. Fig. 3 depicts the human operator and slave torques. Notice that forces measured at the end effector of each robot are converted to torques. Fig. 4 displays the smooth control effort. Fig. 5 shows the behavior of the extended error variable. Finally, Fig. 6 depicts the performance of the system when there exists a time delay of $T = 0.6 \text{s}$, that is, a $RTT = 1.2 \text{s}$. Obviously, the slave tracks the master commands and maintains a stable intermittent contact with the environment.

8. Conclusions

A simple, and yet straightforward approach based on second order sliding mode-impedance control has been synthesized, along with complete order observers, for linear teleoperators. It has been proved that this approach is suitable when there is a lack of velocity and acceleration sensors, and when the communication time is relatively large and unknown. The stability analysis shows that the observer-based controller scheme guarantees chattering-free control inputs and exponential tracking impedance. Closed loop error dynamics gives rise to a desired impedance model, which establishes tracking in free motion, and bounded tracking, modulated by the environmental forces, in constrained motion. Experimental results validate the proposed scheme.
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References