Abstract—Spatial subchannels in multiuser Multiple-Input Multiple-Output/Orthogonal Frequency-Division Multiplexing (MIMO/OFDM) systems are invested. With the goal of maximizing the overall system throughput, we derive the criterion of subcarrier allocation and introduce an adaptive resource allocation algorithm. Furthermore, a time-frequency blockwise loading design which may decrease the computational complexity is suggested. The simulation results show that the algorithm can achieve good performance and high transmission rate efficiently.

Keywords— MIMO/OFDM; adaptive resource allocation; spatial subchannel; time-frequency block

I. INTRODUCTION

The next-generation mobile communication systems are required to provide high-data-rate and high quality transmission over hostile radio channels. It is widely understood that deploying adaptive resource allocation technique can improve the performance and capacity significantly.

OFDM has become a popular technique for transmission of signals over wireless channels, which can convert a frequency-selective channel into a parallel collection of frequency flat subchannels[1]. In addition, MIMO systems have been acknowledged as one of the most promising techniques to achieve dramatic improvement in physical-layer performance [2, 3].

So far, several papers have dealt with adaptive resource allocation for MIMO/OFDM systems. In [4], the eigenvalue decomposition method (EVD), based on the joint optimal transmit and receive antenna weights, was proposed to maximize the Signal-to-Noise Ratio (SNR). In [5], the joint optimal beamforming and power allocation design was proposed to further improve the performance of MIMO/OFDM systems. All of the previous work only dealt with the spatial subchannel related to the largest eigenvalues.

In this paper, we consider all the spatial subchannels, develop a criterion which is used to allocate a subcarrier among users, and present an adaptive spatial subchannel allocation algorithm for multiuser MIMO/OFDM systems. Our objective is to maximize the overall data throughput of the system, while guaranteeing prescribed error performance, under the constraint of fixed transmit-power.

Throughout the paper, we will adopt the following notational conventions: Bold upper and lower case letters denote matrices and column vectors, respectively; \([x]\) denotes the maximum integer which is less than or equal to \(x\); \((\cdot)^T\), \((\cdot)^\dagger\), \((\cdot)^H\) denote the complex conjugate, transpose and conjugate transpose, respectively.

II. SYSTEM MODEL

In this paper, a downlink adaptive multiuser MIMO/OFDM system equipped with N subcarriers, \(M_T\) transmit-antennas at base station and \(M_R\) receive-antennas for each of K users is considered. The block diagram of our system is shown in Figure 1.

Let \(H_{k,m}\) be the channel frequency response matrix of the \(k\)-th user’s at \(m\)-th subcarrier. It was indicated in [3] that a number of parallel channels can be constructed from the channel matrix \(H_{k,m}\) by properly configuring the antenna weights. This is because \(H_{k,m}\) can be decomposed into

\[
H_{k,m} = U_{k,m} S_{k,m} V_{k,m}^H = \sum_{i=1}^{\text{rank}(H_{k,m})} u_{k,m}^i s_{k,m}^i (v_{k,m}^i)^H \tag{1}
\]

Where \(u_{k,m}^i, v_{k,m}^i, s_{k,m}^i\) are the left and right singular vectors, with \(s_{k,m}^i\) denoting the singular values that are arranged in a descending order. Denote the transmitting beamforming vector and receiving weight vector for the \(i\)-th spatial subchannel of the \(m\)-th subcarrier as \(W_{k,m}^i\) and \(W_{k,m}^i\), respectively. Then, \(W_{k,m}^i = v_{k,m}^i\) and \(W_{k,m}^i = u_{k,m}^i\), \(k \in \{1, 2, \cdots, K\}\), and each subcarrier can be divided into multiple parallel spatial subchannels.
III. ADAPTIVE MULTIUSER RESOURCE ALLOCATION

With the goal of maximizing the overall data throughput required to meet the target BER, we develop an OFDM subcarrier allocation approach based on perfect channel information and present an adaptive spatial subchannel allocation algorithm for multiuser MIMO/OFDM systems.

A. Multiuser subcarrier allocation scheme

Assuming that the m-th subcarrier was allocated to the k-th user, the optimization problem of spatial subchannels on m-th subcarrier can be described as follow:

\[
\max \sum_{i=1}^{J_{k,m}} b_{k,m}^i \quad (2)
\]

Subject to

\[
\begin{align*}
BER_{k,m}^i & \leq BER_{\text{target}} \\
\sum_{i=1}^{J_{k,m}} P_{k,m}^i & = P_{k,m} \\
J_{k,m} & \leq \text{rank}\left(\left[H_{k,m}\right]^H H_{k,m}\right) 
\end{align*}
\]

(3)

Where \( H_{k,m} \) is channel matrix, \( J_{k,m} \) is the number of spatial subchannels available, \( b_{k,m}^i \) is the number of bits in i-th spatial subchannel, \( P_{k,m}^i \) is the transmit power, and \( BER_{\text{target}} \) is the target BER.

If M-ary quadrature amplitude modulation (MQAM) is employed, the BER for an AWGN channel is given by[6]

\[
BER_{k,m}^i = 0.2\exp\left(-\frac{1.5P_{k,m}^i \left(s_{k,m}^i\right)^2}{\sigma^2 (2^{\frac{N}{2}} - 1)}\right) 
\]

In the moderate to high SNR case, we may ignore the -1 term in equation (4). Not considering the integer constraint of \( b_{k,m}^i \), we can get the solution of the problem of (2) using Lagrange multiplier methods. Then, the optimal allocation of power to spatial subchannel of the m-th subcarrier can be written as

\[
P_{k,m}^i = P_{k,m} / J_{k,m} 
\]

(5)

To simplify the design, let the number of spatial subchannels be \( J_{k,m} \) be the minimum of \( M_{k}, M_{r} \).

For high SNR, Equation (5) is a good approximation to real solution while have a certain degree of error for low SNR. Then, the bit allocation is

\[
b_{k,m}^i = \log_2 \left(1 + \frac{3P_{k,m}^i \left(s_{k,m}^i\right)^2}{2\sigma^2 J_{k,m} \ln(5R_{\text{target}})}\right) 
\]

(6)

Define \( \xi_{k,m} = -\frac{3P_{k,m}^i}{2\sigma^2 J_{k,m} \ln(5R_{\text{target}})} \).

Equation (6) can be rewritten as

\[
b_{k,m}^i = \log_2 \left(1 + \xi_{k,m} \left(s_{k,m}^i\right)^2\right) 
\]

(7)

The bit rate on m-th subcarrier is

\[
b_{k,m}^i = \sum_{i=1}^{J_{k,m}} b_{k,m}^i = \log_2 \left(\det\left(I_{M_r} + \xi_{k,m} \left(H_{k,m}\right)^H H_{k,m}\right)\right) 
\]

(8)

Then, the multiuser subcarrier allocation criterion is

\[
k = \arg\max_{k \in \{1, \ldots, K\}} \left\{\det\left(I_{M_r} + \xi_{k,m} \left(H_{k,m}\right)^H H_{k,m}\right)\right\} 
\]

(9)

B. Multiuser MIMO/OFDM system Spatial SubChannel Allocation (SSCA)

Introduce Boolean variable \( C_{k,m} \) to indicate whether allocate m-th subcarrier to user k or not. The objective of adaptive resource allocation for multiuser is to achieve maximum system throughput, then the corresponding optimization problem is described as follows:

\[
\max \sum_{m=0}^{N-1} \sum_{k=1}^{K} C_{k,m} b_{k,m} 
\]

(10)

Subject to

\[
\sum_{m=0}^{N-1} \sum_{k=1}^{K} C_{k,m} P_{k,m} \leq P \\
BER_{k,m} \leq BER_{\text{target}} 
\]

(11)

The optimization problem can be decomposed into two steps: first, assign each subcarrier to the user which having maximum transmission rate, and then allocate the overall...
power to all subcarriers; second, distribute each subcarrier’s power to its spatial subchannels and load bits.

1) Multiuser Subcarrier and Power Allocation

If the m-th subcarrier will be allocated, then, let each users’ power on m-th subcarrier, \( P_{i,m} \), \( i = 1, \cdots, K \), be a certain constant value, and then \( \xi_{i,m} \) is constant. According to the allocation criteria of Equation (9), the specific allocation process is

\[
C_{k,m} = \left\{ \begin{array}{ll}
1 & \text{k = argmax } \left\{ \det \left( I_{M_r} + \xi_{i,m} \left( H_{i,m} \right)^H H_{i,m} \right) \right\} \\
0 & \text{others}
\end{array} \right.
\]

(12)

After allocate all subcarriers to users, we distribute overall power to each subcarrier. As each subcarrier is only allocated to one user, we don’t consider the impact of user k after completion of subcarrier allocation, i.e.,

\[
b_m = C_{k,m} b_k, \quad P_m = C_{k,m} P_k, \quad m = 1, \cdots, N
\]

Then

\[
b_m = \log_2 \left( \det \left( I_{M_r} + \xi_{i,m} \left( H_{i,m} \right)^H H_{i,m} \right) \right)
\]

(13)

Equation (13) can be rewritten as

\[
b_m = f(P_m)
\]

(14)

Not considering the integer constraint of \( b_m \), we divide total power into \( N_p \) parts (i.e. \( \Delta P = P/N_p \)), let the gradient of \( b_m \) be \( \varphi = \Delta b/\Delta P \), select the subcarrier which has the maximum gradient, and increase power \( \Delta P \) on it. Then, repeat this process until the power allocation meets the target value \( P \).

Because \( f(P_m) \) is a concave function, the allocation algorithm above can achieve the optimal allocation of power to all subcarriers. This algorithm is a greedy algorithm with high computation complexity. A simple suboptimal solution is the average power allocation.

\[
P_m = P/N, \quad m = 0, \cdots, N-1
\]

(15)

2) Power and Bit Allocation to Spatial Subchannels of Single Subcarrier

According to the result of Equation (5), the power is distributed to spatial subchannels on each subcarrier, and the bits are allocated according to the result of Equation (6), at the same time, the integer-bit restriction is under consideration.

The detailed allocation process is described as follows: (remove the user subscript \( k \) for simplification)

a) Initialization

\[
H_m = U_m S_m V_m^H = \sum_{i = 1}^{\text{rank}(H_m)} u_i^j s_i^j v_i^j^H
\]

\[
J_m = \text{rank}(H_m), \quad s_i^j \geq s_i^{j+1} \geq \cdots \geq s_i^N > 0
\]

\[
P_m = P_m/J_m, \quad i = 1, \cdots, J_m
\]

num = \( J_m \)

b) Recursion allocation

(i) for \( i = 1 \) to num

\[
R' = \log_2 \left( 1 - \frac{3P_m (s_i^j)^2}{2\sigma^2 \ln(5R_{\text{target}})} \right)
\]

\[
b_i^j = \left\lfloor R' \right\rfloor
\]

end

(ii) for \( i = 1 \) to num

if \( b_i^j \leq 0 \) then

\[
J_m = J_m - 1
\]

end

if num > \( J_m \) \( \text{and } J_m \geq 1 \), then

\[
P_i^j = P_m / J_m, \quad \text{num} = J_m \quad \text{return to (i)}
\]

else end

Through these two steps, we allocate each subcarrier to the user, finish the allocation of power to all subcarriers, and allocate the bits to spatial subchannels of every subcarrier.

C. The simple algorithm based on time-frequency block

The type of fading experienced by a signal propagating through a mobile radio channel depends on the nature of the transmitted signal with respect to the characteristics of the channel. Coherence bandwidth is used to characterize the channel in the frequency domain, and coherence time is the parameter which describes the time varying nature of the channel. If the channel coherence bandwidth is greater than the bandwidth of the transmitted signal and the coherence time is greater than the symbol period of the transmitted signal, then the received signal will undergo flat fading. The Simple Blockwise Loading Algorithm (SBLA) in OFDM systems is proposed by Rainer Grunheid et al.[7], where the adjacent subcarriers are assigned to one block. We extend this idea to both time and frequency domain, and design the time-frequency block according to coherence time and coherence bandwidth. Then, with a suitable time-frequency transmission signal block, we can decrease the computation complexity significantly with a little degradation in performance.

Let one time-frequency block include nt OFDM symbol durations and nb subcarriers. If the OFDM system has N subcarriers, the number of allocable blocks for each process is \( \varphi = N/n_b \). Let N can be divided exactly for simplicity.

Each time-frequency block is only allocated to one user during the resources allocation process, so a similar Boolean variable \( B_{k,i} \) is introduced to indicate whether allocate i-th block to user k or not. Assuming the i-th time-frequency block is allocated to user k, the optimization problem is defined as

\[
\max \sum_{i=0}^{\varphi-1} \sum_{k=1}^{K} B_{k,i} \tilde{b}_{k,i}
\]

s.t.
Where $P_B$ is the whole transmit power, $b_{k,i}$ is the number of bits transmitted in the $i$-th time-frequency block, $\bar{p}_{k,i}$ is the transmit power, and $BER_{\text{target}}$ is the target BER.

The adaptive resource allocation process based on time-frequency block is similar to which based on subcarrier, but the former’s computation is only about $1/(n_in_b)$ of the latter’s with a little performance losses.

IV. SIMULATION RESULTS

A. Simulation setup

In our simulation systems, we use the IMT-2000 Vehicular Model A channel model suggested by ITU-R M.1225. It is a six-path Rayleigh fading model. The detailed parameters are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I. SIMULATION ENVIRONMENTS AND PARAMETERS</th>
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<tbody>
<tr>
<td>Carrier frequency</td>
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<tr>
<td>Bandwidth</td>
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<tr>
<td>Mobile velocity</td>
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<tr>
<td>Multipath delay</td>
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<td>Path 1</td>
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<td>Path 2</td>
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<td>Path 3</td>
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<td>Path 5</td>
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<td>Path 6</td>
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Let transmit antenna number $M_T=4$ and receive antenna number $M_R=4$. The bandwidth is divided into $N=1024$ tones and the cycle prefix length $CP=216$ for OFDM system, so the OFDM frame $T_f=51.2\mu s$, and the bandwidth of subband is $\Delta f=19.5\ kHz$. The Vehicular Model A channel coherence bandwidth $B_c=0.54\ MHz$ which is far large than the bandwidth of OFDM subband. Then, the frequency-selective channel is converted to frequency-flat subchannels.

Denote the average transmit SNR on the subcarriers as $P/(N\sigma^2)$, and the system throughput as bits/OFDM symbol.

B. Simulation results of the adaptive resource allocation algorithm based on single subcarrier

Beamforming technique often delivers the message on the spatial subchannel with the greatest eigenvalue, while the proposed scheme deploys all the subchannels available. Figure 2 shows the system data throughput under these two different methods.

![Figure 2. System throughput comparison of Beamforming and proposed scheme](image)

Set the target BER $R_{\text{target}}=10^{-6}$, and the number of users be one to eliminate the impact of multiuser. In Figure 2, the dotted line shows the system throughput when using average allocation of power to subcarriers, and the solid line shows the case of optimal power allocation discussed in chapter III.B.1). The letters ‘P-AV’ and ‘P-OP’ denote the average and optimal power allocation respectively. From figure 2, we can see that the system throughput of the proposed scheme is higher than the beamforming method at high SNR, and converse at low SNR. The cause is that we simplify the BER formula of M-QAM modulation during the optimal planning, and this may lead to large error at high SNR and little error at low SNR.

![Figure 3. System throughput under different number of users](image)

Figure 3 plots the system throughput variance of the optimal power distribution on subcarriers with proposed algorithm under different number of users. The target BER constraint are $R_{\text{target}}=10^{-6}$ and the number of users is 1, 4, 8, and 16.

From Figure 3, we can see that with the increase of the user number, the system throughput increase. This is due to the multiuser diversity. The diversity gain is achieved from the fact that with many users who experience independent fading, there is a high possibility that a user has good channel condition, and the performance is better with the increasing of the numbers of users.
users. On the other hand, as the number of users increases, the system throughput is getting closer to the highest point, and the increase step size is getting smaller.

C. Simulation results of the adaptive resource allocation algorithm based on time-frequency block

In the following simulation, we adopt the adaptive resource allocation algorithm with optimal power allocation discussed in chapter III.B.1), and the number of users is 4.

Define the basic time-frequency block unit as one OFDM symbol period & one subcarrier, then, one time-frequency block include \( n_t \times n_b \) basic block unit. Suppose that the i-th time-frequency block is allocated to user k, and the channel matrix of basic block unit is \( H_{k,m} \). In one time-frequency block, the channel matrix can be consider constant, so the i-th block’s channel matrix \( H_{k,i} \) is

\[
H_{k,i} = \frac{1}{n_t \times n_b} \sum_{m=1}^{n_b} H_{k,m}
\]  

(17)

Figure 4. System throughput under time-frequency block design

Figure 4 shows the system throughput when using time-frequency block. The target BER constraint is \( \text{BER} = 10^{-6} \). The parameters in time and frequency domain are \( n_t \& n_b = 1 & 1, 20 & 10, 60 & 40 \) (one OFDM symbol period & one subcarrier) respectively.

From Figure 4, we can see that with the increase of the time-frequency block size, the system throughput decrease gradually. The cause is that when using the block as allocation unit, the process step increases, the adaptive adjustment process is more ‘rough’. At the same time, the block size mainly affects the accuracy of block channel matrix which obtained from formula (17). In our proposed algorithm, this may lead to worse BER performance, and not affect the allocation process directly, so, the system throughput decrease is not much lower.

Fig 5 shows the BER when using time-frequency block. In time domain, the channel coherence time is about \( T_c / T_t = 45 \), and in the frequency domain, the channel coherence bandwidth is about \( B_c / \Delta f = 27 \). So \( n_t \& n_b = 40 & 20 \) is the turning point of system BER performance. From Figure 7, we can see that when the block size is less than the turning point, the degradation of BER performance is little, however, the performance worsen when exceed the turning point. So it is best to choose block size near the turning point, and guarantee some performance margin. For example, in this simulation, the suitable block size is \( n_t \& n_b = 20 & 10 \).

V. CONCLUSION

In this paper, we have present a practical resource allocation algorithm based spatial subchannel over multiuser MIMO/OFDM systems. We showed, through theoretical analysis and computer simulation, that under the system BER constraint, our algorithm achieve high system throughput performance which is nearly as good as the optimal solution, especially at high SNR.

REFERENCES