An inventory Model for deteriorating items with Weibull Deterioration with Time Dependent Demand and Shortages

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Abstract

This paper deals with in developing an inventory model for deteriorating items, the rate of deterioration follow the Weibull distribution with two parameters. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The numerical example is given to illustrate the model developed. The model is solved analytically by maximizing the total profit.

Keywords: Inventory, EOQ Model, deteriorating items, Weibull distribution of two parameters, shortages, linear demand dependent of time.

Introduction

Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with shortages have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs from depletion by direct spoilage while in store.

Ghare and Schrader¹ developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Shah and Jaiswal², Aggarwal³, Dave and Patel⁴. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip ⁵, Mishra⁶ and Deb and Chaudhuri⁷. Some of the significant recent work in this field have been done by Chung and Ting⁸, Fujiwara⁹, Hariga and Benkherouf¹⁰, Wec¹¹, Jalan et al.¹², Chakraborty and Chaudhuri¹³, Giri and Chaudhuri¹⁴, Chakraborty, et al.¹⁵ and Jalan and Chaudhuri¹⁶. Structural properties of an inventory system with deterioration and trended demand. Burwell¹⁷ developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et al.¹⁸ and inventory model with price and time dependent demand was developed by you¹⁹.

In general holding cost is assumed to be known and constant. But in realistic condition holding cost may not Goh²⁰ considered various functions to describe holding cost. In this paper we developed an EOQ Model for deteriorating items with deterioration rate, where deterioration rate follows two-parameters Weibull distribution and demand is consider as linear function of time. In this model shortages are completely back logged.

Assumptions and Notations

The fundamental assumptions are used to develop the model. The demand rate is dependent on time t. The demand rate is D(t) = (a+bt) or b(t)

The deterioration rate is proportional to time. The cycle length is T. The ordering quantity is q. The ordering cost is A. The selling price per unit item is s. The deterioration cost per unit item per unit time cost C₁. and the deterioration rate is proportional to time.

The inventory holding cost per unit item per unit time is h. C₁ is the shortage cost per unit item per unit time.

The deterioration of time as follows by Weibull parameter (two) distribution 0(t) = αβt^(β-1)

Where 0<α< 1 is the scale parameter and β >0 is the shape parameter.

Formulation and Solution

The length of the cycle is T. during time t₁ inventory is depleted due to deterioration and demand of the items. At the time t₁ the inventor level becomes zero and shortages occurring in the period (t₁, T) which is completely backlogged. Let I(t) be the inventory level at time t (0≤ t < T).

The differential equation Can be defined when the instantaneous state over (O,T) are given by

\[
\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -bt \quad O \leq t \leq t₁
\]

(1)

\[
\frac{dI(t)}{dt} = -b(t) \quad t₁ \leq t \leq T
\]

(2)

With I (t₁) = 0 at t = t₁

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From equation (1) we get

\[ I(t) = b \left[ 1 - \frac{t}{T} \right] \left[ \frac{t}{T} \left( t^2 - t^2 \right) + \frac{\alpha}{\beta + 2} \left( t_1^{(\beta + 2)} - t^{(\beta + 2)} \right) \right] \quad (3) \]

From equation (2) we get

\[ I(t) = \frac{b}{2} \left[ t_1^2 - t^2 \right] \quad t_1 \leq t \leq T \]

(4)

The holding cost during the time period 0 to \( t_1 \)

\[ H = \int_0^{t_1} I(t) \, dt \]

The total holding cost during the time period 0 to \( t_1 \)

\[ H = h \int_0^{t_1} b(1-\alpha \cdot t^{\beta}) \left[ \frac{1}{2} \left( t_1^2 - t^2 \right) + \frac{\alpha}{\beta + 2} \left( t_1^{(\beta + 2)} - t^{(\beta + 2)} \right) \right] \, dt \]

Now holding cost will be

\[ H = bh \left[ \frac{2t_1^2}{6} - \frac{\alpha}{2(\beta + 1)} \left( t_1^{(\beta + 3)} - \alpha t_1^{(\beta + 3)} \right) \right] - \frac{\alpha}{2(\beta + 1)} \left( t_1^{(\beta + 3)} - t^{(\beta + 3)} \right) \]

\[ H = bh \left[ \frac{2t_1^2}{6} + \frac{\alpha}{2(\beta + 1)} \left( t_1^{(\beta + 3)} \right) \right] - \frac{\alpha}{2(\beta + 1)} \left( t_1^{(\beta + 3)} - t^{(\beta + 3)} \right) \]

(5)

The total holding cost during the time period 0 to \( t_1 \) is given by

\[ D_T = C_{D_1} \frac{T}{t_1} \left( \frac{T}{t_1} \right) \, dt \]

\[ D_T = C_{D_1} \frac{T}{t_1} \left( \frac{T}{t_1} - \frac{(\beta - 1)}{2} \right) \left( \frac{T}{t_1} \right) \, dt \]

\[ D_T = C_{D_1} \frac{T}{t_1} \left( \frac{t_1^{(\beta + 3)}}{2(\beta + 1)} \right) \left( \frac{T}{t_1} \right) \, dt \]

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(6)

Now the total shortage cost during the time period \( t_1 \) to \( T \) is given by

\[ Sh = -C_{D_1} \frac{T}{t_1} \left( \frac{T}{t_1} - t^2 \right) \, dt \]

\[ Sh = -C_{D_1} \frac{T}{t_1} \left( \frac{T}{t_1} - t^2 \right) \, dt \]

(7)

From Equation (5), (6) and (7) the total profit per unit time can be defined

\[ P(T, t_1) = s(\beta h) - \frac{1}{T} \left[ A + h \left( \frac{t_1^2}{3} - \frac{t_1^{(\beta + 3)}}{(\beta + 3)} \right) \left( \frac{T}{t_1} \right) - \frac{\alpha}{(\beta + 3)} \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) - \frac{\alpha}{(\beta + 3)} \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \right] \]

(8)

Our main objective to maximize the profit function \( P(T, t_1) \), the necessary condition for maximize the profit are

\[ \frac{\partial P(T, t_1)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial P(T, t_1)}{\partial t_1} = 0 \]

\[ - \frac{1}{T} \left[ A + bh \left( \frac{t_1^2}{3} - \frac{t_1^{(\beta + 3)}}{(\beta + 3)} \right) \left( \frac{T}{t_1} \right) - \frac{\alpha}{(\beta + 3)} \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \right] = 0 \]

\[ C_{D_1} \frac{T}{t_1} \left( \frac{t_1^{(\beta + 3)}}{2(\beta + 1)} \right) \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) = 0 \]

(9)

And

\[ - \frac{1}{T} \left[ bh \left( \frac{t_1^2}{3} - \frac{t_1^{(\beta + 3)}}{(\beta + 3)} \right) \left( \frac{T}{t_1} \right) - \frac{\alpha}{(\beta + 3)} \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \left( \frac{T}{t_1} \right) \right] = 0 \]

(10)
Using the software mathematica-5.1, we can calculate the optimal value of T* and t1* simultaneously by equation no. (8) and equation no. (9). And the optimal value P* (T, t1) of the average net profit is determined by equation no. 7. The optimal value of T* and t1*, satisfy the sufficient conditions for maximizing profit function p* (T, t1) are.

\[
\frac{\partial^2 p}{\partial t^2} (T, t1) < 0 \quad \frac{\partial^2 p}{\partial T^2} (T, t1) < 0 \quad (11)
\]

And \( \frac{\partial^2 p}{\partial T \partial t} (T, t1) \). \( \frac{\partial^2 p}{\partial T^2} (T, t1) > 0 \) And at \( T = T^* \) optimal value \( t_1 = t_1^* \) (12)

**Numerical example**

Example - 1. Let us consider \( A = 300, b = 200, h = 1.3, c_1 = 1.1, \alpha = 0.1, \beta = 0.3 \)

Based on above input data and Using the software mathematica-5.1, we calculate the optimal value of P (T, t1), T* and t1* simultaneously by equation no. (8), equation no. (9) and equation no. (10).

\[
P (T, t1) = 1282.61, T^* = 2.75312, t1^* = 1.478321
\]

Example - 2. Let us consider \( A = 350, b = 175, h = 1.2, c_1 = 1.02, \alpha = 0.15, \beta = 0.35 \)

\[
P (T, t1) = 1381.50, \quad T^* = 2.32141, t1^* = 1.37592
\]

**Conclusion**

In this paper we have developed an inventory model for deteriorating items, such as fruits, vegetables and food stuffs from depletion by direct spoilage while kept in store. The rate of deterioration follows the Weibull distribution with two parameters. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The deterioration cost, inventory holding cost and, shortage cost are considered in this model. The numerical examples are given to illustrate the model developed. The model is solved analytically by maximizing the total profit. In the numerical examples we found the maximum value of profit.

**References**