Skill Acquisition, Incentive Contracts and Jobs:

Labor market adjustment to trade

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August 2010

Abstract

This paper examines how global integration influences worker behavior regarding skill acquisition, as well as firm behavior regarding incentive contracts and occupational diversity. The approach integrates several key components of international trade and the wage distribution in developed countries: namely heterogeneous firms, trade in similar goods, and performance payments to workers that endogenously obtain different skill levels. Greater trading opportunities reduce aggregate prices, causing workers to experience a greater marginal utility derived from income, as well as the skills that aid them in fulfilling performance contracts. Firms respond to skill accumulation among the labor force by adjusting the provision of incentive contracts, and the types of jobs they offer. Labor market adjustment to trade liberalization is characterized by a more steep, but less extensive, provision of incentive contracts among the labor force; higher overall wage inequality exhibiting a U-shaped differential; and job polarization across skill-groups.

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1 Introduction

International trade flows reflect selection of individual firms into international markets. Addressing the relationship between trade and factor returns then requires an analysis of how individual firms interact with their workforce, and with the labor supply in general. A burgeoning trade literature has begun to include key aspects of labor market structure to relate trade and wages, such as worker heterogeneity, unemployment or rent-sharing within firms and the so-called exporter premium\(^1\). On the other hand, a large empirical literature among labor economists highlights different elements affecting the wage distribution: namely endogenous skill acquisition, payments via incentive contracts, and the occupation of employment, rather than the particular firm\(^2\). This paper brings these two literatures together by developing a framework that incorporates contracting problems over multiple occupations and worker skills into a global economy with heterogeneous firms.

To be specific, I examine (1) how workers respond to trade liberalization with regard to skill acquisition, and (2) the subsequent adjustment in the composition of jobs and performance contracts that firms offer when the labor supply changes shape. These issues are important for at least two reasons. First, many countries have experienced changes in their aggregate wage distribution characterized by an increasing 90/50 wage gap, but a steady or shrinking 50/10 wage gap. Trends in employment shares across the skill distribution exhibit a similarly complex pattern. Rather than the skill-biased shifts in labor usage, employment has become more polarized in several developed countries. See Goos and Manning (2007) for evidence from the UK; Autor et al. (2006) and Autor et al. (2008) for evidence from the US; Dustmann et al. (2009) for evidence in West Germany; and Goos et al. (2009) for evidence from several European nations. Moreover, incentive contracts have expanded as a method of payment, and determine much of the variation in wages among high skilled workers. See Lemieux et al. (2009). In order to asses the role of openness in these aggregate trends (if any), a framework relating incentive contracts, occupational diversity and trade is needed.

Second, evidence of micro-level adjustments in occupations and performance payments has already begun to emerge. Cuñat and Guadalupe (2005, 2009) find that greater international competition induced UK and US manufacturing firms to change their provision of incentive contracts. Furthermore, Guadalupe and Wulf (forthcoming) and Bresnahan et al. (2002) observe individual firms shifting the occupations they offer, and their skill demands, in response to globalization. Understanding the economic forces that induce firms to respond to international exposure in this way requires an examination of their contracting problem with

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\(^1\)For review of recent work on trade and worker heterogeneity see Davidson and Sly (2010), for trade and unemployment see Davidson and Matusz (2009) and more generally for globalization and wages see Goldberg and Pavcnik (2004, 2007), and Feenstra and Hanson (2003).

\(^2\)For cross country evidence on the importance of worker characteristics and occupation in determining individual wages see Abowd and Kramarz (1999) and Menezes-Filho et al. (2008).
worker and occupational diversity.

The approach integrates firm choices regarding occupational mix and the provision of performance contracts across multiple workers, as in Kim and Sly (2010), into the standard Melitz (2003) heterogeneous firm model of international trade. Workers are fully mobile and heterogeneous with respect to ability. Each worker makes choices regarding skill acquisition and team formation (i.e. matching) to ensure the best employment opportunities possible. Given the behavior of the labor force, firms decide on the composition of jobs and incentive contracts to offer, aimed at attracting the highest quality workforce possible. Competition for recruits induces endogenous heterogeneity in occupational mix across firms.

Though it is often overlooked by trade economists, skill accumulation among the labor force is key to understanding labor market adjustment to trade. Firms choose the employment contracts they offer with the objective of maximizing recruiting potential\(^3\). Suppose that the labor supply were fixed, in that the distribution of skill were exogenously determined. Then firms in a closed economy, open economy or liberalizing economy would face the same recruiting opportunities regardless. The composition of incentive contracts and jobs would not change as countries become more or less integrated. However, if workers change their skill acquisition behavior when markets open, then firms must reconsider their hiring strategies. Hence all three elements of the model – endogenous skill acquisition, incentive contracts and occupational mix – are necessary to understand labor market adjustment to trade.

In the framework presented here, occupations differ within individual firms because workers receive varying levels of direction in how to steer their efforts toward successful performance. This human resource strategy is often called 'developing agent champions' within a team. Each firm has fixed capability to direct workers, and must choose how to allocate this capability across the workforce\(^4\). For simplicity I allow firms to have two different choices about how to interact with their workforce: each firm can offer a diffuse set of occupations, where some jobs are associated with large investments in promoting worker success, along with others that receive little direction; or the firm can choose to offer similar occupations to its entire workforce, with each worker receiving moderate levels of direction in their efforts. The two strategies can be interpreted as relying primarily on a few champion workers with a high probability of success, or many workers with moderate chances of success.

The production technology of each firm depends first on the performance of its workforce, and second on the (heterogeneous) ability of firms to take advantage of labor performances. Firms which draw greater productivity parameters are able to generate more output from each individual worker success. Strictly

\(^3\)Lazear (2000) provides evidence that increasing the provision of incentive contracts generates gains primarily through better recruiting outcomes.

\(^4\)Bandiera et al. (2007) find that managers make specific choices regarding direction given to workers, and that this choice is partially determined by the incentive contract the manager is given.
speaking, the production process consists of a series of random trials across the workforce employed at each firm. Individual workers realize success stochastically, conditional on their effort level and occupation. Total output by a firm is the number of successes among the workforce augmented by its own productivity parameter. Aggregate labor productivity is greater when workers are motivated effectively, and firms that are better able to capture individual performances operate in equilibrium.

Workers pursue jobs that give them the best chances of fulfilling performance contracts. Competition with full mobility leads to positive assignment of workers across occupations and performance incentives. That is, high-skilled workers are employed in occupations associated with the greatest incentive contracts, consistent with evidence in Lemieux et al. (2009) and Bayo-Moriones et al. (2010). Firms anticipate this organization of the labor force and choose which type of employment vacancies to create accordingly.

There are two sets of results presented below. The first set directly relates trade liberalization to the behavior of workers and firms. As in Melitz (2003) global integration induces selection effects and reallocations of production across firms that raise average productivity, and lower aggregate prices. Workers recognize that at lower prices the marginal utility of income, and the skills which aid them in fulfilling performance contracts, are higher. So, openness leads to greater skill accumulation among the labor force. With a more skill abundant labor supply, firms subsequently alter the employment opportunities they offer. Following trade liberalization a greater share of firms offer diffuse sets of occupations, with stronger performance payments given to the relatively skilled members of their workforce. While the provision of incentive contracts is more steep, it is less extensive in an open economy. Trade reduces the share of the labor force receiving performance pay.

The second set of results connects these micro-level adjustments to the overall wage distribution and the distribution of firm productivity. Changes in firm hiring strategies cause employment to become more polarized; i.e. trade increases the employment share of high- and low- skill occupations, while reducing the share of middle-skill occupations in total employment. This change in labor usage and the accompanying shift in performance contracts increase wage inequality across skill groups. Specifically, the top and bottom end of the wage distribution grow relative to the median earnings level. Inequality within the highest and lowest skill groups also rises. Furthermore, the relationship between openness and welfare is ambiguous in general. Although trade lowers aggregate prices for all consumers, changes in the provision of incentive contracts increase volatility in wages on average, harming risk-adverse workers.

Falvey et al. (2010) have shown that workers do skill-update when international competition intensifies, but highlight import competition as the leading mechanism inducing skill acquisition. Also, Falvey et al. (2008) discuss patterns of human capital adjustment in a two sector model of trade with differences in worker ages and abilities. Again, this does not reflect the current setting, but does suggest that workers consider the their employment potential in a global economy when making decisions about skill acquisition.
Openness changes the composition of firms and incentive contracts offered, impacting overall volatility in firm-level productivity via two channels. The first is a Team Composition Effect that reduces the variability in firm-level productivities because workers satisfy (and fail) their performance contracts with more regularity. The second is a Firm Composition Effect that increases the observed variability of firm-level productivity. Selection of low productivity firms out of the market generates larger swings in output as labor performances within firms vary stochastically.

This paper lies at the intersection of trade and labor literatures regarding the wage distribution. Within the "new trade theory", the discussion of factor returns has typically emphasized differences in payments across firms rather than across occupations. See for example Manasse and Turrini (2001) and Yeaple (2005), with empirical evidence in Bernard and Jensen (1997, 1999), Verhoogen (2008), Frias et al. (2009) and Burstein and Vogel (2009). In the current context I abstract from the exporter premium and technologies with skill complementarities to highlight occupational and skill diversity within firms. In addition, this analysis introduces performance contracts as a payment mechanism used to overcome information problems surrounding production. Others have examined trade and the income distribution using fair wage mechanisms (Egger and Kreickemeier (2009)), efficiency wages (Matusz (1996) and Davis and Harrigan (2008)), and intra-firm bargaining/rent sharing (Davidson et al. (2008), and Sly (2009), Cosar et al. (2009), and Helpman et al. (2010)) into trade models with heterogeneous firms. To my knowledge there is no other theoretical framework that addresses performance contracts and heterogeneous firms in open economies. Yet, Grossman (2004) and Vogel (2007) consider trade and the organization of the labor force when performance (rather than effort) cannot always be attributed to individual workers because of information constraints and poor institutional quality, respectively.

This paper also relates to the growing trade and matching literature. Costinot and Vogel (forthcoming) develop tools to address trade with exogenous changes in the distribution of factor endowments in Roy-like assignment models. Antràs et al. (2006) examine how potential for offshoring influences occupational diversity across skill groups with competitive wages. Other contributions include Davidson et al. (1999), Grossman and Maggi (2000), Bougheas and Riezman (2007), and Bombardini et al. (2009). This approach is distinct in that I consider endogenous shifts in the shape of the labor supply following liberalization and highlight both the matching (team formation) and assignment to occupations as allocations problems that labor markets must resolve.

A large literature has examined the distributions of income and employment the world over, and I cannot hope to review its entirety here. This paper fits directly into a recent strand that has emphasized polarization in both wages and employment shares, rather than skill-biased shifts within labor markets. Lemieux (2007)
provides an excellent review of how the nature of wage inequality has changed in recent history. The roles of educational attainment and technology in shaping the income distribution are detailed in Goldin and Katz (2008) and Acemoglu (2002). With the availability of employer-employee matched data sets from several countries, wage distributions have been decomposed into worker, occupation and establishment (firm or plant) effects, that I emphasize here; see for example Abowd and Kramarz (1999) and Menezes-Filho et al. (2008). International factors such as immigration and offshoring have been introduced to explain polarization in the labor markets in developed economies. See for example the 2009 Richard T. Ely lecture from David Card, Borjas (2003), Borjas et al. (1997), Goos et al. (2009) and Feenstra and Hanson (2003) in addition to the references above.

The next section of the paper introduces the model and details the specific behaviors of workers and firms in open economies. Section 3 derives an unique equilibrium where the aggregate features of product markets are stationary and the allocation in the labor market is in the core. Section 3 also presents key results that link firm behavior regarding the provision of incentive contracts and occupational mix to the distribution of labor productivity and wages. Product market adjustment to trade is addressed in Section 4 and labor market adjustment is taken up in Section 5. The effects of openness on the wage distribution and welfare are discuss in Section 6. Section 7 provides a brief discussion of the model and results, while the final section concludes.

2 Model

The basic framework of the model includes two identical countries which produce a single final consumption good comprised of several individual varieties. Firms produce differentiated products, using only labor, with each worker assigned a single occupation. Firms offer a variety of occupations and must choose the mix of jobs, denoted by $j$, to offer. Every firm possesses a specific productivity parameter, $\theta$, that reflects its ability to generate output as individual workers complete their respective jobs. Thus an individual firm can be characterized by a pairing $(j, \theta)$. Heterogeneity across firms will be reflected in the exogenous realization of productivity, and the endogenous selection of jobs to offer.

Firms recruit workers from a heterogeneous labor supply with total mass $L$. Workers differ according to their ability, $a$, with the atomless distribution given by $G(a)$ on $[a_{min}, a_{max}]$. Higher ability agents can and acquire skills, $s$, at a lower marginal cost to their over all utility. During production workers experience disutility from their efforts, $e$, though higher skills make effort less onerous at the margin. Workers are fully mobile utility maximizers that pursue the best employment opportunities available.
Once employed, the relationship between firms and labor is characterized by an agency problem whereby workers need incentives to provide effort that cannot be observed directly by the firm. It will be more convenient to first describe the product market environment in which firms operate. Then I will describe the agency problem that surrounds production and finally the behaviors of individual workers and firms.

2.1 Consumption

Consumers in each economy are workers who use their income to purchase varieties of the final consumption good; demand for an individual product, \( \omega \), is given by \( x(\omega) \). All consumers have common tastes with CES preferences across varieties.

\[
X = \left[ \int_{\omega \in \Omega} x(\omega)^\rho \, d\omega \right]^{1/\rho}
\]

(1)

An individual firm sets the price of its product to \( p(\cdot) \). Subsequently the aggregate price index, \( P \), can be written as a weighted average of the prices of individual products:

\[
P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\beta} \, d\omega \right]^{1/\beta} \quad \text{with} \quad \beta = \frac{1}{1-\rho}
\]

(2)

2.2 Production

Consumers costlessly assemble differentiated products within industries into the final consumption good. Production of varieties occurs in firms, resulting from the successful completion of tasks. Firms hire teams of labor to complete an array of production tasks, but individual workers are assigned to independent occupations. That is, each worker provides effort toward a single task. Let \( n \) denote the number of different occupations that firms offer, and \( l(\cdot) \) denote the mass of teams that a firm hires. In all firms the production process exhibits informational problems such that firms cannot directly observe the efforts of labor\(^6\).

There are two key features of the agency problem surrounding production. First, though firms cannot observe the efforts of workers, they do observe if an individual worker is successful in completing her task. Effort increases the probability that a worker is successful: for a worker providing effort level, \( e \), the probability she accomplishes her task independently is \( p(e) \), where production responds to efforts continuously according to \( p''(e) < 0 < p'(e) \).

\(^6\)In addition to being the simplest framework to expose, the production environment with simultaneous efforts on independent tasks contains the weakest incentives for assortative matching to arise in equilibrium. Hierarchies of agents within firms or other coordinated efforts would reinforce the strategic complementarities derived below. That is, these models of firm productions which are more relevant would generate the same conclusions. None of the results depend on the simultaneity of efforts or independence of tasks, rather they are obtained despite these assumptions. See Kim (2010) and Kim and Sly (2010) for more discussion on this point.
The second feature of the agency problem is that firms have the capability to advise workers in their efforts. Interaction between the firm and its labor force across occupations follows Kim and Sly (2010). Each firm has a fixed capacity $\Gamma > 0$ to direct the efforts of its work force. Better direction improves the probability that effort leads to successful production. For example all of the $n$ workers in a production team are successful with probability $\Gamma \left( \prod_{k=1}^{n} p(e_k) \right)$. More details about how firms divide this capacity into individual occupations, and the specific job mix $j$ that firms adopt, will be provided below.

The interaction between workers and firms could take several forms. Each task can represent a specific feature of a product. For example smart phones now enable internet access, calendar features, texting, applications, cameras and differ in appearance. While each feature might appeal to consumers, firms that devote more attention to particular components, say providing browser services, are better able to direct worker efforts toward the consumer tastes. So, workers assigned to produce the internet capability in the phone are more likely to be successful. Also, the independent tasks can represent separate product lines. Workers will view employment across product lines differently when the firm makes larger investments in advertising for certain products than others. Better exposure can increase the probability that a worker’s effort, and in turn her specific product, will be successfully received in the market place.$^7$

Each success allows the firm to generate output. Let $t(\vec{e},j)$ denote the number of successes that arise when an array of workers exerting effort levels given by $\vec{e}$, employed in corresponding occupations belonging to job mix $j$. Upon completion of any individual task, a specific firm produces $\theta$ units of output. Hence the total output of a particular team of $n$ workers at a specific firm $(j, \theta)$ is given by

$$\theta t(\vec{e},j)$$  

(3)

Equation (3) indicates that total production at each firm depends on its own productivity, the number of teams hired, and the realized performances of individual workers. Worker performances reflect their own skill and effort decisions, as well as the occupations offered by firms. So to complete the description of production I turn next to the behaviors of workers and firms.

### 2.3 Firm Behavior

There is an unbounded mass of potential firms that can choose to engage in production freely. Firms make a series of preliminary choices in anticipation of product and labor market conditions. Following the Melitz

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$^7$Though multiproduct firms are quite prevalent, and teams in different product lines are likely to work independently, I adopt the interpretation where different tasks correspond to different features of single products to avoid the complications of endogenous product scope, or firms that face several demand curves across various products.
standard, firms make sunk investments that reveal their productivity from a known distribution. If a firm decides to participate in any market it incurs fixed costs each period to maintain employment vacancies.

Hiring decisions are characterized by a mass of workers to employ, as well as a specific mix of occupations to offer. Once the firm has recruited a workforce, it sets rewards to motivate efforts. Finally the firm engages in production and sets prices for each market in which it participates. This section provides details about each of these firm activities.

### 2.3.1 Firm Entry and Market Participation

Entry by a firm is a two-stage process. Firms must first incur a sunk investment cost of $f_c$, after which the firm realizes its productivity parameter, $\theta$, drawn from the known distribution $Z(\theta)$. The density of potential firm productivities, $z(\theta)$, is defined over $(0, \infty)$. Subsequently, if a firm decides to engage in production, each period the firm must pay a fixed cost $f$ in order to create and maintain employment vacancies for its workforce. Participating in foreign markets requires the firm to pay a fixed $f_x$ each period that it exports. Fixed costs are expressed in terms of labor teams that must be hired.

In addition to the fixed costs to access foreign markets, firms incur transportation costs for each unit of output sent abroad. Transportation costs are given by $\tau$ and exhibit the typical iceberg nature; a firm must ship $\tau > 1$ units to the foreign market for a single unit to arrive. I make the standard assumption regarding the size of trade costs: $\tau^{\beta-1} f_x > f$. This assumption partitions firms according to their productivity parameter $\theta$, such that the most productive firms export while the least productive firms do not$^8$.

### 2.3.2 Firm Job Mix and Hiring

Once a firm knows its own productivity level it is able to predict how competitive it can be in product markets, and so decides on the size of a work force it should recruit. Given the heterogenous supply of labor each firm is also concerned with the quality of the workforce it hires. As workers sort into occupations, the job mix that firms make available will determine potential recruiting outcomes. Hence the firm must decide how to allocate its fixed capability, $\Gamma$, to direct effort across its workforce.

Let $\gamma^k_j$ denote occupation $k$ within job mix $j$. Occupations are defined so that a worker putting forth effort level $e$ performs successfully with probability $\gamma^k_j p(e)$. When a firm offers a few occupations with large investments in directing effort (high $\gamma^k_j$), the corresponding workers will be relatively more likely to generate output. Yet, the constraint on how well firms can address the agency problem dictates that workers in the remaining occupations receive little investment from the firm in directing their efforts, and are less likely to

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$^8$See evidence of this sorting on productivities in Bernard and Jensen (1999) and Roberts and Tybout (1997).
perform (effort held constant). Thus each firm faces a trade-off between occupational mixes: rely primarily on a few workers with a high probability of success, or many workers with moderate chances of success.

Specifically let the first occupational mix \((j=1)\) spread the firm’s fixed capability to direct workers similarly across occupations. Suppose the second mix of jobs \((j=2)\) is relatively diffuse, with some occupations that are more likely to translate effort into successful production, as well as some occupations that are not likely to generate output independently. See Figure 1. The relative diversity of occupations, indexed by \(j=2\), corresponds to the Monotone Likelihood Ratio Property about a particular occupation \(k\), where \(1 < k < n\) and \(\gamma_1^k \neq \gamma_2^k\). Ranking occupations within each job mix in order of magnitude, indexed by \(k\), the relative diversity between occupational mixes satisfies the following definition.

**Definition 1** Occupational mix indexed by \(j=2\) is said to be more diffuse such that for any \(h \geq 1\).

\[
\frac{\gamma_{2}^{k-h-1}}{\gamma_{1}^{k-h-1}} < \frac{\gamma_{2}^{k-h}}{\gamma_{1}^{k-h}} \quad \text{and} \quad \frac{\gamma_{2}^{k+h}}{\gamma_{1}^{k+h}} < \frac{\gamma_{2}^{k+h+1}}{\gamma_{1}^{k+h+1}}
\]

Workers responsible for generating output are hired in teams. The size of teams is set arbitrarily to \(n\). So, regardless of the job mix offered, the fixed capability to address the agency problem requires\(^9\)

\[
\Gamma = \prod_{k=1}^{n} \frac{\gamma_j^k}{\gamma_j^{k-1}} \quad \text{for} \quad j = 1, 2
\]

The hiring strategy of a firm describes the composition of employment. Regarding size of the workforce, 

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\(^9\)As is standard when contracting with multiple agents, I impose the following regularity conditions on the production environment: first, \(0 \leq \Gamma p(e) < 1\), and second, for any potential effort levels, \(e_1, e_2\), put forth by workers, \(p''(e_1) p''(e_2) p(e_1) p(e_2) - [p'(e_1) p'(e_2)]^2 > 0\). These assumptions imply that firms can never perfectly direct their workforce, and that strategic interactions exist between the workers in any team.
the mass of teams \( l(\cdot) \) demanded by a particular firm, \((j, \theta)\), is given by

\[
l(j, \theta) = f + \frac{q}{\theta l(\varepsilon, j)}
\]  

(5)

Note that \( nl(\cdot) \) is the total mass of workers hired by the firm. Though all firms face the same information problems during production, those characterized by higher values of \( \theta \) exhibit better labor productivity on average, and those that subsequently recruit and motivate a better workforce experience greater productive efficiency.

The final type of occupation that firms offer is a job without risk, necessary to perform the fixed set of tasks necessary for entry \((f_e)\), to maintain employment vacancies \((f)\) and fixed exporting tasks \((f_x)\). I assume this job requires minimal effort to complete\(^{10}\). After a firm has entered and recruited a workforce, it needs to motivate workers to provide effort. Rewards for successful task completion are described next.

2.3.3 Firm Payments to Workers

Each firm must provide incentive contracts where performance requires unobserved effort and is uncertain. An employment contract specifies rewards for each type of signal received: the firm chooses the payments \( v^s \) when the entire team is successful, \( v^w \) to the workers who perform and \( v^f \) to those who fail when only a fraction of the team is successful, and \( v^o \) to each worker when no one succeeds.

These payments can vary continuously, and be made commensurate with the skill of workers recruited. However I classify the wage schemes in the typical manner. A payment scheme is said to be Joint Performance Evaluation (JPE) if \((v^s, v^f) > (v^w, v^o)\). Under JPE there are strong team incentives since a worker is made better off by good performance of her partner. The firm is said to use Relative Performance Evaluation (RPE) if the optimal contract satisfies \((v^s, v^f) < (v^w, v^o)\). RPE provides strong competitive incentives since workers are best off when they outperform their partner.

Although the firm has freedom in how it chooses to reward workers, I do impose a limited liability restriction. Workers cannot be forced to make payments to the firm, regardless of their performance. Incentive contracts offered by the firm must satisfy

\[
v^s, \ v^w, \ v^f, \ v^o \geq 0
\]

\(^{10}\)The assumption that fixed production tasks requires little effort implicitly assumes that these are low-skill occupations. Later when deriving labor market adjustment to trade liberalization, this assumption will only impact the provision of incentive contracts across the skill distribution. The evidence that incentive contracts are most prevalent among the highest skilled in the labor force is fairly robust; see Lemieux et al. (2009) and Bayo-Moriones et al. (2010). Thus I maintain the assumption that fixed production tasks represent occupations with low-skill intensity.
The remaining constraint on performance payments that firms can offer reflects the incentives of workers to accept contracts in a competitive labor market. Worker behavior is described in a later section.

I will use the notation $V(s, j, \theta)$ to represent the optimal performance contract offered to an array of workers $s$ by a specific firm $(j, \theta)$. Further, let $\tilde{V}(\cdot)$ represent the payments to workers following the realization of performance. Firms also hire labor into an occupation to complete the fixed production tasks necessary for entry and to maintain employment vacancies. I set the competitive wage for these workers to be the numeraire\textsuperscript{11}.

### 2.3.4 Firm Prices and Profits

Consumer preferences for variety in (1) imply that firms face an iso-elastic demand curve for their products. The optimal behavior of firms is to charge prices that are fixed mark-ups over unit costs. A firm that recruits an array of workers $\bar{s}$, to occupations in job mix $j$, and with productivity $\theta$ charges the price\textsuperscript{12}

$$p(s, j, \theta) = \frac{1}{\rho} \frac{\tilde{V}(s, j, \theta)}{\theta t(e, j)}$$

At its optimal price level, a particular firm realizes (ex post) profits in the domestic market equal to

$$\Pi^D(s, j, \theta) = X_P \left( \frac{\rho P \theta t(e, j)}{\tilde{V}(s, j, \theta)} \right)^{\beta - 1} - f$$

If a firm decides to export to the foreign market it earns additional profits equal to foreign revenues net of production and exporting costs. Exporting firms charge higher prices in foreign markets to cover transportation costs and subsequently earn (where asterisks denote foreign aggregate variables)

$$\Pi^X(s, j, \theta) = X^* P^* \left( \frac{\rho P^* \theta t(e, j)}{\tilde{V}(s, j, \theta)} \right)^{\beta - 1} - f_x$$

Perfectly competitive labor markets with full worker mobility will equalize the expected payment per performance, $\frac{V(s, j, \theta)}{t(e, j)}$, across firms. No worker will accept employment where greater incentives exist elsewhere. Since all firms anticipate the same costs for worker performance, the ex ante relative outputs and revenues

\textsuperscript{11}Thus an employment contract specifies \{v*, v^w, v^f, v^o, 1\}, with each $v^p$ paid only to production workers, $L^p_j$, while investment workers used to perform entry tasks, $L^I_j$, earn a wage equal to unity.

\textsuperscript{12}For notational convenience I have expressed the price charged for goods produced by a specific team within a firm having productivity $\theta$. Firms could also be thought to charge a single price that is a fixed markup over average unit costs realized over all production teams, $(1/l(j, \theta)) \int \frac{V(s, j, \theta)}{t(e, j)} dl(j, \theta)$, or varying prices slightly at each moment in time for units cost that arise for the team active at that instant. Because firms will make all investment and hiring decisions prior to the realization of unit costs, only the constant expectation for either interpretation will be crucial to deriving an equilibrium.
of any two firms in a particular market are functions solely of their productivity parameters. For example, relative outputs and revenues anticipated between any two firms in the domestic market can be written

$$\frac{x(j, \theta)}{x(j, \theta)} = \left(\frac{\theta}{\theta}\right)^\beta \quad \text{and} \quad \frac{r(j, \theta)}{r(j, \theta)} = \left(\frac{\theta}{\theta}\right)^{\beta-1} \quad (9)$$

The relationships in (9) demonstrate that firms with higher values of $\theta$ are expected to produce relatively more output and earn more revenues in anticipation of recruiting outcomes. A similar relationship between profits across firms with different productivities exists. The fact that anticipated payments, $\frac{V(\tilde{x}, \tilde{j}, \tilde{\theta})}{\tilde{t}(\tilde{x}, \tilde{j})}$, are constant across firms does not preclude them from paying different wages in equilibrium. Firms that are fortunate in their recruiting can offer smaller incentive contracts as skilled workers are easier to motivate. Subsequently, the realization of labor performance causes wages to vary, even across identical firms and workers. Yet expected payments per performance must remain constant as labor are fully mobile and seek the best employment opportunities.

### 2.4 Worker Behavior

Once employed, each worker maximizes her indirect utility function $B_a$ by selecting optimal levels of effort and skill acquisition. Workers are rewarded based on their performance relative to other workers in their production teams, so it will be notationally convenient to define an object that represents for any worker that acquires skill $s$, the probability that the rest of her team is able to produce. Let $\epsilon(s)$ be the probability that every worker in a team with abilities given by $\bar{s}$ completes their task, excluding worker $s$. Similarly define $\eta(s)$ as the probability that any other worker is able to perform$^{13}$. Then a worker with ability $a$, in occupation $\gamma_j^k$, and facing aggregate prices $P$ solves the following problem:

$$\max_{e, s} B_a = \gamma_j^k p(e) \left\{ \epsilon(s)U(v^s, P) + (1 - \epsilon(s))U(v^{w^s}, P) \right\}$$

$$+ \left[ 1 - \gamma_j^k p(e) \right] \left\{ \eta(s)U(v^f, P) + (1 - \eta(s))U(v^{w^f}, P) \right\} - \frac{e}{s} - \frac{s}{a}$$

Laborers with greater innate ability $a$ can acquire skills at a lower marginal cost to their overall utility. Upon attaining a particular education level, better skills reduce the marginal disutility of effort. All laborers share risk adverse attitudes toward income so that the sub-utility function satisfies $U_1(\cdot) > U(0, p) = 0 > U_{11}(\cdot)$.

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$^{13}$Computing the expected performances of independent workers yields

$$\epsilon(s) = \prod_{s' \in \mathbb{Z}, \ s' \neq s} \gamma_j^k p(e_{s'}) \quad \text{and} \quad \eta(s) = 1 - \prod_{s' \in \mathbb{Z}, \ s' \neq s} \left[ 1 - \gamma_j^k p(e_{s'}) \right]$$
3 Equilibrium

This paper focuses on long-run equilibria where product and labor market allocations are stable. That is, I consider steady-state equilibria in the product market where the composition of firms, prices and aggregate profits are stationary, given that the allocation of the labor force is stable. Specifically a product market equilibrium is defined as a distribution of active firms for each choice of occupational mix, \(\mu_j(\theta)\), the mass of active firms choosing each job mix, \(M_j\), and a productivity cutoff \(\theta^e_j\) that defines which firms export to the foreign market. Together the distribution and mass of firms serving the domestic market determine aggregate prices, for any equilibrium allocation of the labor force.

A labor market equilibrium is defined as a core allocation of the labor force, where matching into teams and assignments to occupations are stable. An equilibrium in the labor market consists of sets of skill levels assigned to each occupation, \(\Sigma^k_j\), matching correspondences between workers that are employed in the same team (and thus the same firm), \(C^k_j : \Sigma^1_j \rightarrow \Sigma^k_j\) for \(k = 2 \ldots n\), worker efforts levels and skill acquisitions, \(e\) and \(s\), and incentive contracts with performance payments to workers given by \(V(\bar{s}_j, j, \theta)\).

3.1 Distribution of Active Firms

Upon entry the firm realizes its own productivity \(\theta\) and then must consider whether or not to produce, given the fixed cost to create employment vacancies. The firm will decide to engage in production as long it anticipates a labor force skilled enough to generate non-negative profits for either hiring strategy. For a stable allocation of labor across occupations, let \(\bar{s}_j\) be the array of workers a firm can anticipate to recruit in equilibrium\(^{14}\). Total profit for a firm is \(\Pi(\cdot) = \Pi^D(\cdot) + \Pi^X(\cdot)\). So rational behavior by firms defines a zero-anticipated-profit cutoff for each occupational mix, \(\theta^o_j\), which satisfies

\[
\Pi(\bar{s}_j, j, \theta^o_j) \equiv 0
\]

(ZAP)

If a firm draws a productivity below \(\theta^o_j\) for both \(j=1,2\) it exits immediately. For each job mix, the ex post distribution of active firms is defined by the ZAP cutoff. Active firms have productivities distributed according to \(z(\theta)\), truncated at the zero-anticipated-profit cutoff, or

\[
\mu_j(\theta) = \begin{cases} 
\frac{z(\theta)}{1 - Z(\theta^o_j)}, & \text{for } \theta \geq \theta^o_j \\
0, & \text{otherwise}
\end{cases}
\]

\(^{14}\)Anticipated recruiting outcomes are discussed below following the derivation of a labor market equilibrium.
Firms are willing to serve foreign markets only if they can anticipate non-negative profits from abroad, net of fixed border penetration costs and variable transport costs. The productivity cutoff between exporters and non-exporters in equilibrium is defined by a zero-anticipated-exporting-profit condition

\[ \Pi^X(\bar{\theta}_j, j, \theta^{ex}_j) \equiv 0 \]  

(ZAXP)

The distribution of exporting firms is equivalent to \( \mu_j(\theta) \), truncated at \( \theta^{ex}_j \). Hence the probability that a firm actively serves the foreign market is \( b^{ex}_j = [1 - Z(\theta^{ex}_j)]/[1 - Z(\theta^*_j)] \).

From the equilibrium distribution of active firms, those implementing each hiring strategy can be summarized by a representative domestic firm with productivity \( \theta^D_j \), where

\[ \theta^D_j = \left[ \frac{1}{1 - Z(\theta^{ex}_j)} \int_{\theta^{ex}_j}^{\infty} \theta^{\beta-1} z(\theta) d\theta \right]^{1/(\beta - 1)} \]

and a representative exporting firm, \( \theta^X_j \), given by

\[ \theta^X_j = \left[ \frac{1}{1 - Z(\theta^{ex}_j)} \int_{\theta^{ex}_j}^{\infty} \theta^{\beta-1} z(\theta) d\theta \right]^{1/(\beta - 1)} \]

Aggregating over all firms, and using symmetry between countries, the distribution of firms offering job mix \( j \) active in any market is summarized by a representative firm with productivity \( \bar{\theta}_j \) equal to

\[ \bar{\theta}_j = \left[ \frac{1}{M_j (1 - b^{ex}_j)} \left\{ M_j \theta^D_j^{\beta-1} + b^{ex}_j M_j (\tau^{-1} \theta^X_j)^{\beta-1} \right\} \right]^{1/\beta - 1} \]

Firms with productivity above the ZAP cutoff are willing to create employment vacancies and engage in production for as long as it survives. The constant probability of firm destruction each period is \( \delta \), so that the value of entry is given by

\[ V(\theta)^{ent} = \max \left\{ 0, \frac{1}{\delta} \Pi(\bar{s}_j, j, \theta), \frac{1}{\delta} \Pi(\bar{s}_j, j, \theta) \right\} \]

Prior to the realization of their specific productivity level, firms are willing to pay the sunk investment cost so long as the probability of successful entry and the anticipated profits are great enough. Hence the free-entry
conditions for active firms are
\[
\frac{1 - Z(\theta^o_j)}{\delta} \Pi(s_j, \bar{\theta}_j) = f_c
\]

Given the relationships defined in equation (9), the zero-anticipated-profit condition can be expressed in terms of average firm profits. So, as in Melitz (2003), the intersection of the free entry conditions (FE) and the zero-anticipated-profit conditions (ZAP) determine unique cutoff productivity levels, \( \theta^o_j \). These cutoffs imply a unique equilibrium distribution of firms that are willing to create employment vacancies and then engage in production.

### 3.2 Mass of Active Firms

A steady-state equilibrium requires that the mass of active firms offing each job mix, \( M_j \), is stationary over time. The mass of new firms \( M_j^{ent} \), with fraction \( b_j \) who decide to hire workers and produce, must exactly replace firms that expire. That is, in equilibrium, \( b_j M_j^{ent} = \delta M_j \). At the time of entry all firms share the same expectations regarding aggregate prices, \( \bar{P} \), their potential productivity draw, \( \bar{\theta}_j \), probability of becoming and exporter, \( b_j^{ex} \), and recruiting quality \( \bar{s}_j \). So applying standard methods reveals that the mass of firms choosing each job mix is tied to the equilibrium payments to labor according to\(^{15}\)

\[
M_j = \frac{\bar{R}_j}{r(\theta_j)} = \frac{V(s_j, \bar{\theta}_j)L_j^P + L_j^{ent}}{\beta[\Pi^D(s_j, \bar{\theta}_j) + b_j^{ex}\Pi^X(s_j, \bar{\theta}_j)]}
\]

where the total mass of labor employed in each job mix is given by the array of workers employed in production, \( L_j^P = \int_{\cup_{k=1}^{n} \sum_j s_j} 1 da \) plus investment tasks \( L_j^{ent} \).

Together the distribution and mass of firms using each hiring strategy are sufficient to describe prices: the aggregate price index is

\[
P = \left( \sum_{j=1,2} M_j p(s_j, \bar{\theta}_j) (1-\beta) + \sum_{j=1,2} b_j^{ex} M_j^* \tau (1-\beta) p(s_j^*, \bar{\theta}_j^*) (1-\beta) \right)^{1/(1-\beta)}
\]

Having derived the conditions for a steady-state equilibrium to arise in product markets I turn next to the characterization of the labor market.

\(^{15}\)The mass of firms is determined before individual recruiting outcomes are known. As long as workers hold a diversified portfolio, any excess profits or losses that arise following the assignment of workers to firms are retained by the labor force. Hence the link between anticipated aggregate revenue, \( \bar{R} \), and the mass of labor employed in each industry is upheld.
3.3 Effort and Skill Acquisition

A worker with ability \( a \) chooses the amount of skills to acquire, \( s \), and subsequently an effort level, \( e \), that maximizes utility for her employment status and aggregate prices. The first order conditions describing worker behavior can be greatly simplified by noting that effort levels are always decreasing for any \( v^f, v^o > 0 \). No firm will reward a worker if she fails to send a positive signal, regardless of the optimal wage scheme or types of occupations offered\(^\text{16}\). Then substituting \( v^f = v^o = 0 \), the optimal effort levels of a worker in any occupation \( \gamma^k_j \) satisfies

\[
\frac{1}{s} \frac{1}{p'(e_s)} \equiv \gamma^k_j \left[ \epsilon(s) U(v^s, P) + (1 - \epsilon(s)) U(v^u, P) \right]
\]

(12)

Clearly, effort is increasing in each performance payment; i.e. the effort supply curve is upward sloping. Furthermore, agents that acquire more skill will exert greater effort during production, for any wage scheme and occupation. The amount of skill accumulated is endogenous and chosen by a worker given her innate ability. Optimal skill accumulation is given by

\[
s_a \equiv a^{1/2} \left[ \gamma^k_j \left[ \epsilon(s) U(v^s, P) + (1 - \epsilon(s)) U(v^u, P) \right] + e_s \right]^{1/2}
\]

(13)

Workers that foresee employment in a job with larger performance incentives will choose to obtain greater skill. It is also evident from equation (13) that lower prices raise the marginal utility of income, and so workers respond by obtaining more skill. I will return to this point when discussing labor market adjustment to trade liberalization. For now I move directly to the performance payments offered by firms.

3.4 Incentive Contracts and Wages

The firm’s problem of setting optimal performance payments can be understood as a two-stage process: first the firm chooses the least-cost contract that incentives workers, and second decides on the vector of effort it wishes to elicit. An optimal incentive contract minimizes the expected unit costs of production, encouraging effort so long as the expected performance justifies the expenditure. In other words, equilibrium performance payments must equate the slope of the firm’s expected unit cost function, with the slope of the workers’ behavior regarding effort in equation (12). Implicit differentiation of each equation provides a relationship between payments, \( \frac{dv^o}{dv^f} \), consistent with the rational behavior of both firms and workers. Equating the slopes

\(^{16}\)Under limited liability the participation constraint for any agent is non-binding. In this instance it is well known that principals optimally select \( v^f = v^o = 0 \).
to find the optimal performance payments yields

$$\frac{U'(v^w, P)}{U'(v^s, P)} = \frac{\sum_{k=1}^{n} \gamma_k^p(e_s)}{\gamma_k^p(e_s) - \Gamma \prod_{k=1}^{n} p(e_s)} > 1 \quad (14)$$

Optimal payments will yield relative utilities that are greater than one as long as the worker whose incentive compatibility constraint that binds the provision of incentives has a less-than-average chance of success; i.e. \( \gamma_k^p(e_s) \leq \frac{\sum_{k=1}^{n} \gamma_k^p(e_s)}{n} \). In general a firm cannot offer a single incentive contract such that every worker’s incentives to provide effort are just binding. Yet, the firm is most concerned with incentivizing workers where the marginal return to effort is the greatest. With decreasing returns, the greatest marginal returns come from motivating low-skill workers who otherwise exert little effort. Hence profit maximizing firms will offer contracts that just align the incentives of a worker who has a less-than-average chance of success.

Workers are risk averse, and so (14) implies that the optimal payment scheme satisfies \( v^s > v^w \). Firms offer incentive contracts with Joint Performance Evaluation, rewarding team successes more heavily than individual performance. Incentive contracts with JPE avoid discouraging low ability workers as they are likely to be outperformed by their more skilled teammates. Note that the optimal wage scheme is JPE for any chosen mix of jobs.

With the least cost method to incentivize workers from equation (14), firms must then choose what level of effort they wish to elicit. Performance payments are made to minimize expected unit costs. Solving the minimization problem yields

$$\frac{dV(s, j, \theta)}{d\tilde{e}} \frac{1}{V(s, j, \theta)} = \frac{dt(\tilde{e}, j)}{d\tilde{e}} \frac{1}{t(\tilde{e}, j)} \quad (15)$$

i.e. the percentage increase in payments necessary to motivate effort must be equivalent to the percentage increase in expected performance. Though JPE is always the optimal type of incentive contract, the actual value of each payment varies across teams. Each firm makes performance payments commensurate with the skill of the workers recruited.

### 3.5 Matching of Workers into Teams

Under a Joint Performance Evaluation payment scheme workers view the successes of their partner as strategic compliments for their own. Looking at the indirect utility function, \( B_a \), the relative benefits of own
skill and the skill level of any other teammate \( s \) to agent \( a \)'s expected payoff are given by

\[
\frac{\partial^2 B_a}{\partial s \partial \dot{s}} = \gamma^k_j \rho' (e_s) \frac{de_s}{ds} \frac{de(s)}{ds} [U(v^s, P) - U(v^w, P)] > 0
\]  

(16)

The difference in the utility of incomes under JPE, \( \frac{de_s}{da} \), and \( \frac{de(s)}{ds} \) are all positive. Then using the fact that \( \frac{ds}{da} > 0 \) always, equation (16) reveals that agent payoffs are supermodular in agent types. Using the well known results from Legros and Newman (2007), supermodularity of individual payoffs is sufficient for positive assortative matching to arise for any distribution of ability. Thus \( C^k_1 \) and \( C^k_2 \) are non-decreasing and unique matching correspondences between workers employed in the same occupational mix. The final component of an equilibrium is the assignment of skills (and implicitly abilities) to occupations.

### 3.6 Assignment of Workers to Occupations

Workers are fully mobile. They are free to pursue occupations that best reward their skill at any firm using either job mix. All workers prefer occupations associated with high values of \( \gamma^k_j \). Higher skilled workers can provide more effort at a lower cost to their overall utility. As a consequence those with greater skill benefit proportionally more from employment guaranteeing better chances of success. To see this formally note that, given the discrete improvements to expected performance across occupations, the relative benefits of supervision across agent types are

\[
\frac{\partial}{\partial s} \left( \frac{\Delta B_a}{\Delta \gamma^k_j} \right) = \rho' (e_s) \frac{de_s}{ds} e(s) [U(v^s, P) - U(v^w, P)] + \rho' (e_a) \frac{de_a}{ds} U(v^w, P) > 0
\]  

(17)

Worker payoffs, \( B_a \), are supermodular in occupations and skill, which is sufficient for positive assignment of workers to occupations according to ability.

The highest (lowest) skilled workers are employed in occupations with the greatest (smallest) performance incentives and guidance from firms. According to the definition of occupational diversity, it must be that \( \gamma^1_2 \) and \( \gamma^2_0 \) are the least- and most- skill intensive occupations, respectively. In fact, with positive assignment the monotone ratio likelihood property is a sufficient condition for the entire diffuse set of occupations to employ relatively higher and lower skilled workers about the median occupation, in terms of stochastic dominance. Conversely, the hiring strategy with similar occupations will employ relatively more middle-skill workers, in terms of stochastic dominance\(^{17}\). Firms that implement either hiring strategy will employ workers with

\[^{17}\text{Formally the relative skill intensity across occupational mixes is summarized by } \int_{\Sigma_j \leq k} dG(a) \leq \int_{\Sigma_j < k} dG(a) \text{ and } \int_{\Sigma_j > k} dG(a) \leq \int_{\Sigma_j \geq k} dG(a), \text{ and the fact that skill acquisition is monotonically increasing in ability.}\]
Many skill levels, however they can be compared directly by considering the relative distributions of skill employed in equilibrium.

All firms hire exactly one worker for each occupation in their chosen job mix. Hence employment must be balanced in the sense that the sets of skills assigned to each occupation have equal mass.

\[ |\Sigma^k_j| = |\Sigma^h_j| \quad \text{for all } k, h = 1...n \text{ and } j = 1, 2 \]  

Occupation assignments dictate the skills of workers that firms expect to recruit. With team formation fixed by worker matching behavior, a firm’s expectation is formed regarding the rank of worker skills within each occupation\(^\text{18}\). All firms pay the same expected wage per performance so teams are recruited randomly. Firms expect to draw the median skill level in each occupation (i.e. the average rank) so that the anticipated workforce is composed of teams described by the array

\[ \bar{s}_j = [\text{med}(\Sigma^1_j), \text{med}(\Sigma^2_j), \ldots, \text{med}(\Sigma^n_j)] \]  

### 3.7 Full Equilibrium in the World Economy

A full equilibrium is a vector \( \{\mu_j(\theta), M_j, \theta^x_j, \Sigma^k_j, C_j : \Sigma^1_j \rightarrow \Sigma^n_j, V(\bar{s}, j, \theta)\} \) defined for each job mix, \( j \), and occupation, \( k \). The preceding conditions generate the following result.

**Proposition 1** *A full equilibrium in an open economy exists and is unique.*

**Proof.** See appendix\(^\text{19}\).  

The equilibrium allocation derived above reflects worker and firm behavior in a environment where final labor recruiting and production outcomes are uncertain. Hiring outcomes reveal the skill of workers engaged in production, but not their actual performances. Yet, observations of firm characteristics such as output, labor productivity, or the wage bill reflect both the anticipated outcomes which drive the economy toward equilibrium, and the *ex post* realizations among the workforce. Then what does a full equilibrium in the world economy look like?

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\(^{18}\)It is not possible, for example, for a firm to draw workers with skills equal to \( \sup \Sigma^k_j \) and \( \inf \Sigma^h_j \) within the same team. Moreover the firm is not concerned with the average skill assigned to each occupation because for general distributions of ability, the expected values may not correspond to the same ranking across occupations, and hence cannot be considered as potential equilibrium recruits.

\(^{19}\)An equilibrium in the open economy can be understood as a simple pigeon-hole problem in the assignments over \( \Sigma^k_j \). Firms face information and organization constraints so that employment must be balanced across occupations. An equilibrium hinges on showing that firms are willing to implement hiring strategies such that a fixed point exists where the pigeon-hole problem is resolved.
Realized labor productivity within firms using both occupational mixes is illustrated in Figure 2a\textsuperscript{20}. The zero-anticipated-profit cutoff will bind the distribution of active firms using both potential job mixes. Firms draw productivity parameters from the same distribution, regardless of their eventual hiring strategy. So firms must be indifferent between the anticipated labor performances for either occupational hiring. If the economy is at a stable equilibrium. The entry behavior of risk neutral firms leads to an equal expected team performance across job mixes. On the other hand, firms that implement different hiring strategies will face different variability in labor performance, based on compositions of workers hired. Perhaps surprising, firms offering a diffuse job mix face a smaller variability in labor performance. When firms offer employment in diffuse occupations they recruit a few high skill workers that consistently perform, as well as a few very low skill workers who consistently do not perform.

**Proposition 2** Firms which offer similar occupations \((j=1)\) exhibit a greater variability in labor performance than firms that offer a diffuse set of occupations \((j=2)\).

**Proof.** Each worker performs independently where individual successes have a Binomial Distribution. Given that (1) effort is increasing in ability/skill, (2) there is positive assignment of workers across occupations, and (3) the probability of successful labor performance is increasing in \(\gamma\) and effort, the following relationship holds: 

\[
\sum_{k=1}^{h} \int \left[ \gamma_{k} F(e) dF_{2}(e) \right] < \sum_{k=1}^{h} \int \left[ \gamma_{k}^{1} F_{1}(e) dF_{1}(e) \right]
\]

for all \(h\), where \(F_{j}(e)\) is the distribution of effort levels defined implicitly for the abilities of workers assigned to occupations in mix \(j\). This criterion, plus identical means in labor performance, is equivalent to the definition of Third-Order Stochastic Dominance. Then lower variability of labor performance is evident from the fact that under any concave utility function, with decreasing absolute risk aversion, recruiting workers in occupations corresponding to \(j=2\) is preferred.

\textsuperscript{20}Figure 2 was generated by simulating labor performances across 10,000 firms, with productivities parameters drawn from a Pareto distribution. Each firm draws teams of 10 workers from a uniform distribution where team formation corresponds to positive assortative matching across occupations. Successes for each worker arrive according to a Poisson process.
(See Bawa (1975).) It is worth noting that a more restrictive property concerning the variances in labor performances also holds. It follows from the binomial distribution for the success of each worker that the variance in labor performance at firms employing occupational mix \( j \) is \( \sigma_j^2 = \sum_{k=1}^{N} \gamma_k^j p(\tilde{e}_j) - [\gamma_k^j p(\hat{e}_j)]^2 \).

So the difference in variances is \( \sigma_1^2 - \sigma_2^2 = \sum_{k=1}^{N} \left[ \gamma_k^1 p(\tilde{e}_2) \right]^2 - \sum_{k=1}^{N} \left[ \gamma_k^2 p(\hat{e}_1) \right]^2 \). Using the fact that the each variance is a convex transformation of realized labor performance, stochastic dominance ensures that \( \sigma_1^2 - \sigma_2^2 > 0 \). \( \blacksquare \)

Figure 2a illustrates within-firm variation in labor productivity based solely on team performance. Firms also differ in their capability \((\theta)\) to produce output from individual successes. Variation in labor productivity across all firms is illustrated in Figure 2b. The distribution of output is centered on \( \theta t(\bar{e}, j) \), over the equilibrium distribution of firm-specific productivities, \( \mu_j(\theta) \). The narrow band represents one standard deviation of labor performances when recruiting diverse workers, and the wide band is one standard deviation in labor performance among firms recruiting similar workers. Deviations from the anticipated output reflect the inherent randomness of both recruiting and performance. The following proposition describes aggregate volatility in labor productivity.

**Proposition 3** Across all firms, the variance in observed labor productivity is increasing in average the productivity of firms operating in equilibrium, defined as the the Firm Composition Effect, and increasing in the share of firms offering similar occupations (according to \( j = 1 \)), called the Team Composition Effect.

**Proof.** Firm level output is given by \( \theta t(\bar{e}, j) \) so that the observed variance in labor productivity across firms using either job mix is \( \sigma_0^2 \sigma_j^2 + \bar{\theta}^2 \sigma_j^2 + t(\bar{e}, j)^2 \sigma_0^2 \). The Firm Composition Effect is evident by considering an increase in average firm productivity \( \bar{\theta} \). Aggregate volatility also reflects the share of teams using each job mix; across all firms the variance in labor productivity is \( M_2l(2, \bar{\theta}_2)[\sigma_0^2 \sigma_j^2 + \bar{\theta}_j^2 \sigma_j^2 + t(\bar{e}, 2)^2 \sigma_0^2] + M_1l(1, \bar{\theta}_1)[\sigma_0^2 \sigma_1^2 + \bar{\theta}_1^2 \sigma_1^2 + t(\bar{e}, 1)^2 \sigma_0^2] \). From proposition 2 we have \( \sigma_2^2 < \sigma_1^2 \). The Team Composition Effect can be seen by noting that aggregate volatility is increasing in \( \frac{M_1}{M_1 + M_2} \). \( \blacksquare \)

The Firm Composition Effect is seen by moving rightward across Figure 2b. When average firm productivity is high there are greater swings in output as worker performances vary randomly. The Team Composition Effect is evidenced by the thickness of the bands around the anticipated level of output. For any productivity level the standard deviation in output is smaller when hiring a diverse workforce. So, aggregate volatility will lessen as more firms offer a diffuse set of occupations.

Workers are compensated for their performances. Hence the distribution of labor productivity across occupations and skill groups informs about the distribution of wages paid in the world economy. Note that the firm hiring strategy aimed at a diverse set of workers corresponds to a more steep provision of incentive.
contracts. Firm hiring strategies then have the following implications for the distribution of wages.

**Proposition 4** Wage inequality across skill groups will be greater when the share of firms that offer a diffuse set of occupations is relatively large. Wage inequality within the highest and lowest skill groups is also increasing in the share of firms offering a diffuse set of occupations.

**Proof.** The distribution of income across all workers is summarized by

$$
\sum_{j=1,2} M_j l(j, \theta_j) \int_{\cup_{k=1}^{N_j}} \gamma_k p(e)[\epsilon(s)v^* + (1 - \epsilon(s))v^w]dF(e).
$$

Note $\epsilon(s)$ is strictly increasing. Third-Order Stochastic Dominance of labor performance was demonstrated in the proof of proposition 2. This implies that output, and subsequently wages, for a team employed in occupations according to $j=1$ Lorenz dominates payments to workers in occupations across $j=2$. See Lambert (1993). So, as a larger share of the workforce is employed at firms offering a diverse set of occupations (high $M_2 l(j, \theta_2)$ in equilibrium) the aggregate wage distribution becomes more unequal in terms of Lorenz Dominance. Recall that in equilibrium, firms that offer occupations according to $j=2$ will employ relatively more high and low skill workers, in terms of stochastic dominance. Thus inequality within these groups is also rising in the employment shares of occupations corresponding to $j=2$. The result can also be established using the variance in wages to measure inequality by direct calculation of the income distribution above.

The wage distribution is illustrated in Figure 3. The two dashed lines trace Lorenz curves for workers employed in either job mix, where the more equal distribution of income corresponds to $j=1$. The solid line represents the aggregate income distribution obtained in equilibrium, and is a weighted average (by employment shares) of wages paid in each occupational mix. Lorenz Dominance will be instrumental when examining the welfare effects of trade. Yet the fact that the distributional consequences of firm hiring

![Figure 3: Provision of Incentives across Occupations](image)

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strategies allow for direct comparisons of variance in (log-)wages will help connect the results to empirical evidence in the literature.

This section has highlighted the consequences of firm behavior regarding incentive contracts and job mix in the world economy. In what follows, I will examine how trade liberalization alters the equilibrium behavior of firms. The results above provide a direct link between trade and the real features of an economy that we are interested in.

4 Trade and Product Market Adjustment

In this section I briefly demonstrate how product markets adjust as international trading opportunities become larger. This partial equilibrium response characterizes how firm entry and exporting behavior is affected by falling trade costs, and how aggregate prices change when global competition intensifies. Reallocations in the product market act as a catalyst for labor market adjustments; workers consider aggregate prices when they choose a skill level to acquire, and the hiring potential of firms weighs on their job offerings. The general equilibrium response to trade includes these labor market adjustments and I return to them immediately below.

Let the skill distribution be fixed in a world equilibrium and consider trade liberalization that results from falling transportation costs, $\tau$. As markets become ‘closer’ the opportunities to export and the average profits earned abroad both increase. So as in Melitz (2003) the fall in trading costs shifts the zero-anticipated-profit condition up such that the resulting equilibrium distribution of firms exhibits higher average productivity via selection effects and reallocations across firms. (It is important to note that the ZAP condition shifts for both potential job mixes, so that $\theta_j$ increases for $j=1,2$.) Selection arises because the expansion of exporting firms puts upward pressure on real wages paid to those responsible for the fixed production tasks. In more open economies low productivity firms cannot justify the expense to maintain employment vacancies $(f)$ and must exit.

Selection of some domestic firms changes the provision of performance pay across the labor force. As more firms enter the export market, the employment share of low skill occupations assigned to complete fixed tasks of production must rise. Since workers in these occupations do not receive incentive contracts the following result is obtained. (See proof in appendix.)

**Proposition 5** Trade liberalization reduces the extent of the labor force which is offered incentive contracts, and raises the employment share of low-skill occupations.

Regardless of the occupational mix that firms employ, lower trade costs induce reallocations of labor across
firms and occupations according to proposition 5. More extensive low-skill employment relates to discussions about the potential for openness to destroy "good jobs" and create "bad jobs" in their place. However, this sort of reallocation of employment across firms and job types is not the most relevant adjustment in the labor market. First, changes in employment from production tasks to fixed tasks occur only at a particular skill margin, rather than having a significant impact across the whole labor force. Second, these changes in employment separate the labor force into only two skill groups, low and high, while observed changes in employment across the skill distribution exhibit more interesting and complex patterns\(^\text{21}\). The composition of jobs over the entire labor force is determined by firms’ recruiting strategies. The next section relates trade and the composition of employment by considering the skill acquisition behavior that guides firm hiring practices.

5 \hspace{1em} \textbf{Trade and Labor Market Adjustment}

A partial consequence of openness in product markets is to reduce consumer prices. The general equilibrium response to trade must include worker behavior regarding skill acquisition since lower prices raise the marginal utility of income, and hence the skills used to earn wages. Workers respond to lower prices in their skill acquisition behavior according to the following lemma.

\textbf{Lemma 1} \hspace{0.01cm} \hspace{0.01cm} \hspace{0.01cm} \hspace{0.01cm} The distribution of skills employed in each occupation for aggregate prices \(P\) stochastically dominates the distribution of skills employed in the same occupations for any higher price level, \(\hat{P} > P\).

\textbf{Proof.} Looking to optimal skill acquisition in equation (13), differentiation with respect to \(P\), holding the occupation of a worker fixed, reveals that workers increase their skill level when facing lower prices. Then considering the distributions of skill in any occupation \(\Sigma_j^k\) for price level \(P\) and \(\hat{\Sigma}_j^k\) for price level \(\hat{P}\) we have

\[ \int_{\Sigma_j^k \cap \hat{\Sigma}_j^k} s_a dG(a) \leq \int_{\Sigma_j^k \cap \hat{\Sigma}_j^k} \hat{s}_a dG(a) \]

As the labor force changes shape the pre-trade allocation is no longer stable. Firms are confronted by a labor force with greater skill and so face new hiring possibilities. Renewed competition among workers for the best occupation assignments and teammates also changes the organization of the labor force. Lemma 1 compares the distributions of skills within occupations at different price levels to account for resorting across changing employment opportunities. For any equilibrium allocation, every occupation will be filled by higher skilled workers in terms of stochastic dominance.

\(^{21}\) It is also important to recognize that the real wage paid to low-skill workers increases with trade. The accompanying reduction is skill requirements also benefit worker utility. Hence it is not apparent that employment in a newly created "bad" job necessarily lowers the welfare of a worker. Section 7 provides more discussion on this point.
Skill accumulation among the labor force will most benefit the firms that hire from the top of the skill distribution. These must be firms that offer a diffuse set of occupations with the strongest incentive contracts available. Equilibrium in a liberalizing country can only be restored if tighter competition for high-skilled workers mitigates the potential for better recruiting. This point is made more clear by the following proposition.

**Proposition 6** A reduction in trade costs, \( \tau \), will increase the share of firms that offer diffuse sets of occupations, \( \frac{M_2}{M_1 + M_2} \).

**Proof.** See Appendix.

Propositions 5 and 6 are the key results in describing labor market adjustment to trade. Openness reduces the extent to which performance contracts are offered, and increases the steepness of incentive contracts across the entire labor force. Because of equilibrium sorting patterns in the labor market, the provision of incentive contracts is tied to specific occupations and skill groups. Thus a corollary to the hiring strategies of individual firms predicts how aggregate employment across skill groups responds to greater openness.

**Corollary 1** A reduction in trade costs, \( \tau \), leads to job polarization across the skill distribution. Specifically, the employment share of low-skill and high-skill occupations will rise, while the employment share of middle-skill occupations will fall.

This prediction is consistent with recent changes in labor usages across several developed countries. See Goos and Manning (2007), Autor et al. (2006), and Goos et al. (2009). Empirical definitions of occupations by skill include average years of schooling, mean occupational wage or cognitive requirements. Here, high (cognitive) ability workers acquire more skills and earn higher wages (in expectation), so that the predicted impact of trade on employment shares can be interpreted using each standard metric.

6 Trade, Wages, and Welfare

The previous two sections described product and labor market adjustment to openness highlighting the composition of firms and occupations. In this section I use these results to examine the distribution of

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\(^{22}\) This point is reenforced by the fact that higher ability workers accumulate relatively more skill as prices fall, i.e. \( \frac{\partial s}{\partial p} > 0 \).

\(^{23}\) The current framework abstracts from firm hierarchies where occupational titles also inform about the composition of employment. However, the same results would obtain in a setting where workers played a sequential game following the efforts of a leader in the occupation with the best expected performance. The possibility for endogenous hierarchies in the multi-agent framework would generate even stronger incentives for assortative matching as derived above. (Again see Kim (2010) and Kim and Sly (2010) for more discussion on this point.) Considering different occupations within a firm hierarchy, the impact of trade on employment shares across the skill-distribution also corresponds to job titles.
wages and aggregate volatility as the world economy grows. Without a full parameterization of the model I cannot address the wage gaps at the 90/50 or 50/10 percentiles directly. It will be easiest instead to describe the wage distribution across skill groups. Recall that in terms of stochastic dominance, the diffuse mix of occupations encompasses the highest and lowest skill jobs to receive performance contracts.

Adjustment in the mixes of occupations and performance contracts that workers receive leads to changes in the distribution of wages across skills. Given that openness tends to reduce the share of the labor force receiving performance payments, and increases the share of firms offering steep performance incentives, we have the following implications for the wage distribution.

**Proposition 7** Greater openness increases overall wage inequality. Specifically, the wages of the highest and lowest skill groups increase relative to the median wage. In levels, the highest and lowest skill groups gain in terms of real wages, while the real wages of middle-skill workers have an ambiguous relationship with openness.

Proposition 7 describes a U-shaped differential in the distribution of income across skills. Autor et al. (2008) find evidence of this pattern in the US after 1990. Goos and Manning (2007) show that the UK has exhibited polarization in wages for a much longer period. Dustmann et al. (2009) find a U-shaped changes in the West German wage distribution during the 1980s. Polarization of earnings is an event distinct from simple increases in the skill-premium that have occurred during separate time periods. One may be concerned that variability in performance payments, such as bonuses, has caused U-shaped inequality in annual income, but that earning mobility may have offset the rising inequality over time. (By focusing only on steady-state equilibria the theoretical framework exposes itself to this concern.) However, Kopczuk et al. (2010) show that annual earning inequality trends in the US match the pattern of long-term earnings closely.

The key mechanisms underlying polarization in the labor market are skill accumulation and real price savings in an open economy. Globalization generally describes increases in several activities that cross borders: offshoring, global production sharing, immigration, importing of intermediate goods, as well as exporting of final goods. Yet, shifts in labor demands due to offshoring or immigration are not competing hypotheses to the market access/skill accumulation mechanism presented here. Each may operate in conjunction as the world economy integrates.

A shift in the employment shares changes the distribution of income across skills. As workers are reallocated across occupations the distribution of income within skill groups also changes. In particular, openness increases the employment share of the highest and lowest occupations, thereby raising the degree of heterogeneity in performance payments at each end of the skill distribution. See Figure 1. Hence the
following relationship between trade and wage dispersion can be established.

**Proposition 8** A reduction in trade costs increases wage dispersion within the highest and lowest skill groups receiving performance contracts.

Polarization also appears in residual, or within-group, wage dispersion; see Autor et al. (2008). A problem arises when trying to link the results presented here to the empirical findings pertaining to residual wages because of the role of skill accumulation. Lemieux (2006) demonstrated that changes in workforce composition can generate spurious findings with respect to residual wages because of mechanical issues in estimation. The results of proposition 8 follow from worker skill acquisition behavior in an open economy, and represent actual (not mechanically perceived) changes in wages among skill-groups. By comparing net increases in residual wages at the 90/10 percentiles to the gross changes at the 50/10 and 90/50 percentiles, Autor et al. (2008) demonstrate that polarization in residuals actually occurred in the US after 1990. The variation in wages within skill groups described by proposition 8 is due to differences in performance payments. Lemieux et al. (2009) confirm that this is a key source of variation of wages among similarly skilled workers, especially high-skill groups. Hence the preceding results are consistent with the recent dynamics of the US income distribution.

Rising inequality within groups following trade liberalization has been observed by Attansio et al. (2004), though they focus on tariff reductions in Columbia as opposed to reductions by its trading partners. More closely related is the analysis of Helpman et al. (2010), who argue that international market access will raise inequality within groups because exporting firms will screen workers more intensively, and pay higher wages to better matches. These results speak directly to the evidence regarding exporter premium and wage dispersion across firms in Bernard et al. (1995), Bernard and Jensen (1997), Dunne et al. (2004) and Frás et al. (2009). Here rising wage inequality within groups is a consequence stochastic worker performances effects and occupational assignments. Menezes-Filho et al. (2008) and Abowd and Kramarz (1999) highlight the importance of worker effects and occupation of employment relative to establishment effects across several countries. Still worker, occupation and establishment (firm) characteristics all appear crucial to explaining the distribution of wages. The worker and occupational focus here should be viewed as complementary to the emphasis on match-specific elements in Helpman et al. (2010).

Observed wages respond to trade via several mechanisms: skill accumulation, payment schemes, effort levels, etc. The welfare effects of trade are less complicated because these determinants of the wage distribution represent the endogenous behavior of workers and firms in the world economy.

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24 In Lemieux et al. (2009) and in proposition 8, rising wage dispersion among low-skill workers is mitigated by the fact that few low-skill workers receive performance payments.
With risk adverse agents only the provision of incentive contracts and aggregate price level influence welfare. There are two countervailing forces at work when the world economy becomes more integrated. First, trade reduces the aggregate price level. Since the competitive wage of low-skill workers in fixed production tasks is the numeraire, openness increases the real wage of all workers. Second, trade changes the provision of incentive contracts such that on average workers face more volatility in their income levels. (This is a consequence of the Lorenz Dominance of the pre-trade wage distribution.) Thus, with risk adverse workers populating each country the welfare effects of trade cannot be established generally. Krishna and Senses (2009) find evidence that import competition in the US causes sustained increases in earnings volatility among workers, with significant welfare consequences. Moreover, these observed changes in volatility reflect persistent increases in the variability of earnings, consistent with a new steady-state composition of performance incentives being offered\(^\text{25}\).

Figure 4 illustrates the pre- and post-trade Generalized Lorenz curves as characterized by Shorrocks (1983). The price effect of trade shifts the more unequal post-trade Lorenz curve up, raising the possibility that the Generalized Lorenz curves cross twice. In this instance the welfare consequences of trade are ambiguous, see Lambert (1993). Clearly, trade is more likely to be welfare improving when the price effect is large.

A last point to discuss is the relationship between trade and variability in production. Recall that trade will raise the average productivity of firms operating in equilibrium, and increase the share of firms that hire production teams into diffuse occupations. Then using proposition 3 we have the following.

**Proposition 9** *Greater openness reduces the variability in labor productivity observed within firms on av*

\(^{25}\) Another potential mechanism by which openness can influence earnings volatility is put forth by Rodrik (1997). He suggests that by increasing the elasticity of demand for goods, trade exposure increases the elasticity of demand for labor. As a result market shocks generate relatively larger swings in employment and wages.
average via the Team Composition Effect, but increases variability in labor productivity observed across firms via the Firm Composition Effect.

The specific hiring strategies used by firms, and strategic changes following trade liberalization, are not likely to be observed by an econometrician. The last proposition can help identify the mechanisms at work in the model. Reductions in the volatility of labor productivity among firms that persist in open economies would provide indirect evidence that firms adjust the provision of incentive contracts and occupations offered.

7 Discussion

Some of the specific features of the model were chosen for expositional reasons while some are necessary to describe an equilibrium allocation in the labor market. Equations (16) and (17) are sufficient conditions for positive matching within teams and positive assignment of workers to occupations. Workers perform independently, so the firm cares only about the efforts of individual workers. The production process itself exhibits no complementarities between the skills of workers. Positive assortative matching in equilibrium team formation is a result of the strategic behaviors about efforts endogenous to the model. Positive assignment of workers, on the other hand, is due to the modeled proportional increase in signal quality that comes from better jobs. If better employment guaranteed only a fixed (additive) increase in the propensity to succeed, positive assignment may not arise. However, a more intensive provision of incentive contracts among higher skilled workers is the empirically relevant scenario; see Lemieux et al. (2009) and Bayo-Moriones et al. (2010). The assumptions regarding occupational diversity are made with this fact in mind.

Firm heterogeneity in the model above reflects differences in the ability to capture worker performance, but no differences in the capacity of firms to harness worker skills. As a result all firms are equally competitive in attracting teams of labor, and recruiting occurs randomly across firms offering the same occupations. The model could easily include the possibility that firms can make investments that allow them to better address the agency problem; i.e. pay a fixed costs to obtain a higher value of $\Gamma$. As a result, firms that make investments in directing worker efforts would attract higher skilled teams, and exhibit greater labor performance relative to those that do not. This approach has been taken by others in various contexts. See Yeaple (2005) and Davidson et al. (2008) for investments in technologies with skill complementarities, and Helpman et al. (2010) for investments in worker screening with complementarities between workers’ abilities.

While these authors have demonstrated that greater foreign market access can incentivize technological and recruiting investments at the firm level, I am concerned presently with adjustments within firms, and

\[ \text{If the production environment were any such that firms offered JPE wage schemes, then the behavior of workers would still lead to positive matching. Under JPE incentive contracts } \frac{\partial \Gamma}{\partial \mu} > 0 \text{ and hence equation (16) remains positive.} \]
Evidence of skill-upgrading (i.e. relatively greater demand for high skills) in exporting firms has been presented in the literature, but this mechanism is absent from the model to highlight the diversity of occupations in open economies.

One final issue to discuss is dynamics in the model. The hiring and entry behavior of firms in equilibrium are responses to their anticipated recruiting potential. Since the firm has to pay fixed costs to maintain employment vacancies regardless of the workforce it recruits, one may be concerned that firms may change their hiring behavior over time. For example a firm that recruits several teams of high quality might find it optimal to expand its workforce in the following period. Of course, this behavior could unravel the equilibrium derived above. While not modeled explicitly, the motivation for assuming firms perennially face uncertain recruiting outcomes is rooted in high levels of labor mobility. Davis et al. (1998) document large changes in firm-level employment within sectors that reallocate nearly a fifth of the labor force each year. Burgess et al. (2000) find additional mobility in the labor market as workers leave firms where employment opportunities persist. The high volume of worker flows into and out of firms suggest that recruiting is a constant issue that must be addressed.

8 Conclusion

When decomposing income distributions in many countries, worker, occupation and residual elements are tantamount to wage variation resulting from firm effects, if not more important. A refined look at wage dispersion across skill groups reveals that changes in inequality and labor usage are often more complex than simple shifts in the skill premium. Many developed countries have experienced prolonged episodes of polarization, or U-shaped differentials across skill levels, in both wages and employment. Furthermore, as labor market outcomes have transformed, so have the payment methods used to clear the market. The provision and composition of incentive contracts appears fluid and crucial to explaining changes in the wage distribution. This paper examined the impact of openness on these features of the labor market in a framework that incorporates contracting problems, occupational diversity and worker heterogeneity.

Labor market adjustment was characterized in terms of worker skill acquisition behavior and firm choices regarding occupations and performance contracts. First, workers respond to openness by skill-updating. Recognizing changes in the composition of the labor force, firms alter their hiring strategies towards employment with a more steep, but less extensive, provision of performance pay, as well as relatively greater usage of high- and low- skill occupations. The implications of labor market adjustment for wage outcomes

27The roles of workforce adjustment and random variation in firm-level productivity in shaping trade patterns is taken up by Cunat and Melitz (forthcoming).
are rising inequality, with middle-skill workers losing relative to those at the top and bottom of the skill distribution. Changes in the provision of incentive contracts introduced more income risk for the average worker, making the welfare consequences of openness difficult to characterize generally.

A key feature of both trade patterns and the wage distribution that is missing from the analysis is industry effects. Though it was not highlighted in the text, there is a large role for industry composition to influence trade adjustment. Firms choose their hiring strategies based on their expectations regarding worker performances; the fact that profits and revenues are convex functions of labor performance was instrumental in deriving their behavior both pre- and post-trade. The convexity of these outcomes is fully described the elasticity of demand for their goods. Hence these elasticities inform about the extent of adjustments to occupational diversity and incentive contracts across different industries. A full analysis of this point is deferred to future study.

References


Appendix

Existence and Uniqueness of Full Equilibrium: proof of proposition 1

Given the job compositions in Definition 1, a labor market equilibrium is characterized by unique effort levels for each worker that satisfy equation (12); skill acquisition behavior governed by (13); wage schemes for each any team recruited according to equations (14) and (15); for any mix of jobs, worker payoffs are supermodular in skills (equation 16), so that matches between workers are positively assorted; expected performance payments are supermodular in skill and occupations, so that workers are positively assigned to incentive contracts as in (17). These conditions define an unique core allocation for any aggregate price level P.

A steady-state equilibrium in product markets can be established for any stable allocation of the labor force using methods identical to Melitz (2003). A full equilibrium exists and is unique only if stable product and labor market equilibria coincide. Connecting product and labor market allocations into a full equilibrium requires firms to be willing to offer employment opportunities such that an assignment across occupations is balanced, with full employment.

Firm hiring strategies are bound in equilibrium by the ZAP condition for each job mix. In equilibrium firms must be indifferent between the labor performance they expect from the teams they anticipate to recruit. The Indifference Condition is

$$\Pi(\bar{s}_1, 1, \theta_1^p) = \Pi(\bar{s}_2, 2, \theta_2^p)$$  

(IC)

The willingness of firms to offer different occupational mixes is described by the Indifference Condition. However firms are only able to offer job mixes that satisfy definition 1. Regardless of the mix of occupations that firms choose, they will hire one worker per job type. So, there is a series of Balance conditions that must be satisfied by firm hiring strategies in equilibrium. Using $\gamma_{kj}$ as the reference occupation for each job mix without loss of generality, and assuming an atomless distribution of worker abilities, the Balance Condition requires that for all occupations $k = 1...n$, in job mix $j = 2$, the following holds

$$|\Sigma_2| = |\Sigma_2^k|$$  

(BC.1)

Employment in the alternate job mix must also be balanced and evenly divide the remaining mass of workers. The series of Balance conditions for job mix $j = 1$ across all occupations $k$ is

$$|\Sigma_2| = \frac{n(L - L^{ent})}{L_2^p} - \frac{L_1^p}{L_2^p} |\Sigma_1|$$  

(BC.2)

Full employment requires that production workers for each job mix $L_j$ and workers assigned to fixed tasks of production $L^{ent}$ cover the entire labor force. Non-production workers are assigned to investment tasks necessary for any production level. So the wage bill of these worker must equal aggregate profits:

$$L^{ent} = M_1 \Pi(\bar{s}_1, 1, \bar{\theta}_1) + M_2 \Pi(\bar{s}_2, 2, \bar{\theta}_2)$$

Then full employment must satisfy

$$L - L^{ent} - L_1^p - L_2^p = 0$$

$$L - M_1 \Pi(\bar{s}_1, 1, \bar{\theta}_1) - M_2 \Pi(\bar{s}_2, 2, \bar{\theta}_2) - \sum_{k=1}^{n} |\Sigma_1^k| - \sum_{k=1}^{n} |\Sigma_2^k| = 0$$  

(FE)

Given positive assignment over the number $2n + 1$ different occupations available there are $2n$ cutoff values that need to be determined to describe a full equilibrium (Walras’ Law). Full employment across all occupations is one condition of a full equilibrium. The Balance Conditions provide $2n - 2$ implicit functions of the ability cutoff levels; the missing conditions arise because the mass of agents in one occupation cannot be related to itself, and the balance condition does not apply to occupations necessary to complete fixed tasks of production. The final equilibrium condition is the Indifference Condition regarding job mix offered by firms. (The Indifference Condition provides an implicit function of the equilibrium cutoff values for each occupation per the definitions of $s_j$.) Therefore it only remains to show that with Full Employment the
Incentive Compatibility and Balance conditions determine unique ability cutoffs between occupations. To verify existence of an equilibrium we find a correspondence, \( F(\cdot) \), that maps the \([a_{\min}, a_{\max}]^{2n}\) space onto itself, such that any fixed point is an equilibrium. Then we demonstrate that the correspondence indeed contains a fixed point.

Since the correspondence must be defined such that any fixed point is an equilibrium we write \( F(\cdot) = [F_{IC}, F_{BC2}, F_{BC3}, ... , F_{BCn-1}, F_{FE}] \) to accommodate each equilibrium requirement. We need to define four types of correspondences: one for indifference, one for balance within job mix \( j=2 \), one for balance in job mix \( j \geq 2 \), and one for full employment. Let

\[
F_{IC}(\vec{a}_1, \vec{a}_2) = \{ (\vec{y}_1, \vec{y}_2) \in [a_{\min}, a_{\max}]^{2n} : \\
|\Pi(\vec{y}_1, 1, \vec{\theta}_1) - \Pi(\vec{y}_2, 2, \vec{\theta}_2)| \leq |\Pi(\vec{y}_1, 1, \vec{\theta}_1) - \Pi(\vec{y}_2, 2, \vec{\theta}_2)|, \\
\forall (\vec{y}_1, \vec{y}_2) \in [a_{\min}, a_{\max}]^{2n} \} \quad (A.4)
\]

so that the correspondence maps vectors of abilities into the least non-negative value of the Indifference Condition. Also, for occupations \( k = 1...n \) define

\[
F_{BC2}^k(\vec{a}_1, \vec{a}_2) = \{ (\vec{b}_1, \vec{b}_2) \in [a_{\min}, a_{\max}]^{2n} : \\
\left|\Sigma_2^1(\vec{b}_1, \vec{b}_2) - \Sigma_2^k(\vec{b}_1, \vec{b}_2)\right| \leq \left|\Sigma_2^1(\vec{b}_1, \vec{b}_2) - \Sigma_2^k(\vec{b}_1, \vec{b}_2)\right|, \\
\forall (\vec{b}_1, \vec{b}_2) \in [a_{\min}, a_{\max}]^{2n} \} \quad (A.5)
\]

and

\[
F_{BC3}^k(\vec{a}_1, \vec{a}_2) = \{ (\vec{b}_1, \vec{b}_2) \in [a_{\min}, a_{\max}]^{2n} : \\
\left|\Sigma_2^1(\vec{b}_1, \vec{b}_2) - \Sigma_2^k(\vec{b}_1, \vec{b}_2)\right| \leq \left|\Sigma_2^1(\vec{b}_1, \vec{b}_2) - \Sigma_2^k(\vec{b}_1, \vec{b}_2)\right|, \\
\forall (\vec{b}_1, \vec{b}_2) \in [a_{\min}, a_{\max}]^{2n} \} \quad (A.6)
\]

so that these correspondences map vectors of abilities into the least non-negative values of the Balance Conditions. Finally let

\[
F_{FE}(\vec{a}_1, \vec{a}_2) = \{ (\vec{f}_1, \vec{f}_2) \in [a_{\min}, a_{\max}]^{2n} : \\
|L - M_1\Pi(\vec{s}_1, 1, \vec{\theta}_1) - M_2\Pi(\vec{s}_2, 2, \vec{\theta}_2) - \sum_{k=1}^{n} \Sigma_1^k(\vec{f}_1, \vec{f}_2) - \sum_{k=1}^{n} \Sigma_2^k(\vec{f}_1, \vec{f}_2)| \leq \\
|L - M_1\Pi(\vec{s}_1, 1, \vec{\theta}_1) - M_2\Pi(\vec{s}_2, 2, \vec{\theta}_2) - \sum_{k=1}^{n} \Sigma_1^k(\vec{\gamma}_1, \vec{\gamma}_2) - \sum_{k=1}^{n} \Sigma_2^k(\vec{\gamma}_1, \vec{\gamma}_2)|, \\
\forall (\vec{f}_1, \vec{f}_2) \in [a_{\min}, a_{\max}]^{2n} \} \quad (A.6)
\]

The are several things to note about these correspondences. First, they are everywhere hemi-continuous. Second, they are each defined over the convex, compact set of abilities \([a_{\min}, a_{\max}]\). Lastly, all together, \( F(\cdot) \) maps \([a_{\min}, a_{\max}]^{2n}\) onto \([a_{\min}, a_{\max}]^{2n}\). So by Kakutani’s theorem, these correspondences have a fixed point. Each correspondence is defined to map vectors of ability into the least non-negative values of the equilibrium conditions. If each correspondence maps to a fixed point, then it must return cutoff vectors that satisfy equal expected profits across job mixes, equal mass of employment across occupations, and full employment. Because such a fixed point must exist, so must an equilibrium. The last step is to establish
uniqueness.

A sufficient condition for uniqueness is that the Jacobian matrix of the equilibrium conditions is sign-definite. With positive assignment, the range of abilities employed in each occupation is convex so that we can describe the equilibrium using the upper-ability cutoffs for each occupation\(^{28}\); define the upper-ability cutoff for occupation \(\Lambda^k_j\) as \(A^k_j\), and for workers performing fixed tasks, \(A_f\). Without loss of generality, write the column-elements of the Jacobian matrix as the derivative of each equilibrium condition, with respect to \(A^k_j\) for job mix \(j = 2\) and then \(j = 1\), in ascending order of \(k\), and finally \(A_f\). Then, write the row-elements with the Indifference Condition (IC) stacked over the Balance Conditions (BC.2 then BC.1) and the Full Employment Condition (FE) last.

Then the Jacobian matrix of the system of equilibrium conditions is

\[
\begin{bmatrix}
\frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \ldots & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \frac{d\Pi(\hat{s}_1, \theta^e_2)}{d\bar{A}_1} & \ldots & \frac{d\Pi(\hat{s}_1, \theta^e_2)}{d\bar{A}_1} & \ldots & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} \\
G(\bar{A}_1) & -G(\bar{A}_2) & 0 & 0 & 0 & \ldots & 0 \\
G(\bar{A}_2) & 0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
G(\bar{A}_1) & 0 & 0 & \frac{-L^p_1}{L^0_1}G(\bar{A}_1) & 0 & \ldots & 0 \\
G(\bar{A}_2) & 0 & 0 & 0 & \frac{-L^p_2}{L^0_2}G(\bar{A}_2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \ldots & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \ldots & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2} & \ldots & \frac{d\Pi(\hat{s}_2, \theta^e_2)}{d\bar{A}_2}
\end{bmatrix}
\]

As we are only interested in the sign-definiteness of the matrix above, it is helpful is write it simply in terms of the signs of each element.

\[
\begin{bmatrix}
+ & + & \ldots & - & - & \ldots & + \\
+ & - & 0 & 0 & 0 & \ldots & 0 \\
+ & 0 & - & 0 & 0 & \ldots & 0 \\
+ & 0 & 0 & - & 0 & \ldots & 0 \\
+ & 0 & 0 & 0 & - & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
- & - & - & - & - & \ldots & -
\end{bmatrix}
\]

It is straightforward to verify that the determinants of the principal minors of this matrix alternate in sign, so that the Jacobian must be a negative-definite matrix. This verifies uniqueness of a full equilibrium.

**Trade and the Provision of Performance Pay: proof of proposition 5**

Workers employed to complete fixed entry tasks \(L^{ent} = L^{ent}_1 + L^{ent}_2\), are engaged in the investment activities that firms undertake prior to production. The market clearing condition for these workers ensures that their wage bill is equal to aggregate profits: \(L^{ent} = M_1\Pi(\hat{s}_1, 1, \hat{\theta}_1) + M_2\Pi(\hat{s}_2, 2, \hat{\theta}_2)\). The wage paid to these workers is set to be the numeraire, so an increase in the total wage bill following trade liberalization corresponds necessarily to an increase in employment, \(L^{ent}\), and with full employment and decrease in the mass of labor receiving performance pay.

First note that lower trade costs, \(\tau\), increase exporting profits and the opportunities to export such that profits of a representative firm rise; i.e. \(\frac{d}{d\tau}[\Pi^D(\hat{s}_j, j, \hat{\theta}_j) + \Pi^X(\hat{s}_j, j, \hat{\theta}_j)] > 0\). From (10), higher average profits with trade are associated with fewer firms operating in a more open equilibrium. Thus aggregate profits respond to trade via an increase in average profits and a decrease in the mass of firms operating.

\(^{28}\)Recall that the definition in (MLRP) does not make pairwise comparisons for all occupations so that each \(A^k_j\) may demark a cutoff between occupations in the same, or alternate job mixes. Such a definition is not be necessary to establish a unique equilibrium. Arbitrary orderings are permitted to maintain generality.
Trade reduces the extent of performance pay, and increases the employment share of low skill occupations, \( L_j \), as long as the percentage change in the mass of firms is less that the percentage increase in profits of the representative firm for \( j=1,2 \). Calculating directly,

\[
\left| \frac{dM_j}{d\Pi(s_j, j, \tilde{\theta}_j)} \right| = \frac{[\Pi^D(s_j, j, \tilde{\theta}_j) + b^{ex}\Pi^X(s_j, j, \tilde{\theta}_j)] - f - b^{ex}f_r}{[\Pi^D(s_j, j, \tilde{\theta}_j) + b^{ex}\Pi^X(s_j, j, \tilde{\theta}_2)]} < 1
\]

**Trade and the Composition of Occupations: proof of proposition 6**

The relative change in firm hiring strategies following trade liberalization can be seen by totally differentiating equation (10) to obtain \( \frac{dM_1}{M_1} - \frac{dM_2}{M_2} > 0 \), since trade necessarily reduces the number of firms of each type. Then using lemma 1, and totally differentiating across occupations, we have

\[
-d \left[ \frac{dM_j}{M_j} \right] = \sum_{k=1}^{n} \beta-1 \frac{\rho P^\delta t(e, \tilde{\theta}_j)}{V(s_j, \tilde{\theta})} \beta-2 \left[ XP + b^{ex}X^*P^{*\tau}\beta-2 \right] \frac{dt(\cdot)}{ds^k} \frac{ds^k}{d\tau} \\
\frac{1}{\beta} \left( \frac{\rho P^\delta t(e, \tilde{\theta}_j)}{V(s_j, \tilde{\theta})} \beta-1 \left[ XP + b^{ex}X^*P^{*\tau}\beta-1 \right] \\
= \sum_{k=1}^{n} \frac{dt(\cdot)}{ds^k} \frac{ds^k}{d\tau} (\beta-1) \left[ XP + b^{ex}X^*P^{*\tau}\beta-2 \right] \frac{\rho P^\delta t(e, \tilde{\theta}_j)}{V(s_j, \tilde{\theta})} \left[ XP + b^{ex}X^*P^{*\tau}\beta-1 \right]
\]

So the percentage change in the mass of firms using each hiring strategy is an increasing, concave function of team labor performance, \( t(e, j) \), with a decreasing coefficient of Absolute-Risk Aversion. Since team performances from occupations \( j=1 \) Third-Order Stochastically dominate performances in \( j=2 \), it must be that \( \frac{dM_1}{M_1} - \frac{dM_2}{M_2} > 0 \). Following trade liberalization and skill updating by the labor force, fewer firms operate in equilibrium, but relatively more firms that offer diffuse occupations remain.