Full Length Research Paper

Fuzzy portfolio optimization using Chen and Huang model: Evidence from Iranian mutual funds

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Accepted 19 April, 2012

In this study, we examine the performance of mutual funds in Iran in order to determine the best allocation of money to equity mutual funds base on the Chen and Huang (2009) model. First, funds are categorized based on the four criteria through cluster analysis: return rates, standard deviation, Sortino ratio, and turnover rate. In the Chen and Huang (2009) model, the Treynor ratio has been used while we use Sortino ratio. The clustering process shows that mutual funds can be categorized into three groups which we called, inferior performance funds, aggressive funds, and good performance funds. Chen categorized the funds in Taiwan into four groups. Future return rates and future risk are presented as triangular fuzzy numbers and then portfolio optimization problem is developed in two ways: maximizing the future expected return subject to the given greatest future risk, and minimizing the future risk subject to a required lowest future expected return. We suggest that risk-averse investors consider the minimization of the risk as objective and take returns as constraint. Risk-seeking investors can maximize the returns at a given level of risk.

Key words: Portfolio optimization, equity mutual funds, performance indices, cluster analysis, fuzzy set.

INTRODUCTION

The mutual fund industry is considered as one of the most dominant players in the world economy and is an important constituent of the financial sector and Iran is no exception. Mutual funds provided diversified portfolios and professional fund management to a large number, particularly small investors. As mutual funds are managed by professionals, they are considered to have a better knowledge of market behaviors. They also maximize gains by proper selection and timing of investment. From the data given by the financial information processing center of Iran (www.Fipiran.ir ), over the last two years (2009 to 2011) mutual funds in Iran have given impressive return and many funds outperformed the market. The remarkable performance of this industry has attracted many investors to invest in mutual funds and researchers to study and examines the performance of the fund, over this period.

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security data in the past. However, it is hard to ensure this kind of assumption for the real ever-changing financial markets. The fuzzy set, proposed by zadeh (1978), is a powerful tool used to describe an uncertain financial environment where not only the financial markets but also the investment decision makers are subject to vagueness, ambiguity or fuzziness.

One of the criteria proposed by Chen and Huang (2009) for cluster analysis is the Treynor ratio which we have in the present study replaced with the Sortino ratio. The main advantage Sortino ratio has is that it penalizes only those returns falling below a minimum acceptable rate (MAR), or required return rate.

The remainder of the paper is organized as follows; brief background of portfolio selection approaches and performance evaluation techniques; portfolio selection model and cluster analysis; numerical results for Iranian equity mutual funds; conclusion.

LITERATURE REVIEW

In 1952, Markowitz (1952, 1959) published his pioneering work and laid the foundation of modern portfolio analysis. The core of the Markowitz mean variance model is to take the expected return of a portfolio as investment return and the variance of the expected return of a portfolio as investment risk.

Markowitz formulated the problem of portfolio optimal selection among n assets as a constrained quadratic minimization problem (Markowitz, 1952). Markowitz relied upon the assumption that an investor wants to achieve a predetermined expected return, while, at the same time, minimizing his/her risk:

\[ \min \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \]
\[ \sum_{i=1}^{n} x_i r_i = r^* \]
\[ \sum_{i=1}^{n} x_i = 1 \]
\[ x_i \geq 0 \]

(1)

Where \( x_i \) is the percentage of money invested on asset i (the decision variables), \( n \) the number of assets available, \( r_i \) the expected return of asset i, \( r^* \) the desired expected return, and \( \sigma_{ij} \) the covariance between assets i and j.

The Markowitz model, despite its theoretical importance, has never been extensively used to make large-scale portfolios. Because of the computational difficulties that a large-scale quadratic programming problem with a dense covariance matrix has, there are some restricting assumptions, such as symmetric distribution of the returns. So the model has experienced much development in two directions: alternative portfolio selection models’, and ‘equilibrium models’. Some of the alternative models, with minor changes, are Mean-Semivariance, Mean-absolute deviation, Mean-Variance-Skewness, or Mean-variance with logical constraints; and, if more fundamental changes are considered, we can include Robust optimization, Markov chain, Multi-objective decision-making, Possibility and Fuzzy theory, or Minimax modelling of portfolio optimization. For the equilibrium models, the capital asset pricing models (CAPM) developed by Sharpe (1964, 1970), Lintner (1965), or the arbitrage pricing theory formulated by Ross (1976) and developed by Huberman (1982) can be named.

Sharpe (1966) introduced a measure for the performance of mutual funds by considering both systematic and unsystematic risks (called total market risk) as the measurement of excess return. Sharpe ratio is still one of the most widely used portfolio ratio measure due to its simplicity. Treynor (1965) suggested that a portfolio’s performance can be measured using the ratio of the mean risk premium to the systematic risk of the portfolio over the evaluation period rather than using total risk. Jensen (1968) proposed the Jensen Index as a more absolute performance measure than either the Sharpe or Treynor Indices. Jensen’s alpha measures that part of portfolio performance which is not explained by its CAPM beta. Frank Sortino and Van der Meer (1991) proposed the Sortino ratio as an extension of the Sharpe ratio. While the Sharpe ratio takes into account any volatility in return of an asset, Sortino ratio differentiates volatility due to up and down movements. The up movements are considered desirable and not accounted in the volatility. In addition, data envelopment analysis has also been used to evaluate the performance of mutual funds (Murthi et al., 1997; Basso and Funari, 2001).

Since the 1960s, fuzzy set theory has been widely used to solve many problems in financial risk management. By using fuzzy approaches, experts’ knowledge and investors’ subjective opinions can be better integrated into a portfolio selection model. Zadeh (1978) proposed fuzzy decision theory. Ostermark (1998) proposed a dynamic portfolio management model where the fuzzy decision principle can be combined. Tanaka et al. (2000) formulate fuzzy decision problems based on probability events. Watada (2001) presented another type of portfolio selection model based on the fuzzy decision principle. This model is directly related to the mean-variance model, where the goal rate for an expected return and the corresponding risk are described by logistic membership functions. Carlsson et al. (2002) studied a portfolio selection model in which the return rate of securities follows the possibility distribution. Ammar and Khalifa (2003) introduced the formulation of fuzzy portfolio optimization problem based on the Markowitz’s mean-variance model. In their model, the future return rates and variances are denoted by triangular fuzzy numbers. Lacagnina and Pecorella (2006) proposed a
multistage stochastic soft constraints fuzzy model to solve a portfolio management problem. Enriqueta et al. (2007) presented a fuzzy downside risk approach for managing portfolio problems in the framework of risk-return trade-off using interval-valued expectations. Xiaoxia (2007) gave a new definition of risk for random fuzzy portfolio selection. Recent advances of portfolio selection model considered integration of various multi-criteria decision making models such as fuzzy analytic hierarchy process (AHP) in Tiryaki and Ahlatcioglu (2009) and expert systems such as Smimou et al. (2008). Chen and Huang (2009) presented another type of portfolio selection model. In their model, cluster analysis is used to categorize mutual funds. Then the optimal investment proportions in each category are determined through use of fuzzy optimization model based on Ammar and Khalifa (2003) model.

METHODOLOGY

Here, we first calculate evaluation measures for each fund. The process of categorizing is then followed based on the measures. The aim is to induce the characteristics of funds into groups to help investors in selecting which mutual funds to invest in.

Evaluation indices

Return rate

The higher the return rate, the better the performance of the fund is.

\[ R_i = \frac{NAV_{t1} - NAV_{t0}}{NAV_{t0}} \times 100 \% \]  
(2)

Where I is the number of funds, \( R_i \) is the return of portfolio I (in percentage), \( NAV_t \) is the net asset value of funds at current evaluation period, \( NAV_{t0} \) is the net asset value of funds at previous evaluation period.

Turnover rate

A high turnover rate shows the aggressive operation of the funds, and hence causes an increase in transaction costs, while a low turnover rate shows stability in the performance of the funds. On average, a high turnover rate does not really benefit return rate or other indices.

\[ \text{Turnover rate} = \frac{\text{The amount of values in transactions (buy or sell)}}{\text{Average value of total assets}} \]  
(3)

Sortino ratio

Chen and Huang (2009) applied Treynor ratio as one of the criteria for cluster analysis. This ratio measures the amount of excess return relative to the systematic risk (\( \beta \)).

\[ T_i = \frac{R_i - R_f}{\beta_i} \]  

Where \( i \) is the number of funds; \( T_i \) is the Treynor ratio of portfolio \( i \); \( \beta \) is the beta of portfolio \( i \) over the evaluation period; \( R_i \) is the average holding period returns on the portfolio \( i \) over the evaluation periods; \( R_f \) is the average risk-free return over the evaluation period.

Treynor ratio is a classic performance measure that takes into account any volatility in return of an asset. This ratio also explicitly constructed for situations in which only a small portion of the investor’s wealth is allocated to the mutual fund under consideration (Eling and Schuhmacher, 2007). The beta factor is generally calculated using the correlation between the return of a market index and the return of mutual fund so it requires the choice of a good reference index, because the denominator heavily depends on the selected benchmark.

We applied The Sortino ratio. This is similar to the Sharpe ratio, but instead of using the standard deviation as the denominator, it uses downside deviation to distinguish between “good” and “bad” volatility. The downside deviation is calculated as follows:

\[ \text{Downside deviation (DDP)} = \sqrt{\frac{\sum_{i=1}^{n} \min(0, r_i - r_t)^2}{n}} \]  

Where \( n \) is number of weeks, \( r_i \) is weekly return of portfolio, \( r_t \) the target return or minimum acceptable return (MAR) and \( DD_P \) the downside deviation of returns of portfolio \( P \) below the minimum acceptable return. In this paper risk free rate have been taken as MAR.

The Sortino ratio is defined as follows:

\[ \text{Sortino ratio} = \frac{R_p - R_t}{DD_P} \]  
(4)

Where \( r_p \) is the expected return of portfolio \( P \), \( r_t \) the target return or minimum acceptable return (MAR).

Standard deviation

The standard deviation reports a fund’s volatility, which indicates the tendency of the returns to rise or fall drastically in a short period of time. A security that is volatile is also considered higher risk because its performance may change quickly in either direction at any moment.

\[ \sigma_i = \sqrt{\frac{\sum_{i=1}^{t} (R_{ij} - \bar{R})^2}{t-1}} \]  
(5)

Where \( \sigma_i \) is the weekly standard deviation for portfolio \( i \); \( R_{ij} \) is the return rate of portfolio \( i \) on the \( j \)th week; \( \bar{R} \) the average return rates of \( t \) weeks.

Since the measurement unit differs across the four evaluation indices, we should normalize the data. The normalization process helps to ensure the result of clustering is on the same basis, and thus is more accurate. Normalization then is achieved through the formula that follows:

\[ \bar{a}_{ij} = \frac{\bar{a}_i - \min_j(\bar{a}_{ij})}{\max_j(\bar{a}_{ij}) - \min_j(\bar{a}_{ij})} \]  
\( i, j = 1, 2, ..., N \)  
(6)

\( \bar{a}_i \) is the Normalization of fund \( i \); \( \bar{a}_{ij} \) is the average unit of fund \( i \) during a 1-year period; \( \min_j(\bar{a}_{ij}) \) is the minimum unit of one index among all funds; \( \max_j(\bar{a}_{ij}) \) is the maximum unit of one index
among all funds.

**Clustering of funds**

Due to the large amount of equity funds, the determination of the investment proportion of each fund is difficult. Moreover, investors generally are not supposed to distribute their wealth in hundreds of funds. Therefore, the categorization is performed based on the four evaluation criteria so that funds with similar performance are placed in the same group and the investor can choose from these groups based on their own preferences.

Cluster analysis classifies a number of observations into two or more mutually exclusive groups based on a set of variables. The objective of clustering is to group observations with a high degree of similarity into one group, and to discriminate observations with a high degree of variances into different groups. The clustering of funds in this research at the first stage uses Ward’s method to compute the distance between clusters, while the distance between objects is computed based on Euclidean distance. The number of clusters, k, is determined using a tree diagram then funds are categorized using the non-hierarchical method, K means. K-means in computation is similar to analysis of variance (ANOVA), and the objectives are:

1) To minimize the variance of objects in one cluster
2) To maximize the variance among clusters

**Asset allocation model**

The assumptions of the study are:

1) Only a single period is considered for the models.
2) Investors have a specific amount of money to invest at the present time.
3) The herding behavior of investors/fund managers does not exist.
4) Short sales are not allowed.
5) Investors have similar expectations about future outcomes over the entire horizon of one period.
6) There are no transaction costs, taxes, dividends or interest charges that occur during evaluation period, and there is no inflation.
7) The correlation between clusters can be neglected.

**Optimization based on fuzzy models**

The future expected return and the future risk can be estimated when the historical return of securities are known. Similarly, given the past return of each cluster, we can only approximate values as its future expected return and future risk. The return and future variance of each cluster are used in form of fuzzy numbers instead of being presented in a crisp way.

Fuzzy return rates are denoted as a triangular fuzzy number, \( \tilde{R} = (l, m, n) \), whose membership function is as follows:

\[
\mu_{\tilde{R}}(t) = \begin{cases} 
\frac{t - l}{m - l} & , l \leq t \leq m \\
1 & , t = m \\
\frac{(n - t)}{(n - m)} & , m \leq t \leq n \\
0 & , t \leq l \text{ or } t \geq n 
\end{cases}
\]  

(7)

The \( \alpha \)-level confidence of in terms of interval values corresponding to the triangular fuzzy number is:

\[
\tilde{R}_\alpha = [R^-_\alpha, R^+_\alpha] = [(m - l)\alpha + l, n - (n - m)\alpha]; \quad \forall \alpha \in [0,1]
\]

Where \((m-l)\alpha+1\) and \(n-(m-n)\alpha\) are the lower and upper bounds of the \(a\)-level confidence. Similarly, fuzzy variance is denoted by \(\tilde{\sigma} = (l, m, n)\) with the membership function:

\[
\mu_{\tilde{\sigma}}^2(t) = \begin{cases} 
\frac{t - l}{m - l}, & l \leq t \leq m \\
1, & t = m \\
\frac{(n - t)}{(n - m)}, & m \leq t \leq n \\
0, & t \leq l \text{ or } t \geq n
\end{cases}
\]  

(8)

\[
\tilde{\sigma}_\alpha = [\sigma^-_\alpha, \sigma^+_\alpha] = [(m - l)\alpha + l, n - (n - m)\alpha]; \quad \forall \alpha \in [0,1]
\]

The portfolio selection problem based on the mean–variance model with fuzzy return rates and fuzzy variances is:

Maximize \( E(R_p) = \sum_{i=1}^{N} \tilde{R}_i x_i \)

Minimize \( \sigma_p^2 = \sum_{i=1}^{N} x_i^2 \tilde{\sigma}_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \tilde{\sigma}_{ij} \)

Subject to:

\[
\sum_{i=1}^{N} \tilde{R}_i x_i \geq L
\]

\[
\sum_{i=1}^{N} x_i \tilde{\sigma}_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \tilde{\sigma}_{ij} \leq M
\]

\[
\sum_{i=1}^{N} x_i = 1
\]

\[
x_i \geq 0 \quad i = 1, ..., N
\]  

(9)

As stated earlier, one of the objectives of clustering through K means method is to minimize the variance of objects in one cluster. In view of this feature, the correlation between clusters can be ignored and Equation 9 can thus be rewritten as:

Maximize \( \sum_{i=1}^{N} (\tilde{R}_i)_\alpha x_i \)

Minimize \( \sum_{i=1}^{N} x_i^2 (\tilde{\sigma}_i^2)_\alpha \)

Subject to:

\[
\sum_{i=1}^{N} \tilde{R}_i x_i \geq L
\]

\[
\sum_{i=1}^{N} x_i \tilde{\sigma}_i^2 \leq M
\]

\[
\sum_{i=1}^{N} x_i = 1
\]

\[
x_i \geq 0 \quad i = 1, ..., N
\]  

(10)

The afore-mentioned bi-objective problem can be split into:

P 1) maximize fuzzy return subject to given fuzzy variance
Maximize $\bar{Z}(x) = \sum_{i=1}^{N} (\bar{R}_i)_a x_i$

Subject to:

$$\sum_{i=1}^{N} x_i^2 (\bar{\sigma}_i^2)_a \leq M$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0 \quad i = 1, \ldots, N$$  \hspace{1cm} (11)

P (2) minimize fuzzy variance subject to given fuzzy return

Minimize $\tilde{Z}(x) = \sum_{i=1}^{N} x_i^2 (\bar{\sigma}_i^2)_a$

Subject to:

$$\sum_{i=1}^{N} \bar{R}_i x_i \geq L$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0 \quad i = 1, \ldots, N$$  \hspace{1cm} (12)

The afore-mentioned two sub-problems can be reformulated with the $\alpha$-level confidence of fuzzy numbers.

P (1) can be written in the following form:

Maximize $\tilde{Z}^\alpha = \sum_{i=1}^{N} (\bar{R}_i)_a x_i$

Subject to:

$$\sum_{i=1}^{N} (\bar{\sigma}_i^2)_a x_i \leq M$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0 \quad i = 1, \ldots, N$$  \hspace{1cm} (13)

P (2) can be written in the following form:

Minimize $\tilde{Z}^\alpha = \sum_{i=1}^{N} (\bar{\sigma}_i^2)_a x_i$

Subject to:

$$\sum_{i=1}^{N} (\bar{R}_i^+_a x_i \geq L$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0 \quad i = 1, \ldots, N$$  \hspace{1cm} (14)

Both of the problems could be solved by satisfying the Kuhn–Tucker conditions based on the lower and upper bounds, separately, at different $\alpha$-level confidences. The optimal solutions of Equations 13 and 14 at $\alpha$ level will be $[z^\alpha_\alpha(x), z^\alpha_+ = \tilde{z}^\alpha(x)]$.

PORTFOLIO SELECTION

Data

Our initial sample contains 79 equity mutual funds. We exclude funds that have been operating for less than one year and finally the total sampled data are 39 cases. The period of analysis is 23 October, 2010 to 23 October, 2011, representing one year of weekly fund data. The required data for calculating weekly returns, Sortino ratio, standard deviation and turnover rate of mutual funds have been taken from financial information processing center of Iran (www.fipiran.ir).

A longer time frame potentially improves the precision of the parameter estimates and the generalizability of the results. However, due to short life of mutual funds in Iran, the time period of study has been considered one year.

Clustering of funds based on statistical analysis

Before cluster analysis, we perform the normalization for all cases to ensure that all indices are in the same unit, that is, $[0, 1]$. After the normalization, we apply the results to the cluster analysis. For cluster analysis, this research uses the two-stage method. In the first stage, the Ward method of amalgamation rules is applied to determine the number of clusters, $k$. Next, the K-means of non-hierarchical method is applied to proceed with the cluster analysis. Based on the tree diagram in Figure 1, we can divide the cases into 3 clusters.

Funds can be clustered in three groups based on the results from ward method, that is, $k = 3$. The results from K means method are given in Table 1. The mean value of each index for a cluster is presented in Table 1. The first cluster is called the inferior performance funds so investment does not occur in these. The second cluster is called good performance funds and the third group aggressive funds:

Cluster 1: inferior performance funds. Funds in cluster 1 are categorized as inferior performance funds, since their return rates are quite low, and the deviation relatively high. Also low Sortino ratio in this first cluster displays great downside risk and poor performance.

Cluster 2: good performance funds. The risk of the second cluster is lower than the other clusters but the return of this cluster is between the other two. Also, both the Sortino and turnover ratio is between those of the first and the third clusters.

Cluster 3: aggressive funds. Cluster 3 is named as aggressive funds, since they have rather high return rates as well as a high standard deviation. Although investors may profit from the high return rates, they also endure the high risk. The Sortino ratio and the turnover ratio of this cluster are higher than the other clusters that represent the aggressiveness of this group.

Asset allocation for portfolio selection

In this stage, first, we use actual data instead of normalized numbers to reflect the actual situation. Then determine the optimized investment ratio in each cluster. Table 2 illustrates the return rates and the risk of funds 2, and 3. The other two evaluation
measures are not used in the optimization model for a high or low turnover rate does not necessarily reflect a better performance and merely indicates how aggressively the fund is performing. The Sortino ratio too, calculates the excess portfolio return due to every unit of downside risk. Consequently these two criteria are only used to aid in clustering of mutual funds based on their performance. The return rates and risks in Table 2 represent the center (mean value) of each cluster, when ignoring turnover rate and Sortino index.

Obviously, funds in Cluster 3 have the characteristics of high return rate with high risk, while Cluster 2 is opposite. To reflect the nature of uncertainty in the portfolio selection, the variables are represented as triangular fuzzy numbers. Assume that the spread of each side of the triangular fuzzy number is set as one standard deviations of each variable, which is sufficient to describe the uncertainty of each variable in the cluster. To demonstrate this, the membership functions of return rate and risk of Cluster 3 are formulated as 15 and 16, respectively. Similarly, the membership functions for Cluster 2 can be defined.

\[
\mu_{R_3}(t) = \begin{cases} 
(t - 0.321)/0.138, & 0.321 \leq t \leq 0.458 \\
1, & 0.458 \leq t \leq 0.597 \\
(t - 0.321)/0.138, & 0.597 \leq t \leq 0.597 \\
0, & 0.321 \leq t \leq 0.458 \\
0, & t \geq 0.597 \\
0, & t \leq 0.321 
\end{cases}
\]

\[
\mu_{\sigma^2_3}(t) = \begin{cases} 
(t - 338.538)/107.003, & 338.538 \leq t \leq 445.541 \\
1, & 445.541 \leq t \leq 552.544 \\
(552.544 - t)/107.003, & 552.544 \leq t \leq 552.544 \\
0, & 338.538 \leq t \leq 445.541 \\
0, & t \geq 552.544 \\
0, & t \leq 338.538 \\
0, & t \geq 552.544 
\end{cases}
\]

The optimization problem based on Markowitz model (Markowitz, 1952) consists of two parts: maximizing the fuzzy returns based on a certain fuzzy variance and minimizing the fuzzy risk at a given level of fuzzy return. Both problems have been solved at different \( \alpha \)-level confidences, as follows:

1) Maximize fuzzy return subject to a given fuzzy variance

The objective is to find the optimum investment proportion of each cluster to obtain the maximal fuzzy return at a given fuzzy risk. The symbol \( M \) represents the maximum fuzzy variance, which means the upper bound of fuzzy variance that the investors can tolerate. Thus, the optimization problem is:
Table 3. Changes of solutions at different α-level confidences in 400 levels of risk (M).

<table>
<thead>
<tr>
<th>Confidence level (α)</th>
<th>Z</th>
<th>X2</th>
<th>X3</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=0</td>
<td>[0.321 , 0.572]</td>
<td>[0, 0.1582]</td>
<td>[0.8418 , 1]</td>
</tr>
<tr>
<td>α=0.25</td>
<td>[0.355 , 0.542]</td>
<td>[0, 0.1338]</td>
<td>[0.8662 , 1]</td>
</tr>
<tr>
<td>α=0.5</td>
<td>[0.390 , 0.511]</td>
<td>[0, 0.1084]</td>
<td>[0.8916 , 1]</td>
</tr>
<tr>
<td>α=0.75</td>
<td>[0.421 , 0.481]</td>
<td>[0.1281 , 0.0816]</td>
<td>[0.9184 , 0.9772]</td>
</tr>
<tr>
<td>α=1</td>
<td>0.451</td>
<td>0.0532</td>
<td>0.9468</td>
</tr>
</tbody>
</table>

Table 4. Changes of solutions at different α-level confidences in 300 levels of risk (M).

<table>
<thead>
<tr>
<th>Confidence level (α)</th>
<th>Z</th>
<th>X2</th>
<th>X3</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=0</td>
<td>[0.313 , 0.550]</td>
<td>[0.0592 , 0.3019]</td>
<td>[0.6981 , 0.9408]</td>
</tr>
<tr>
<td>α=0.25</td>
<td>[0.342 , 0.516]</td>
<td>[0.0955 , 0.2743]</td>
<td>[0.7257 , 0.9045]</td>
</tr>
<tr>
<td>α=0.5</td>
<td>[0.371 , 0.490]</td>
<td>[0.1290 , 0.2468]</td>
<td>[0.7532 , 0.8710]</td>
</tr>
<tr>
<td>α=0.75</td>
<td>[0.410 , 0.460]</td>
<td>[0.1604 , 0.2189]</td>
<td>[0.7811 , 0.8396]</td>
</tr>
<tr>
<td>α=1</td>
<td>0.431</td>
<td>0.1902</td>
<td>0.8098</td>
</tr>
</tbody>
</table>

Table 5. Changes of solutions at different α-level confidences in 200 levels of risk (M).

<table>
<thead>
<tr>
<th>Confidence level (α)</th>
<th>Z</th>
<th>X2</th>
<th>X3</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=0</td>
<td>NFS</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>α=0.25</td>
<td>[0.316 , 0.478]</td>
<td>[0.2785 , 0.5450]</td>
<td>[0.4550 , 0.7215]</td>
</tr>
<tr>
<td>α=0.5</td>
<td>[0.345 , 0.455]</td>
<td>[0.3140 , 0.4751]</td>
<td>[0.5249 , 0.6860]</td>
</tr>
<tr>
<td>α=0.75</td>
<td>[0.373 , 0.429]</td>
<td>[0.3496 , 0.4272]</td>
<td>[0.5728 , 0.6504]</td>
</tr>
<tr>
<td>α=1</td>
<td>0.401</td>
<td>0.3867</td>
<td>0.6133</td>
</tr>
</tbody>
</table>

Table 6. Changes of solutions at different α-level confidences at L=0.35 level.

<table>
<thead>
<tr>
<th>Confidence level (α)</th>
<th>Z</th>
<th>X2</th>
<th>X3</th>
</tr>
</thead>
<tbody>
<tr>
<td>α=0</td>
<td>NFS</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>α=0.25</td>
<td>[193.77 , 337.918]</td>
<td>[0.0385 , 0.6315]</td>
<td>[0.3685 , 0.9615]</td>
</tr>
<tr>
<td>α=0.5</td>
<td>[178.72 , 217.517]</td>
<td>[0.2759 , 0.6432]</td>
<td>[0.3568 , 0.7241]</td>
</tr>
<tr>
<td>α=0.75</td>
<td>[149.87 , 162.128]</td>
<td>[0.5068 , 0.6567]</td>
<td>[0.3433 , 0.4932]</td>
</tr>
<tr>
<td>α=1</td>
<td>145.875</td>
<td>0.6726</td>
<td>0.3274</td>
</tr>
</tbody>
</table>

Z: \[
\begin{align*}
\text{Max} & \quad (0.130\alpha + 0.180, -0.130\alpha + 0.440)x_2 \\
& \quad + (0.138\alpha + 0.321, -0.138\alpha + 0.597)x_3 \\
\text{Subject to} & \quad (119.957\alpha + 96.930, -119.957\alpha + 336.843)x_2^2 \\
& \quad + (107.003\alpha + 338.538, -107.003\alpha + 552.544)x_2 \leq M \\
& \quad \sum_{i=2}^{3} x_i = 1 \\
& \quad x_i \geq 0
\end{align*}
\]

The aforementioned problem can be solved by satisfying the Kuhn–Tucker condition based on the lower and upper bounds, separately, at different α-level confidences. For doing this, the LINGO software in version 10.0 (2006) is applied to obtain the optimal solution. The optimal investment proportions at M = 400 are found in Table 3. “NFS” means non-feasible solutions with the corresponding condition. From Table 5, the feasible solutions of the problem cannot be found when α=0 and M=200.

Similarly, the optimal solutions can be obtained at different α-level confidences and different level of risk (M). To demonstrate this, the solutions at α = 0.25, 0.5, 0.75, and 1.0 for M= 200, 300 and 400 are listed in Tables 3 to 5, respectively.

(2) Minimize fuzzy risk subject to a given fuzzy return

The aim here is to find the optimal investment proportion for each cluster in order to minimize the fuzzy risk, subject to achieving a
specified level of fuzzy return. The symbol $L$ denotes the minimum fuzzy return in the problem. The fuzzy optimization problem is formulated as follows:

$$Z: \quad \min \ (119.957\alpha + 96.930, -119.957\alpha + 336.843)x_2^3 + (107.003\alpha + 338.538, -107.003\alpha + 552.544)x_3^3$$

$$\text{Subject to:}$$

$$(0.130\alpha + 0.18, -0.130\alpha + 0.440)x_2 + (0.138\alpha + 0.321, -0.138\alpha + 0.597)x_3 \geq L$$

$$\sum_{i=2}^{3} x_i = 1$$

$$x_i \geq 0$$

(18)

Similar with (17), the afore-mentioned problem can be solved at different $\alpha$-level confidences. When $\alpha = 0, 0.25, 0.5, 0.75$, and $1$, the optimal investment proportions of (18) can be obtained at different $L$ levels. For example, in Tables 6 to 8, $L$ is considered 0.35, 0.30 and 0.20, then optimal investment proportion of each cluster for $\alpha=0, 0.25, 0.5, 0.75$ and $1$ is calculated. "NFS" means non-feasible solutions with the corresponding condition.

### RESULTS AND DISCUSSION

After clustering, the first cluster is excluded from the portfolio selection model due to its poor performance. Therefore, we consider Clusters 2 and 3 for the investment decisions.

The asset allocation problem is performed by two alternatives. One considers the maximization of fuzzy return rates subject to a given upper bound of fuzzy risks. The problem is resolved with different levels of risk on five confidence levels, that is, $\alpha = 0, 0.25, 0.5, 0.75$, and $1$. When the $\alpha$ level increases, the interval length of optimal return rates will decrease to reflect the higher confidence. For example, when the $M=400$ and the $\alpha=0$, the investment returns is placed within $[0.321, 0.572]$ and when we consider higher confidence level such $\alpha=0.5$, the interval length of optimal return rates will decrease and is placed within $[0.390, 0.511]$, when $\alpha=0.75$ optimal return rates is placed within $[0.421, 0.481]$ and finally at $\alpha=1$ (highest confidence level) the optimized return equals to 0.451.

For most cases, the investment proportion of Cluster 3 is higher than that of Cluster 2, since Cluster 3 has a better return rate. For every $\alpha$ level, as the upper bound of fuzzy risk $M$ decreases, the investment proportion of Cluster 3 will also decrease. Since funds in Cluster 2 have lower return rates and risks, thus as $M$ decreases, the investment proportion of Cluster 2 increases.

The other approach to the asset allocation problem focuses to minimize fuzzy risk subject to a given lower bound of fuzzy return rates. The problem is also resolved at the same confidence levels as the first approach.

### Conclusions

Small investors face a lot of problems in the share market such as limited resources, lack of professional advice, lack of information etc. Mutual funds are a special type of institutional device or an investment vehicle which help investors to invest their savings in wide variety of securities under the guidance of a team of experts in order to minimize risk. In selecting mutual funds, the past...
performance of funds plays a central role in the expectations of the future performance of funds. Therefore, a number of performance evaluation techniques have been employed.

In this paper, the process of portfolio optimization is conducted focusing on mutual funds in Iran, using the Chen and Huang (2009) model. We apply four criteria to cluster analysis; namely return rate, standard deviation, turnover rate, and the Sortino index. The clustering process shows that mutual funds can be categorized into three groups which we called, inferior performance funds, aggressive funds, and good performance funds.

The asset allocation for portfolio selection is performed in two ways. Risk-averse investors prefer to minimize the risk than maximize return rates, and may consider minimization of risks as the objective and taking returns as the constraint. Risk seeking investors who emphasize on return rates may consider the maximization of return rates as the objective. In addition, to deal with the nature of uncertainty in portfolio selection for future investment, the variables are represented as triangular fuzzy numbers based on fuzzy set theory. According to different confidence levels, the optimal investment proportions can thus be determined.

REFERENCES


