



Comparison of FRF and Modal Methods for Combining Experimental and Analytical Substructures

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25th International Modal Analysis Conference

February, 2007

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
for the United States Department of Energy's National Nuclear Security Administration
under contract DE-AC04-94AL85000.



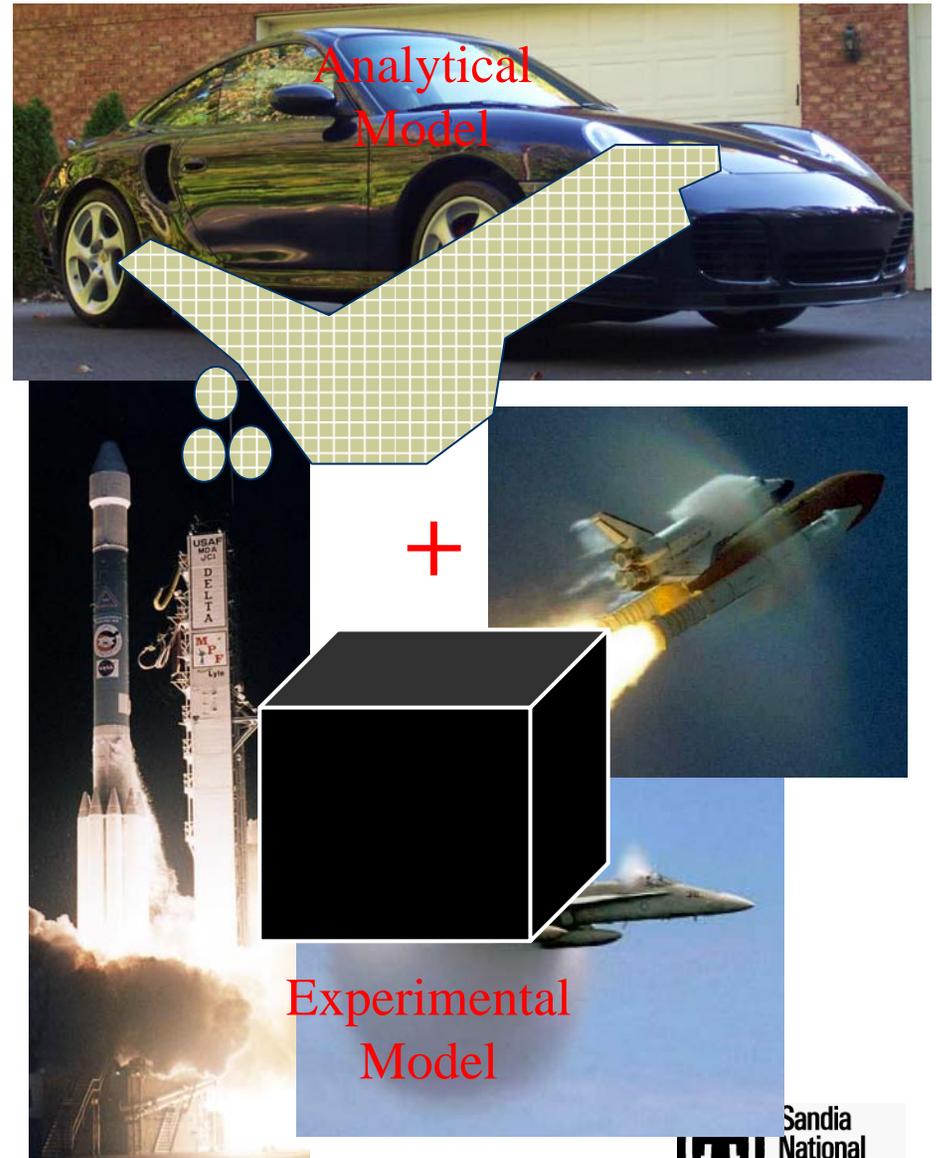


Outline

- ◆ Motivation
- ◆ Modal Coupling vs. FRF Coupling
- ◆ Component Mode Synthesis Theory
- ◆ Methods for connecting subsystems
 - Connection Point Method (CPT)
 - Modal Constraint for Fixture and Subsystem (MCFS)
- ◆ Experimental Results
 - Rigid Fixture Model with CPT constraint
 - Elastic Fixture Model with CPT constraint
 - Elastic Fixture with MCFS constraint
- ◆ Conclusions

Motivation

- ◆ Subcomponents are often designed by a number of independent groups that do not have the information or the resources to model the macro system.
- ◆ In other applications some particular components may be too difficult to model analytically with the required precision.





Modal vs. FRF Coupling

- ◆ Two distinct approaches to substructure coupling exist:
- ◆ Component Mode Synthesis or Modal Coupling:
 - Modal models for substructures combined to find an approximate modal model for the total system.
- ◆ Requires modal parameter estimation to reduce measurements to a modal system model.
 - Measurements must be sufficient to obtain reasonably accurate estimates of each important modal frequency and mode shape at the connection points.
- ◆ Results can sometimes be understood by appealing to well-established Ritz Theory.
- ◆ Disadvantage: It is not always easy to identify an adequate modal model from experimental measurements.
- ◆ FRF Based Admittance or Impedance Coupling:
 - Response measurements (FRF matrices) for substructures combined to find FRFs for total system.
- ◆ Can be performed on raw FRFs (even if modal parameter estimation is not feasible.)
 - However, all connection point FRFs must be measured if MPE is not employed.
 - $6*N_c \times 6*N_c$ set of FRFs!
 - Numerical ill-conditioning may present a formidable challenge.
 - Modal parameter estimation is very desirable to reduce measurement errors for lightly damped systems. [Imregun, Robb, Ewins IMAC-1987]



Component Mode Synthesis

- ◆ Given a set of modal parameters, one has a set of equations of motion (EOM) for a substructure:

$$[I] \{\ddot{\eta}\} + [\omega_r^2] \{\eta\} = [\Phi]^T \{F\}$$

$$\{y\} = [\Phi] \{\eta\}$$

- $\{y\}$ is a vector of physical coordinates
 - $\{\eta\}$ are modal coordinates
 - $[\omega_r^2]$ is a diagonal matrix of natural frequencies squared
 - $[\Phi]$ is a matrix of mass-normalized mode vectors.
- ◆ Consider two independent structures A and B:



- ◆ Their EOM are....



Component Mode Synthesis (2)

$$\boxed{A} \bullet y_c + y_c \bullet \boxed{B}$$

$$\begin{bmatrix} [I]_A & 0 \\ 0 & [I]_B \end{bmatrix} \begin{Bmatrix} \{\ddot{\eta}\}_A \\ \{\ddot{\eta}\}_B \end{Bmatrix} + \begin{bmatrix} [\omega_r^2]_A & 0 \\ 0 & [\omega_r^2]_B \end{bmatrix} \begin{Bmatrix} \{\eta\}_A \\ \{\eta\}_B \end{Bmatrix} = \begin{Bmatrix} [\Phi]_A^T \{F\} \\ [\Phi]_B^T \{F\} \end{Bmatrix}$$

- Connect them by enforcing linear constraints:

$$(y_c)_A = (y_c)_B$$

- All DOF can be expressed in terms of an unconstrained set of generalized coordinates as

$$\begin{Bmatrix} \{\eta\}_A \\ \{\eta\}_B \end{Bmatrix} = [P] \begin{bmatrix} -[\hat{a}_c]^{-1} \hat{a}_u \\ [I] \end{bmatrix} \{\eta\}_u = [B] \{\eta\}_u$$

- The equations of motion in unconstrained coordinates are then:

$$[\hat{M}] \{\ddot{\eta}\}_u + [\hat{K}] \{\eta\}_u = \{Q\}$$

$$[\hat{M}] = [B]^T \begin{bmatrix} [I]_A & 0 \\ 0 & [I]_B \end{bmatrix} [B]$$

$$[\hat{K}] = [B]^T \begin{bmatrix} [\omega_r^2]_A & 0 \\ 0 & [\omega_r^2]_B \end{bmatrix} [B]$$

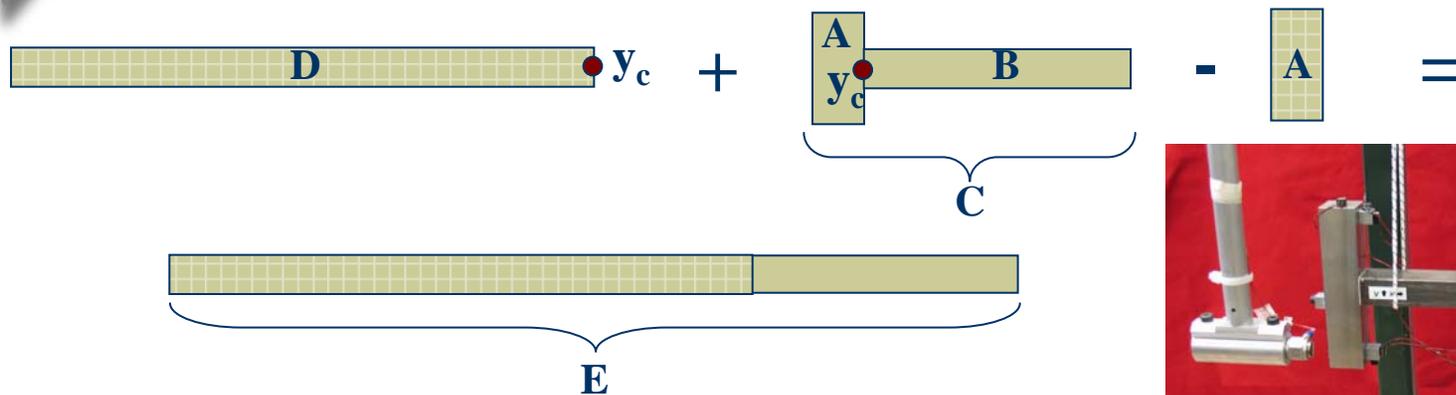
Paper includes a method that chooses constrained DOF so that this matrix is always invertible if such a choice exists.



Component Mode Synthesis (3)

- ◆ One can now find the modes of the combined system, construct FRFs, etc... using these EOM.
- ◆ The mode shapes of the combined system are found in the physical coordinates of A and B.
 - The combined system mode shapes are linear combinations of the mode shapes of the subcomponents A and B.
 - One must assure that enough modes of both A and B are present so that these are a good approximation for the true modes of the combined system.

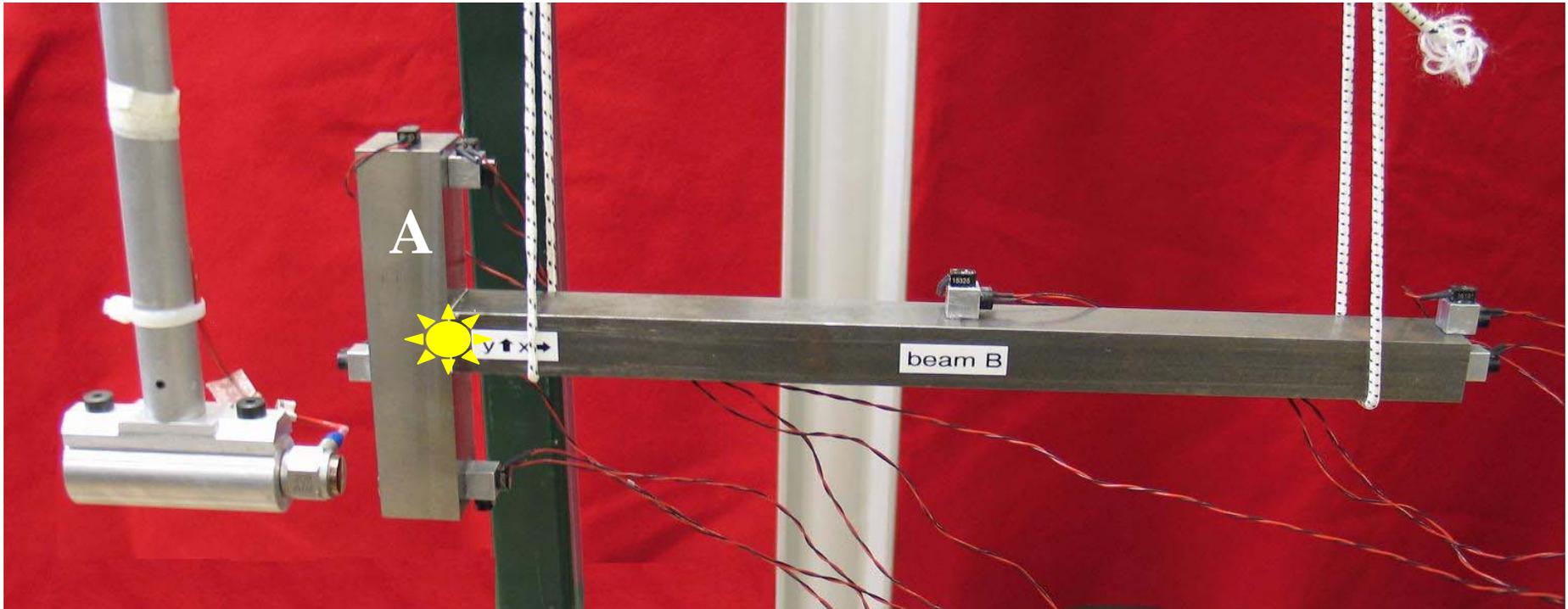
Test System



- ◆ Objective: Join an experimental model of beam B to an analytical model for beam D at point y_c .
 - Measurements are taken on system C consisting of beam B with fixture A attached.
 - Analytical model (Euler-Bernoulli) for fixture A removed from system C resulting in an experimental model for beam B.
 - Analytical model for fixture A
 - Beam B is then combined with an analytical model (tuned Euler-Bernoulli) for beam D to find the combined system Beam E.



Experimental Procedure



- ◆ Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.



Connection Methods – CPT and MCFS

- ◆ One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
 - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:

$$\left\{ \begin{array}{l} \{y^C\}_m \\ \{y^C\}_c \end{array} \right\} \approx \left[\begin{array}{l} [\Phi^A_m] \\ [\Phi^A_c] \end{array} \right] \{\eta^C\} \rightarrow \{y^C\}_c = [\Phi^A_c][\Phi^A_m]^\dagger \{y^C\}_m$$

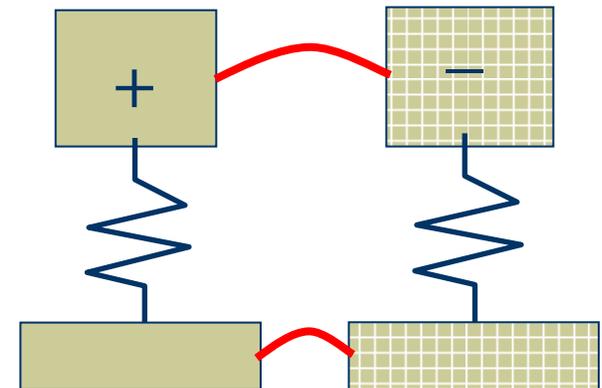
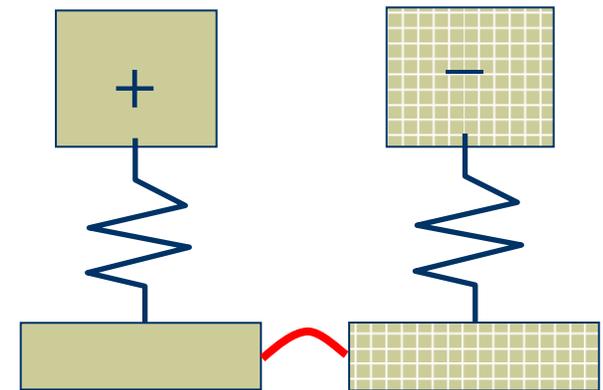
- MCFS Method: Constrain the modal DOF of the Fixture model to their approximation on C: (MCFS stands for Modal Constraint for Fixture and Subsystem)

$$\left\{ \begin{array}{l} \{\eta^A\} = [\Phi^A_m]^\dagger \{y^A\}_m \\ \{\eta^C\} = [\Phi^A_m]^\dagger \{y^C\}_m \end{array} \right\} \rightarrow \{\eta^A\} = \{\eta^C\}$$



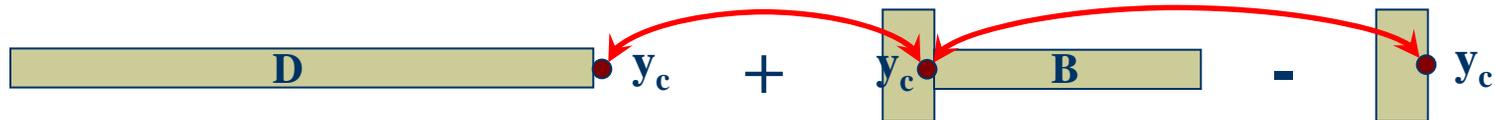
Connection Methods – Rationalization

- ◆ CMS can be very sensitive to errors when removing a substructure from a system.
- ◆ In the problem considered here the fixture response is dominated by (4) modes.
- ◆ Requiring equal 2D motion at the connection point enforces only (3) constraints.
 - $4_{\text{DOF}} * 2_{\text{Systems}} - 3_{\text{Constraints}} = 5_{\text{Remaining DOF}}$
 - Two elastic modes remain in the system. One would hope that these have no effect on the response.
 - It might be preferable to remove these extra modes by adding constraints rather than simply hope that the fixture model is accurate enough so that they cancel.

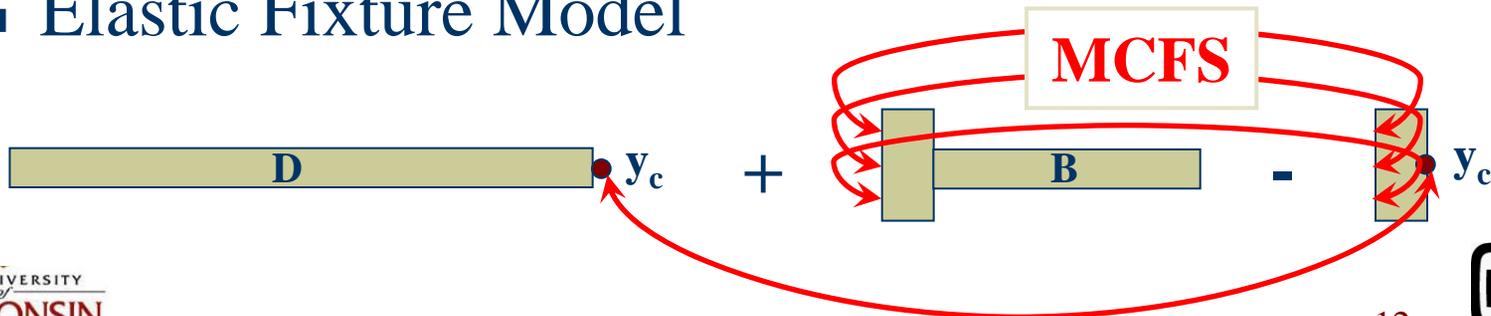


Cases Considered

- ◆ Case 1: **CPT**: Models for A, C and D joined at the connection point.
 - Case 1a: Rigid Fixture Model
 - Case 1b: Elastic Fixture Model

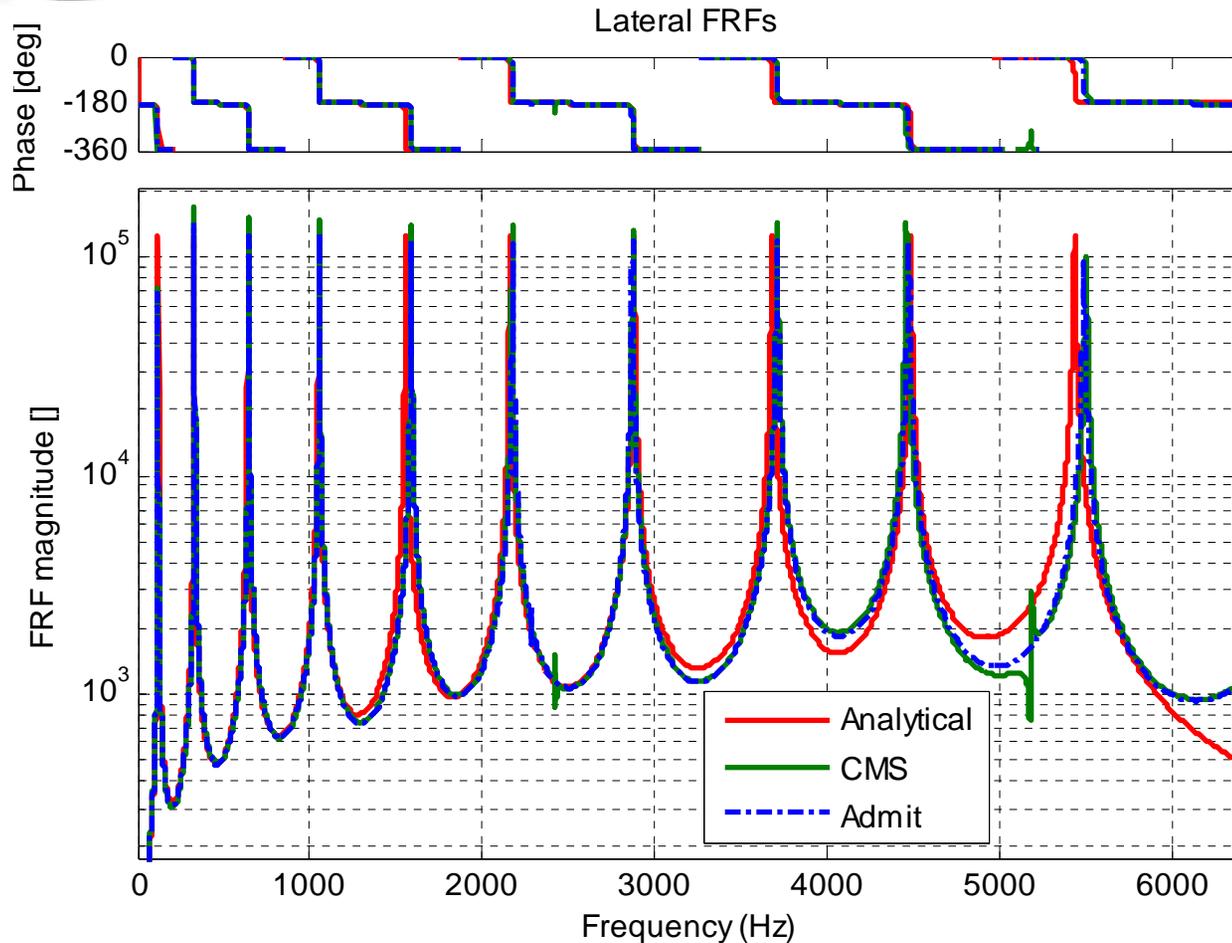


- ◆ Case 2: **MCFS**: Models for A and C joined using MCFS method.
 - Elastic Fixture Model





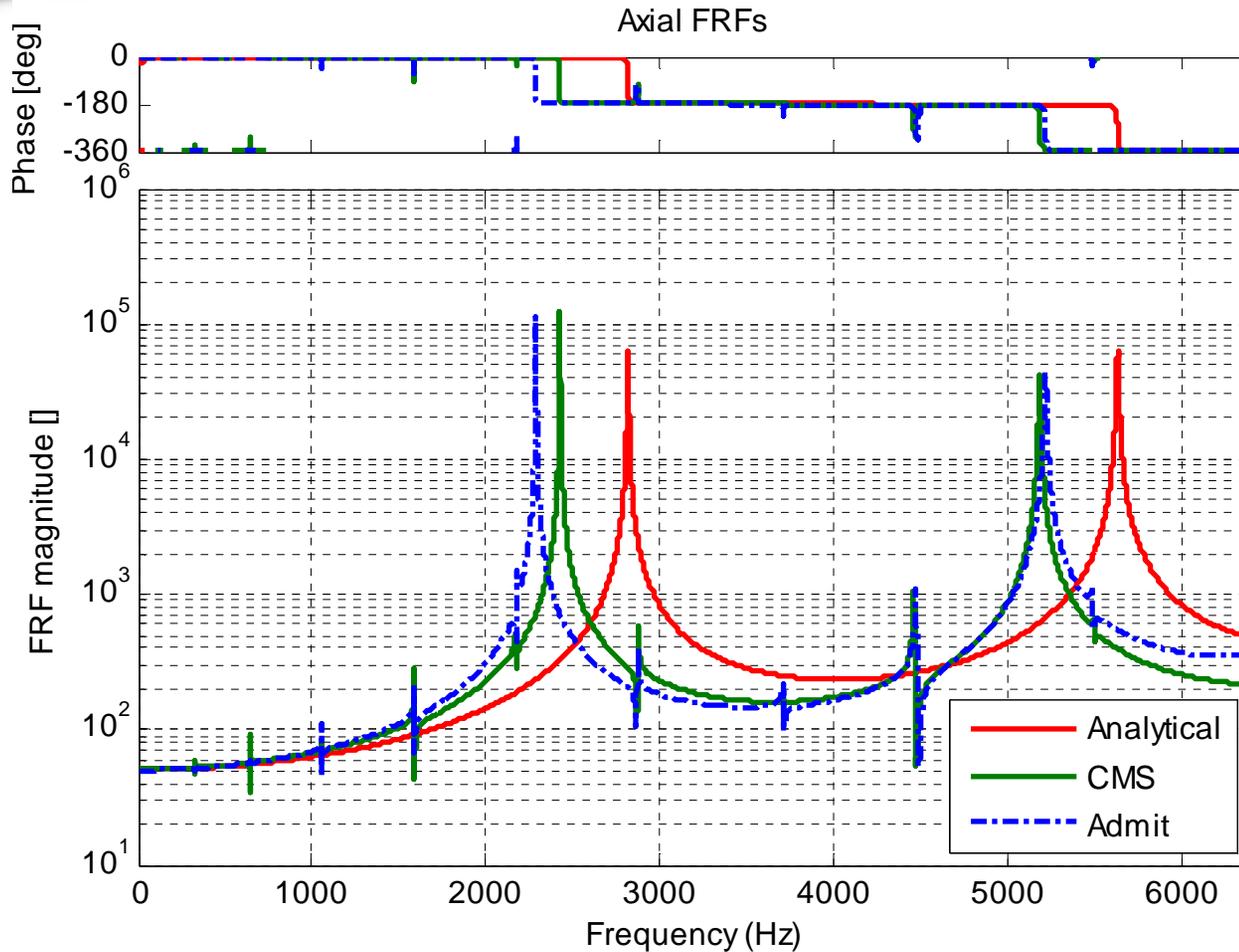
Case 1a: Rigid A, CPT



- ◆ Excellent results obtained in the lateral direction (Bending Modes).
- ◆ Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- ◆ The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.



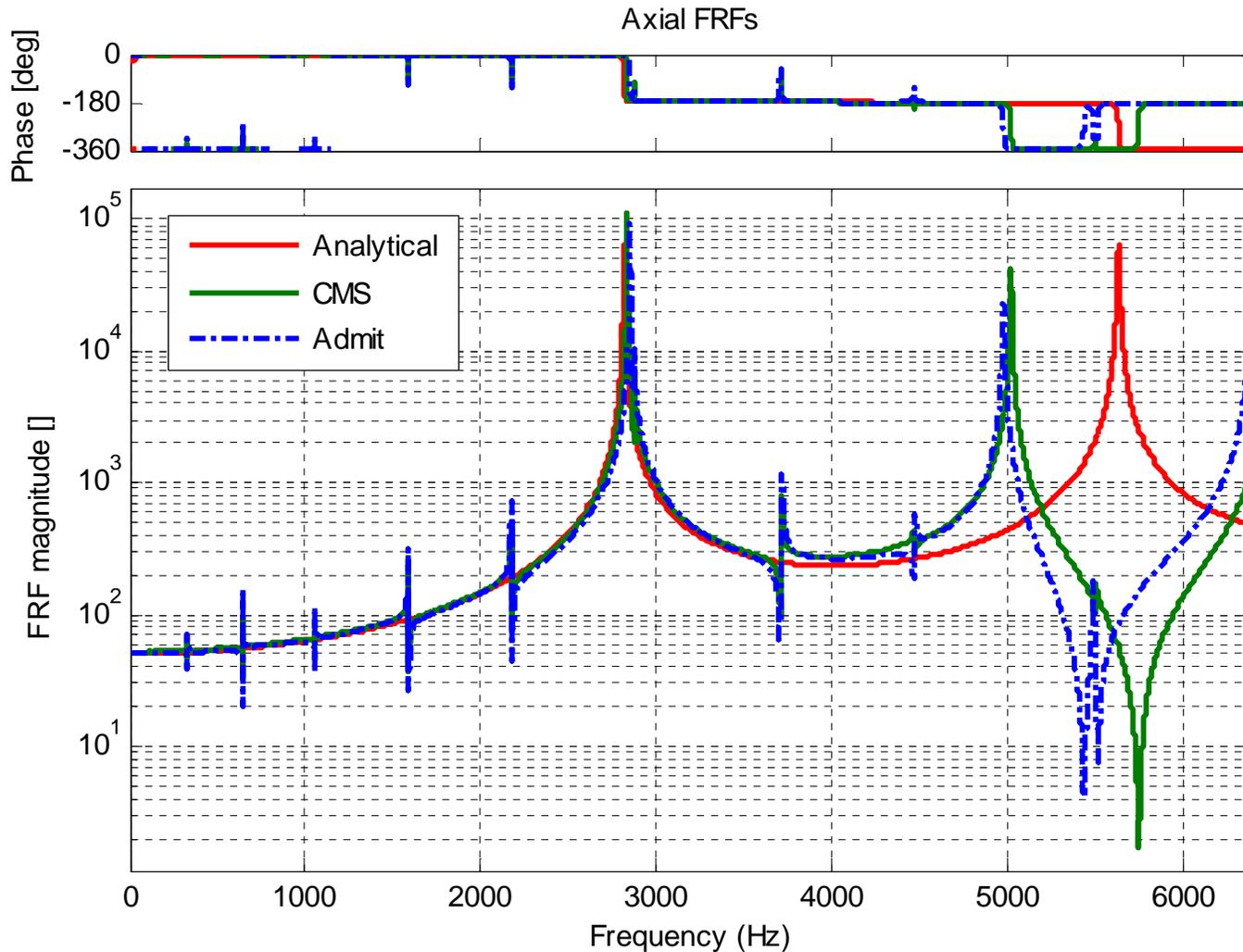
Case 1a: Rigid A, CPT



- ◆ Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
 - These errors are larger than one would expect due to modal truncation alone.
 - Rigid fixture model is not adequate.
- ◆ Both also over-predict the axial motion in the bending modes resulting in contamination at the bending natural frequencies.
 - Possibly due to small curve fitting errors or cross axis sensitivity.



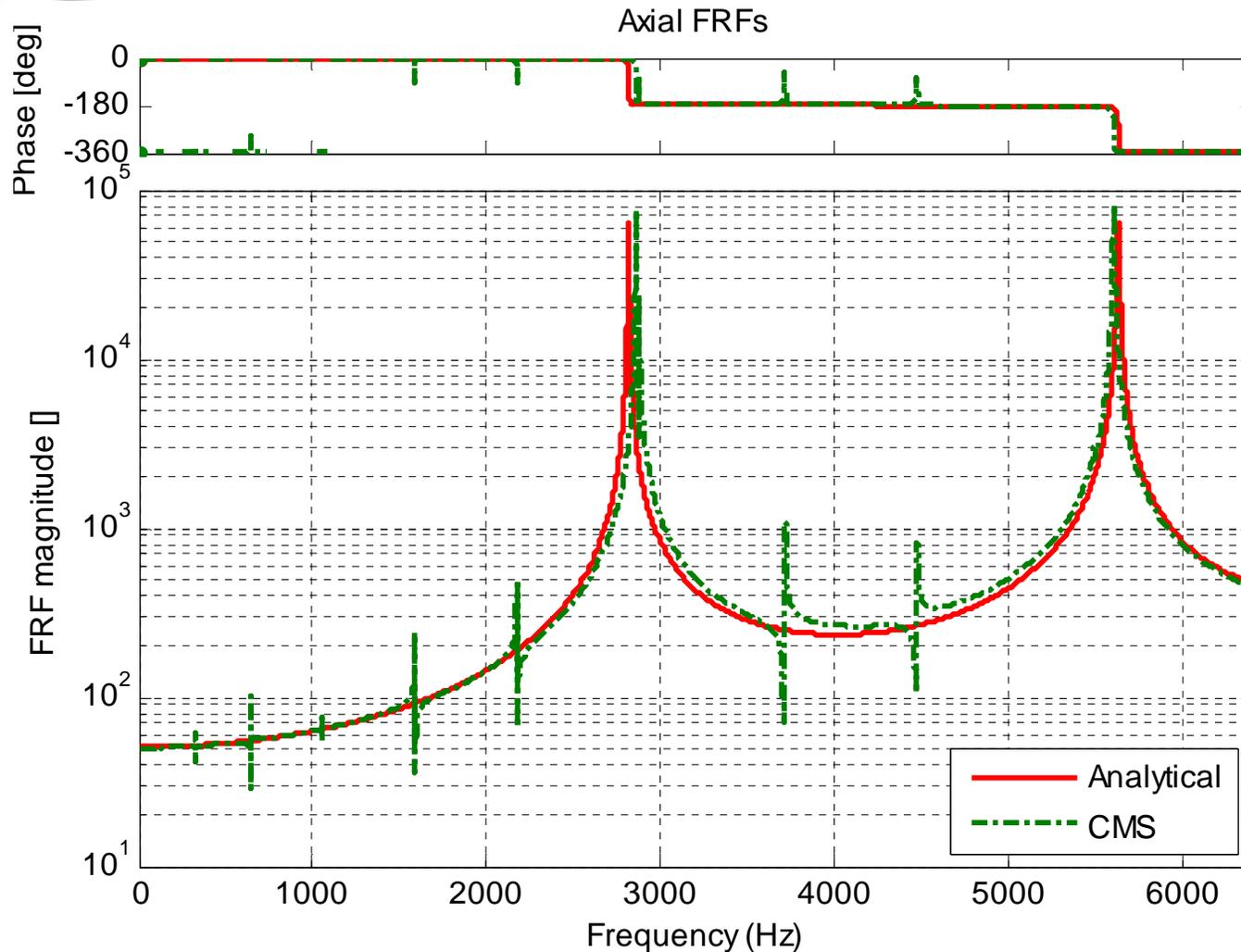
Case 1b: Flexible A, CPT



- ◆ Both Admittance and CMS accurately predict the first axial frequency with the flexible fixture model.
- ◆ Both methods more severely under-predict the second axial mode.
- ◆ Both predict a spurious zero near 5500 Hz.
- ◆ The Lateral FRFs were similar to those shown previously.



Case 2: Flexible A, MCFS Method



- ◆ The CMS method predicts the axial FRF very accurately when the Modal Constraint for Fixture and Subsystem (MCFS) is used.



Physically Realizable Models

- ◆ The E system models obtained were not completely physical.
 - Some of the E system models had complex natural frequencies
 - The E system mass matrix was not always positive definite.
- ◆ This is to be expected since an approximation to the stiffness of the fixture has been removed.
- ◆ When combining structures, the spurious modes all appear at high frequencies, yet they can appear at lower frequencies when removing a substructure.
 - In these cases, the spurious natural frequencies are always near the extremes of the frequency band.
 - Admittance results also show non-physicality (negative eigenvalues of drive point FRFs at some frequencies).

	Complex f_n (Hz)	eig(M) < 0
Case 1a Rigid Fixture, Con. Pt.	0 + i*8.93e-5 0 + i*54100	-0.011
Case 1b Flexible Fixture, Con. Pt.	0 + i*1.36e-4 8951 - i*2450 8951 + i*2450	-1
Case 2 Flexible Fixture, MCFS	0 + i*2.28e-4 13050 - i*4285 13050 + i*4285	-0.086

It could be worse!

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	Complex f_n (Hz)	$\text{eig}(M) < 0$
Case 1a Rigid Fixture, Con. Pt.	$0 + i*8.93e-5$ $0 + i*54100$	-0.011
Case 1b Flexible Fixture, Con. Pt.	$0 + i*1.36e-4$ $8951 - i*2450$ $8951 + i*2450$	-1
Case 2 Flexible Fixture, MCFS	$0 + i*2.28e-4$ $13050 - i*4285$ $13050 + i*4285$	-0.086



Conclusions

- ◆ Lateral (bending direction)
 - Both Component Mode Synthesis (CMS) and FRF based Substructuring (Admittance) can be used to accurately predict the lateral modes of this Experimental-Analytical system.
- ◆ Axial direction:
 - It was necessary to account for the elasticity of the fixture to obtain accurate estimates of the axial modes.
 - When doing so, the Modal Constraint for Fixture and Substructure (MCFS) method improved the accuracy of the predictions when compared to the connection point method (CPT).
 - MCFS allows one a new level of freedom when designing test fixtures for these types of analyses.
 - It would be helpful to have a method of removing test fixtures that assures that a physically meaningful model is obtained.