A Comparison on Features Efficiency in Automatic Reconstruction of Archeological Broken Objects

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Abstract
Automatic reconstruction of archeological broken objects is an invaluable tool for restoration purposes and personnel. In this paper, we assume that broken pieces have similar characteristics on their common boundaries, when they are correctly combined. In this paper we work in a framework for the full reconstruction of the original objects using texture and surface design information on the sherd. The texture of a band outside the border of pieces is predicted by inpainting and texture synthesis methods. Feature values are derived from these original and predicted images of pieces. We present a quantitative and qualitative comparison over a large set of features and over a large set of synthetic and real archeological broken objects.

Categories and Subject Descriptors (according to ACM CCS): I.4.3 [Image Processing]: Registration I.4.5 [Image Processing]: Reconstruction I.4.7 [Image Processing]: Feature measurement, Feature representation, Moments, Texture I.4.9 Archaeologic reconstruction Applications

1. Introduction
Reconstruction of archeological pieces from fragments of parts found in archeological sites is an arduous task if it is manually performed. In order to help the archeologists and the reconstruction personnel, several automatic tools for the reconstruction of broken archeological pieces have been developed until now [KK01], [KS03], [PKT01], [SL02], [WC03], and [SE05].

In [SE05], the authors propose a method that begins with expanding each input piece outwards, by predicting the pixel values in a band outside the border of the pieces with a confidence measure using a technique known as inpainting [CPT04]. Features obtained from the predicted texture outside a piece are correlated with original pictorial specifications of possible neighboring pairs. The idea of matching two or more pieces is based on the fact that common features of neighboring fragments are more strongly related than in fragments that are not neighboring to one another.

In [SE05], an affinity measure of corresponding pieces is defined and alignment of the puzzles pieces is carried out using an FFT based image registration technique. The optimization of total affinity gives the best assembly of the broken pieces. The features used in [SE05] were the mean and the variance of the pixels, however, we will show in this paper that other features can drastically influence the results of the reconstruction. For example, the algorithm described in [SE05], given the input shown in Fig. 1 (a), provides different outcomes when using RGB and gray scale based features, as shown in Fig. 1 (b) and (c) respectively. The use of matching by features method, as compared to matching by shape, is advocated by the fact that in real archeologic broken pieces, usually, parts are missing from the original (see Fig. 1 (c)).

Figure 1: Example of reconstruction of fragments using different features: (a) a typical input broken image; (b) a reconstruction based on RGB features; (c) a reconstructed image using the gray scale component.

In this work, we try to answer the question: what are
the best features to be used in the context of automatic reconstruction of archeological broken objects? Therefore, we present a quantitative and qualitative comparison over a large set of features and over a large set of synthetic and real broken objects. To the best of our knowledge, this is the first work that provides evidence for the most useful features for reconstruction of broken pieces. The features that we consider are local and are computed on small neighborhoods around pixels that are selected for analysis. We also propose a set of curvature field based features. We compute curvature fields by sampling continuous B-spline [IRI] fields. The curvature field serves as an input image, on top of which the described features are evaluated.

2. Features

In this section, we describe the features that we analyze. We compute the mean, the median, the variance, and the canonical moments (up to order six). Each of these features is computed as the mean, the median, the variance, and the canon-

canonical moments of the analyzed object. They are intrinsic to the shape. For clarity, when computing canonical moments we obtain \( m_{10} = m_{01} = m_{11} = 0 \).

2.3. Curvature Field

In this section, we define and describe the computation of the curvature field of an image that we implemented. Let \( I_{x,y} \) denote the values of the pixels of the input image, where \( x \in \{0..m\} \) and \( y \in \{0..n\} \). Here, \( m \) and \( n \) are the input image dimensions. Following [SER06], we model the image \( I_{x,y} \) as a continuous field represented by a uniform open-end cubic B-spline surface \( f(x,y) = \sum_{i=0}^{\Omega} \sum_{j=0}^{\Omega} h_{i,j} B_{i,3}(x) B_{j,3}(y) \), where \( \tau_x \) and \( \tau_y \) are (uniform) knots. The knots are defined such that the domain of definition of the B-spline surface fits exactly the domain of the input image. We symbolically compute B-spline surfaces of the first and second derivatives of \( f(x,y) \). For each pixel \( (x,y) \), we evaluate the derivatives \( f_x, f_y, f_{xx}, f_{xy}, \) and \( f_{yy} \), and assign \( K_a(x,y) = -\text{sign}(f_x) \frac{L_0 f_x^2 - 2f_x f_{xx} f_y + f_{yy} f_x^2}{\sqrt{\sigma_x^2 + \sigma_y^2}} \) where the derivatives are evaluated at the \((x,y)\) parametric location and \text{sign}(z) means the sign of the \( z \) value.

The resulting \( K_a \) has the same dimension as \( I \) and represents the values of the normal curvatures of iso-lines of \( I \). Mainly, we followed the line of symbolic computations of the Gaussian and mean curvature in [SER06], however, we decided to move part of the computations into the discrete domain for computational reasons. Figures 2 (a) and (b) represent an input image and its curvature field respectively.

![Curvature field and inpainting: (a) an archeological image; (b) the curvature field of (a); (c) a cropped region from (b); (d) the cropped region in (c) which is inpainted.](image_url)

(a) (b) (c) (d)

3. A Feature Comparison Method

In this section, we describe the comparison procedure for features. We define a grading procedure of the features when matching two fragments. We employ the grading procedure over a large set of puzzles towards feature comparison. Our technique of features comparison is inspired from the reconstruction technique implemented in [SE05], which we explain shortly in the next paragraph.

Consider the archeological basirelief fragments shown in Figures 3 (a) and (b). In [SE05], each one of the fragments is inpainted, employing [CPT04]. For example, the inpainted region for Fig. 3 (b) is illustrated in Fig. 3 (c) and denoted by \( L \). We will then compare the features of the inpainted region of one fragment with all the other fragments. The inpainting procedure naturally produces confidence maps; the further
away we get from the original piece, the lower the confidence of each predicted pixel is, as shown in Fig. 3 (d). The confidence values decrease from blue to red.

![Figure 3: Reconstruction and grading: (a) and (b) two fragments; (c) inpainting of (b); (d) confidence of inpainting in (c) represented as a distance map.](image1)

In the remaining of this section, we will describe the grading computation of one arbitrary feature measuring the degree of similarity of the feature for a candidate match. The grading procedure of two fragments is composed of two steps. In each one of these steps, one of the fragments is considered fixed, while the other one is rotating. In each step, we compute an intermediary grade. Let’s call this intermediary grade a directional grade. The directional grade consists of computing scores for feature similarity for each pixel in the inpainted regions.

### Per Pixel Score Computation
Consider the two pieces illustrated in Figures 3 (a) and (b). Assume the fragment in Fig. 3 (b) is fixed and the other one floating. For every pixel \((x, y) \in L\) we compute each one of the features described in Section 2. Without loss of generality, we assume that the pieces have been aligned up to translation. The fragment (a) is translated with at most two pixels in each possible direction. For each such translation, the feature of the pixel \((x, y)\) is compared versus the overlapping pixel in the translated piece (a). Therefore, the features of the pixel \((x, y)\) have up to twenty counterparts from the floating fragment (a). Denote the feature value of \((x, y)\) in \(L\) with \(f\) and its counterparts with \(f_{i,j} \mid i,j \in \{-2,\ldots,2\}\). Define an error of matching as the score equals \(\sum_{(x, y) \in L} \sum_{i,j = -2}^{2} \frac{|f_{i,j} - f|}{\sigma} \). The error is transformed in a matching score via a quantization table, i.e. for \(0 \leq \text{error} < 10^{-4}\) the score equals 10, for \(10^{-4} \leq \text{error} < 10^{-3}\) the score equals 9, for \(10^{-3} \leq \text{error} < 10^{-2}\) the score equals 8, and so on. In the end, for \(10^{-2} \leq \text{error} \leq 1\) the score equals 6.

### Directional Grade Computation
For each pixel \((x, y) \in L\), we compute the feature score as explained in the previous paragraph. Then, the directional grade is the average of all the scores of all the pixels in \(L\). This average is weighted using the distance map. The corresponding weight is implemented by \(w(x, y) = \exp(-\text{distmap}(x, y)/\sigma)\), where \(\sigma\) is a constant. Empirically, we choose \(\sigma = 1000\). The global score per fragment of image is computed by \(\text{score}(x, y) = \sum_{(x,y) \in L} \text{score}(x, y) + w(x, y)\).

Finally, the grade of matching of two fragments is the average of the two directional grades. Global grades for large sets of puzzles are achieved by averaging over matching of two fragments matching grades.

4. Experiments
We chose twenty five real pieces, see Fig. 4. Part of the pieces are physically broken, as shown in Fig. 1 (a). For all others, we simulated fragmentation using masks randomly generated.

4.1. Experimental results
In total, seventy five (solved) puzzles were generated (employing masks) and analyzed. We generated masks of 4, 8, and 16 fragments. The features are computed over grayscale, RGB, and YCbCr representations of images. In addition, the features were also computed over the curvature field images described in Section 2.3.

![Figure 4: Examples of synthetic and real images.](image2)

The best features obtained for a subset of four images from all the twenty five images used in experiments are shown in Table 1. The last line summarizes the result per number of fragments used in puzzle.

<table>
<thead>
<tr>
<th>Feature</th>
<th>4 pieces per mask</th>
<th>8 pieces per mask</th>
<th>16 pieces per mask</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{1})</td>
<td>(\text{mean Cr} 0.15)</td>
<td>(\text{mean Cr} 0.11)</td>
<td>(\text{mean Cr} 0.07)</td>
</tr>
<tr>
<td>(w_{2})</td>
<td>(\text{mean Cr} 0.15)</td>
<td>(\text{mean Cr} 0.11)</td>
<td>(\text{mean Cr} 0.07)</td>
</tr>
<tr>
<td>(w_{3})</td>
<td>(\text{mean Cr} 0.13)</td>
<td>(\text{mean Cr} 0.1)</td>
<td>(\text{mean Cr} 0.06)</td>
</tr>
<tr>
<td>(w_{4})</td>
<td>(\text{mean Cr} 0.14)</td>
<td>(\text{mean Cr} 0.1)</td>
<td>(\text{mean Cr} 0.07)</td>
</tr>
<tr>
<td>Total</td>
<td>(\text{mean Cr} 0.14)</td>
<td>(\text{mean Cr} 0.11)</td>
<td>(\text{mean Cr} 0.07)</td>
</tr>
</tbody>
</table>

We summarized the grades of the utility of features in Fig. 5. The features are sorted in decreasing order, from the best feature to the worst. On the vertical axis, we represent the average of all the grades of matching obtained in all the experiments.

The first best twenty features are: mean of component Cr, \(m_{0}\) of component Cr, mean of component Cb, \(m_{20}\) of component Cr, \(m_{22}\) of component Cr, \(m_{24}\) of component Cb, \(m_{20}\) of component Cr, \(m_{24}\) of component Cb, mean of component Y, \(m_{20}\) of component Y, \(m_{24}\) of component Y, mean of gray scale component, \(m_{24}\) of component Y.
gray scale component, mean of G component, $m_{00}$ of component G, and $m_{20}$ of gray scale component.

The curvature based features received relatively low grades. This result follows the phenomenon illustrated in Figures 2 (c) and (d). Fig. 2 (d) is an inpainted result of (c), which is a curvature field. One can see that in the inpainted region, the curvature values are not preserved.

We mention that our research was triggered by an empirical observation that the YCbCr representation of puzzle images provide the basis for best image reconstruction. The phenomenon was observed in Fig. 6 and extensive testing confirmed this empirical observation. Extensive tests were performed using automatic generated random puzzles, such as the one shown in Figures 6 (e).

![Graph with grades per feature. The global grades are averages of all the grades of matching obtained in all the experiments.](image)

**Figure 5:** Graph with grades per feature. The global grades are averages of all the grades of matching obtained in all the experiments.

We show the results of reconstruction of a piece from the Digital Forma Urbis Romae Project collection [URB] in Fig. 7. The original piece (see Fig. 7 (a)) is broken in eight parts (see Fig. 7 (b)) and reconstructed (see Fig. 7 (c)).

![Example of reconstruction of fragments using different features](image)

**Figure 6:** Example of reconstruction of fragments using different features: (a) a broken image; (b) the reconstructed image using the mean of the gray scale component; (c) The reconstructed image using the mean of the Y, Cb, Cr channels; (d) a synthetic image; (e) a mask for fragmenting simulation.

**5. Conclusions and Future Work**

In this work, a framework for measuring utility of features for reconstruction of 2D archaeological pieces was proposed. We conclude that the mean values of the Cr and Cb channels provide the best features statistically. Extensive testing on images of various artificially or real broken pieces validate our conclusions. Further testing of our method for archaeological image reconstruction with the best twenty features will be performed.

**References**


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