M-Algorithm-Based Optimal Detectors for Spatial Modulation

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Abstract — A novel M-algorithm-based detector, namely M-algorithm to maximum-likelihood (M-ML), for spatial modulation (SM) receiver is proposed recently, however, due to the miscalculation of metrics, bit-error-rate (BER) performance of M-ML is far away from optimal actually. In this paper, a modified M-ML detector, which calculates accumulated metrics of each layer and can achieve optimal BER performance, is proposed; Furthermore, an M-algorithm-based constellation-reduction (MCR) method specially for SM in high correlated environment is proposed, MCR is capable of combining with maximum-likelihood (ML) and M-ML to further reduce complexity. The BER performance of M-ML and MCR is simulated, simulation results show that the proposed detectors can achieve almost the same BER performance to ML detector while reducing the computational complexity significantly.

Index Terms — Spatial modulation, M-algorithm, correlated channel, maximum-likelihood

I. INTRODUCTION

To meet the requirement of the development of future mobile communications, the work on wireless communication systems strives for the purpose of high data transfer rate [1]. Although multiple-input multiple-output (MIMO) technique is widely recognized as an important approach to increase data transfer rate, MIMO systems could be incapable of action in the aspects of energy efficiency and detection complexity in the high data rate condition [2]. To solve this problem, spatial modulation (SM) which is known as a single radio-frequency MIMO technique, is proposed in [3]-[5].

Compared with conventional MIMO systems, SM makes use of the difference of MIMO channels to convey information based on a new spatial dimension, thus the conventional two-dimensional (2D) constellation is extended to three-dimensional (3D) constellation as shown in Fig. 1. Due to the high energy efficiency and low detection complexity, SM is regarded as a candidate technique for future wireless communication systems, especially for large-scale MIMO [6].

SM is also an effective technique to overcome inter-channel interference (ICI) and time synchronization among antennas which are necessary for MIMO systems [3]-[5]. Besides, the complexity of optimal detector for SM has an exponential relationship with modulation order and increases linearly with the number of transmit and receive antennas, thus SM can reduce detection complexity significantly compared to conventional MIMO [3]-[5]. However, since some transmit antennas remain inactive in every channel use, SM offers a lower throughput than spatial multiplexing [7]. This implies that SM requires more transmit antennas or higher modulation order for achieving the same spectral efficiency. As a result, the complexity of SM with high data rate is still great.

At the receiver, the optimal detection algorithm needs to estimate both the modulated symbol and index of selected antenna by a joint exhaustive search for recovering the information data [8]. The most obvious and efficient way to reduce complexity is to restrict maximum-likelihood (ML) search space. For example, the sphere decoder (SD) algorithm, applied to both MIMO and SM [9]-[13], avoids the exhaustive search by examining only those points that lie inside a sphere with an appropriate radius.

However, another popular detector for MIMO systems, M-algorithm to maximum-likelihood (M-ML) detector [14], has barely been studied for SM to date. It can reduce the complexity of ML detection by combining with QR decomposition and tree search structure [15], [16]. The main purpose of M-algorithm is that limit the candidate constellation points to a fixed number \( M \) to reduce the exhaustive search space. In [17], a novel M-algorithm-
based detection method called M-ML applied to spatial modulation was proposed, the proposed algorithm has fixed complexity and reduces complexity by using parallel structure to limit the search space of optimum detector. However, follow the procedures described in [17], the M-ML detector performs far away from the optimal detector actually. This is mainly because the metrics of each layer are miscalculated. In this paper, a modified M-ML detector is proposed which corrects the miscalculation of metrics and achieves optimal bit-error-rate (BER) performance.

Considering the characteristics of SM, lower data rate and complexity of detection, it is more suitable to be used as an enhancement of uplink transmission [5]. Therefore, in order to obtain higher data rate of uplink, the terminal side needs to allocate a large number of transmit antennas. However, due to the constraint of terminal size, it is difficult to make the correlation coefficients of the antennas to be low enough. In this high correlated condition, the optimization of signal detection algorithm is barely been studied.

Therefore, to reduce the complexity of SM signal detection in high correlated channel, a novel M-algorithm-based constellation-reduction algorithm called MCR is proposed in this paper. MCR algorithm reduces the constellation search space of ML by using the highly relevant characteristics of transmit antennas while maintaining an optimum performance. Moreover, MCR can be combined with both ML and M-ML to get further complexity reduction.

The rest of this paper is structured as follows. Section II presents a general description of SM system model. The modified M-ML detector and proposed MCR algorithm are discussed in Section III. Section IV shows some simulation results. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

SM system block scheme is presented in Fig. 2. Assume a $N_t \times N_r$ SM system with $L$-ary constellation and complete channel state information at receiver, the steps of SM can be described as follows:

Step 1: Segment the input bit stream into blocks with the length of $\log_2(N_t \times L)$.

Step 2: Each block is mapped into both constellation points and space dimension, where $\log_2 L$ bits select a symbol $s$ from the constellation set $S$ and $\log_2 N_t$ bits select one antenna out of $N_t$ transmit antennas.

Step 3: Every symbol is transmitted by the correspondingly selected antenna, and the other antennas stay idle during the same time slot.

Step 4: The receiver does signal detection in order to estimate both the transmit symbol and the index of selected antenna.

For example, assume a system with 4 transmit antennas and BPSK constellation in Fig. 3. The information bits are firstly segmented into blocks with 3 bits length, then mapped to a $\pm 1$ BPSK symbol and transmitted on one of the four available antennas.

The system model can be formulated as

$$ y = HX + n, $$

where $y$ is the $N_r \times 1$ received symbol vector, $H$ is the $N_r \times N_t$ channel matrix, $X$ is the $N_t \times 1$ transmit symbol vector modulated by $L$-QAM, $n$ denotes the $N_r \times 1$ Additive White Gaussian Noise (AWGN) with zero-mean and variance $\sigma^2$. Furthermore, based on the characteristics of SM, the system model can be simplified into

$$ y = h_i x_i + n = h_i s + n, $$

where $h_i$ is the $i$-th column of $H$ and denotes the complex channel gain between the active transmit antenna and the receive antennas. $x_i$ is the $i$-th element in $X$ and can be further written as the selected symbol $s$ from the constellation set $S$. $1 \leq i \leq N_r$, $s \in S$.

At the receiver, the optimal detection algorithm is ML detector in [8] and can be described as follows,

$$ \{i_{ML}, s_{ML}\} = \arg\min_{i,s} \| y - h_i s \|, $$

$$ = \arg\min_{i,s} \sum_{j=1}^{N_t} |y_j - h_{ij}s|^2 $$

$$ = \arg\min_{i,s} \sum_{j=1}^{N_t} E_j $$

Fig. 3. SM with 4 transmit antennas and BPSK
where \( i_{ML} \) and \( s_{ML} \) are the estimations of \( i \) and \( s \). \( i \) denotes the transmit antenna index. \( s \) is the transmit symbol, and \( \| \| \) represents Frobenius norm operation.

Use 
\[
E_j = \left| y_j - h_{j,s} \right|^2
\]
to denote the metric of the \( j \)-th receive antenna and \( i \)-th transmit antenna. Besides, define 
\[
A_i = \sum_{j,i,j} E_j
\]
to denote the accumulated metrics of the \( k \)-th receive antenna.

III. DETECTORS

In this section, the M-ML algorithm is modified and the proposed detector specially for applying in high correlated channel is described.

As shown in Fig. 4, ML detector of SM system also has tree search structure. However, the difference is that since only one antenna is active during each transmission in SM, there is no need to do QR decomposition to realize layer search. SM regards each receive antenna as a level of the tree structure. Fig. 4 shows the tree search structure of a \( 4 \times 3 \) SM system with BPSK constellation, the black nodes represent the path with smallest accumulated metrics.

The number of the real multiplications required by one detector is calculated to examine the computational complexity. The computational complexity of (3) is
\[
C_{ML}^{N_sN_t} = 8N_sN_tL
\]
Computing \( \left| y_j - h_{j,s} \right|^2 \) requires 8 real multiplications.

Equation (4) shows that the complexity of SM used massive antennas or high-order modulation is still great.

In this paper, the extraordinarily popular Kronecker channel model in [18], [19] is used as a general model to deal with the correlated channels.

Define the transmitter and receiver correlation matrices are \( R_{N_t} \) and \( R_{N_r} \)
\[
R_{N_t} = \frac{EH^H}{Tr[R_{N_t}]}
\]
\[
R_{N_r} = \frac{EH^H}{Tr[R_{N_r}]}
\]
where \( E[\cdot] \) and \( Tr[\cdot] \) denote the expectation and trace operator, \( (\cdot)^H \) is the conjugate (Hermitian) transpose of a matrix. Moreover, the spatial stationarity is assumed, i.e., the correlation coefficients remain invariant as long as the antenna separations remain constant.

In order to simplify the description and simulation, \( k_t \) and \( k_r \) represent the correlation coefficient of transmit and receive antennas, respectively. Besides, assume that the correlation coefficient between each antenna is restricted to be the same real value, and \( 0 \leq k_t, k_r \leq 1 \). For example, the transmitter correlation matrix of a four transmit antennas system can be written as
\[
\begin{bmatrix}
k_t & k_t & k_t \\
k_t & k_r & 1 \\
k_t & 1 & k_t \\
k_t & 1 & 1
\end{bmatrix}
\]

Note that this restriction has no effect on the simulation results and performance of the proposed algorithm.

TABLE I. MODIFIED M-ML ALGORITHM

<table>
<thead>
<tr>
<th>Initialization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_0 = {i,s}</td>
</tr>
<tr>
<td>( M = {M_{1,1}, \ldots, M_{N_t,j}, M_{N_t,j+1} } )</td>
</tr>
<tr>
<td>( M_{N_t} = 1, M_{j+1} \leq M_j )</td>
</tr>
<tr>
<td>Algorithm:</td>
</tr>
<tr>
<td>1: for ( j = 1 ) ( \rightarrow ) ( N_t )</td>
</tr>
<tr>
<td>2: for ( (i,s) \in Q_{j+1} )</td>
</tr>
<tr>
<td>3: ( \delta_j(i,s) = A_j )</td>
</tr>
<tr>
<td>4: end</td>
</tr>
<tr>
<td>5: ( Q_j = \arg(\delta_j(i,s) \text{ smallest} )</td>
</tr>
<tr>
<td>6: end</td>
</tr>
<tr>
<td>7: ( {s_{ML}, i_{ML}} = Q_{N_t} )</td>
</tr>
</tbody>
</table>

where \( A_j = \sum_{k=1}^{j} \left| y_k - h_{k,i,s} \right|^2 \) denotes the accumulated metrics of the \( j \)-th receive antenna.

The number of multiplications required by M-ML is given in [17]. The complexity of M-ML is
\[
C_{ML}^{N_sN_t} = 8N_sN_tL + \sum_{i=1}^{N_t} 8M_i
\]
An important observation from (7) is that M-ML algorithm has the advantages of variable complexity depending on the controllable value of $M$, and associated with the parallel structure, M–ML is a promising way to application.

In this paper, the M-algorithm is also tailored to SM in high correlated environment. However, different from the joint estimation of transmit symbol and antenna index in ML, the proposed MCR, which exploits the high correlation between channels, reduces the search space for constellations above all, and then conduct a joint detection. The proposed MCR algorithm can be described in Table II.

<table>
<thead>
<tr>
<th>Algorithm:</th>
</tr>
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<tbody>
<tr>
<td>1: $\forall i \in {1,2,..,N_t}$</td>
</tr>
<tr>
<td>2: for $s \in S$</td>
</tr>
<tr>
<td>3: $\delta(i,s) = \left| \mathbf{y} - \mathbf{h}_s \right|_f^2$</td>
</tr>
<tr>
<td>4: end</td>
</tr>
<tr>
<td>5: $G = \arg(M_0 \text{smallest } \delta(i,s))$</td>
</tr>
</tbody>
</table>

MCR algorithm retains $M_0$ constellation points with smallest metrics to form a new set $G$, thus the constellation search space is restricted. After that a joint search such as ML or M-ML over $G$ and all transmit antennas can be conducted to estimate $i$ and $s$.

For example, a $2\times 2$ SM system using QPSK constellations and without considering the effect of noise. The following sequence of bits to be transmitted, $q=[0\ 0\ 0\ 0]$, the first two bits can be mapped to a QPSK symbol $s=[0.7071+0.7071i\ 0.7071-0.7071i]$, where $i$ is the imaginary number, and the last bit 0 represents to transmit on the first antenna, the $2\times 2$ channel matrix $\mathbf{H}$ is

$$
\begin{pmatrix}
0.5308 + 0.0147i & 0.9165 + 0.1884i \\
-0.9215 - 0.0734i & -1.2712 + 0.1491i
\end{pmatrix}
$$

Correspondingly, the received vector is $y=[0.3650 + 0.3857i\ -0.5997 - 0.7035i]$. Then, MCR detector selects a transmit antenna arbitrarily (assume to choose the second transmit antenna) to calculate the values of $\left\| \mathbf{y} - \mathbf{h}_2 \right\|_f^2$. The results of calculation are $[0.8376\ 2.7111\ 2.5789\ 3.6557]$.

The two constellation points with the smallest metrics are $s_1=0.7071+0.7071i$ and $s_2=0.7071-0.7071i$, then conduct a ML search over these two points, the search result shows that $s_1=0.7071+0.7071i$ is the transmit symbol and the first antenna is the transmit antenna, therefore, the information bits are $[0\ 0\ 0\ 0]$.

In high correlated channels, the channel responses between transmit antennas become difficult to be distinguished [20]. Hence, take advantage of this characteristic, the transmit antenna can be supposed to be anyone to recover the modulated symbols. Obviously, this detection is not reliable because of the arbitrary choice of antennas. Therefore, to compensate for this loss, a proper number of candidate constellation points are retained to make sure that the correct one is included. After doing this, the size of constellation set is reduced, then joint detection can be done to achieve the optimal performance. This is the main principle of MCR algorithm.

According to ML, the complexity of calculating the metrics for the given $\mathbf{h}$, is $8N_tL$, and the complexity of a joint exhaustive search over the new constellation set is $8N_tN_0M_0$ as well. Therefore, the complexity of MCR combine with ML is

$$
C_{MCR\&ML}^{-N_t, N_0} = 8N_tN_0M_0 - 8N_tM_0^2 + 8N_tL
$$

Compare to ML detector, the reduction of complexity is

$$
R_{MCR\&ML} = \frac{C_{ML}^{-N_t, N_0} - C_{MCR\&ML}^{-N_t, N_0}}{C_{ML}^{-N_t, N_0}} = \frac{(L-M_0)(N_t-1)}{N_tL}
$$

As shown in (9), the proposed algorithm can reduce the complexity of ML in a system with at least 2 transmit antennas (absolutely suitable for SM). Besides, an important observation from (9) is that smaller the value of $M_0$, larger the complexity reduce. When SM systems use large number of transmit antennas so that the value of $(N_t-1)/N_t \approx 1$, the equation (9) can be simplified to

$$
R_{MCR\&ML} = 1 - M_0/L
$$

Therefore, the reduction of complexity for MCR with ML mainly depends on the value of $M_0$.

To conveniently analysis the complexity of MCR with M-ML, define a new set $M=[M_0, M_1, ..., M_r, ..., M_{N_t}]$, where $M_0$ represents the value used by MCR and $M_1 \sim M_{N_t}$ denotes the $M$ values for M-ML, $M_0 \leq L$, $M_1 \leq N_t/M_0$. Note that there is no need to calculate the metrics of the antenna which has been selected by MCR, actually the number of transmit antennas should be $N_t-1$ for M-ML. Then the complexity of MCR with M-ML can be calculated as follows:

$$
C_{MCR\&M-ML}^{-N_t, N_0} = 8(N_t-1)M_0 + \sum_{i=1}^{N_t-1} 8M_i + 8N_tL
$$

The reduction of computational complexity versus ML detector is

$$
R_{MCR\&M-ML} = \frac{C_{ML}^{-N_t, N_0} - C_{MCR\&M-ML}^{-N_t, N_0}}{C_{ML}^{-N_t, N_0}} = 1 - \frac{1}{N_t} \frac{(N_t-1)M_0 \sum_{i=1}^{N_t-1} M_i}{N_t^2N_tL}
$$

Equation (12) shows that the reduction increases with $N_t$ and $L$, therefore this algorithm has the advantage of
applying in large-scale antennas and high-order modulation condition.

Thanks to the introduction of M-algorithm, the algorithms above are all have fixed complexity and parallel structure which are more suitable for practical implementation.

Compare with other SM detectors, for instance, the novel detection algorithm for SM based on the modified sphere decoder (SM-SD) which has variable complexity and serial detection structure and is difficult for practical applications, the algorithms above with fixed complexity and breadth-first parallel structure are all conducive to practical implementation. However, they also have some drawbacks, such as higher average complexity and complicated metric sorting process, however, considering the practical application, M-algorithm-based detectors are still a promising way for SM signal detection.

IV. SIMULATION RESULTS

This section provides simulation results to compare BER performance of the proposed algorithms and optimal detector. In the simulations, a 4×4 uncoded SM system with three different bit rates (r = 4, 6, 8 bpcu) is considered, the Kronecker channel model is used to realize correlated fading environment. Configuration of the simulator can be found in Table III.

| TABLE III: CONFIGURATION OF THE SIMULATOR |
|-------------------------------|------------------|
| Simulation software      | MATLAB            |
| Channel model            | Kronecker         |
| Modulation type          | QPSK,16QAM,64QAM  |
| Frame length             | 1000              |
| Times of simulation loops| At least 10^5     |

The BER performances versus signal-noise-ratio (SNR) are shown in Fig. 5-7. Firstly, a comparison between M-ML and modified M-ML is shown in Fig. 5, where \( k_i = k_r = 0.1 \). MCR with ML and MCR with modified M-ML are compared with the optimal detector in Fig. 6 and Fig. 7, where \( k_i = 0.8 \) and \( k_r = 0.1 \).

In Fig. 5, the values of \( M \) for each rate are \([12 6 4 1]_{r=4} \), \([32 10 4 1]_{r=6} \), \([64 34 16 1]_{r=8} \), an obvious observation is that with the same value of \( M \) the modified M-ML can achieve almost optimal performance while M-ML perform far away from it. The reason is obvious, due to the miscalculation of metrics, M-ML cannot retain the correct candidate points, consequently, the performance decreases substantially. However, the modified M-ML, which uses accumulated metrics to measure each candidate point, can retain the correct points and near optimal performance.

Fig. 6 and Fig. 7 demonstrates that MCR with ML or MML can perform extremely close to ML, note that MCR with M-ML has a performance loss comparable with ML at low SNR, this is because of the characteristic of M-ML [17], the values of \( M \) are not big enough to contain the correct estimates when the power of noise has a great influence on tree search. Moreover, for MCR with ML, the corresponding values of \( M \) are 2, 8, 32 and the reductions of complexity for each rate are all 37.5%; for MCR with M-ML, the corresponding values of \( M \) for each rate are \([2 4 3 2]_{r=4} \), \([8 12 6 4 1]_{r=6} \), \([32 32 16 8 1]_{r=8} \) and the further reduction of complexity are 51.56%, 57.03%, 60.16%, respectively. As can be seen, a higher reduction of complexity can be achieved for higher data rates, this is mainly due to the fact that the complexity of ML detection with high-order modulation increases more significantly than with the M-ML detectors.
According to the simulation results, another significant phenomenon is that the channel correlation has a great negative influence on BER performance, specifically, the larger the correlation coefficient is, the worse the system performs. This is mainly due to the high correlated channel responses of transmit antennas are difficult to be distinguished so that the probability of estimating the correct antenna index declines.

V. CONCLUSIONS

In this paper, the M-ML detector for SM is modified, the modified one, which uses accumulated metrics to measure each candidate point, can obtain optimal BER performance; moreover, another M-algorithm-based detector called MCR is proposed, which can restrict the size of constellation space for joint exhaustive search in correlated environment. More specifically, MCR takes advantage of the correlation between transmit antennas to reduce search space of modulated symbols and does a joint search just over the retained points and all transmit antennas. Furthermore, MCR detectors can be associated with both ML and M-ML to achieve optimal BER performance, as well as reduces the computational complexity significantly when the value of M is appropriately selected.

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