Effect of Terrain Traction, Suspension Stiffness and Grasp Posture on the Tip-Over Stability of Wheeled Robots with Multiple Arms

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Received 22 February 2011; accepted 28 June 2011

Abstract

In this paper, dynamics, postural stability and control of suspended wheeled mobile manipulators for cooperative heavy object manipulation are elaborated considering the effect of grasping posture. The presented model considers a system equipped with multiple manipulators with flexible suspension, which also contains an accurate nonlinear behavior of the tires. Moreover, it includes the vibratory response of the tires as unsprung masses. Therefore, this is one of the most complete models that have been presented for wheeled mobile manipulators to date. First, based on the Newton–Euler formulation for a chain of rigid bodies, the dynamics model of such complicated systems in three-dimensional maneuvers is developed without considering a nonlinear frictional model of tires, which was verified using the ADAMS multibody simulator. Then, a proper nonlinear friction model is added to the developed dynamics to provide a more complete one. Considering pneumatic tires, the Dugoff tire friction model is adopted to describe the longitudinal and lateral forces produced at the contact patch of the wheels. Using the obtained dynamics along with the moment-height stability measure the effect of frictional effects as well as suspension attributes on the postural stability of such systems are accurately investigated for maneuvers over flat and rough terrains. Finally, the effect of grasping posture and relevant configuration of the robot on the stability of the system is examined during a heavy object manipulation task.


Keywords

Wheeled mobile robots, suspension system, postural stability, Dugoff friction model, cooperative manipulation control, grasp condition

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1. Introduction

An autonomous wheeled mobile robot equipped with one or more manipulators combines the dexterous manipulation and locomotion capabilities at the price of a more complex mechanism and new challenges in the design, planning and control. Although wheeled mobile manipulators have been extensively studied to date, in most of the previous investigations it was supposed that the workspace of the robot is a structured laboratory-like area. Consequently, most of the previous studies considered a rigid suspension for the system. However, exploiting a wheeled mobile robotic system in uneven environments, it is inevitable to use some kind of suspension/tire compliance module. In addition, the use of a suspension subsystem increases the safety/durability of the base as well as its mounted arm(s), in response to shocks and jerks transmitted to them due to irregularity of the open terrain. Thus, a suspended wheeled mobile manipulator (SWMM) can be considered as an appropriate system for motion on rough and gentle rugged terrain, as shown in Fig. 1. It is remarked that the mobile mechanism using wheels is one of the most popular mechanisms for mobile robots because the wheel-type mobile mechanism’s energy efficiency is high and the mechanism is simple [1].

Attempts to attain the dynamic equations of motion for mobile manipulators have presented successful results. However, most of them have ignored the compliance characteristics of the platform/tire(s). A systematic method based on the natural orthogonal complement method for the kinematics and dynamics modeling of a 2-d.o.f. automated guided vehicle has been presented by Saha and Angeles [2]. Forward recursive formulation based on the principle of virtual works for the dynamics of multibody systems has been employed to obtain the governing equation of the nonholonomic mobile manipulator systems in Ref. [3]. Kane’s approach has been successfully exploited for the dynamics of multiple cooperating mobile manipulators moving on a flat surface while transporting an elastic object [4]. In Ref. [5], a systematic procedure has been presented to obtain the mathematical model of

![Figure 1. Dual-arm suspended mobile manipulator that can move on rough terrain to manipulate objects.](image-url)
SWMMs using the iterative Newton–Euler approach. In Ref. [5], the system dynamics were derived without considering the frictional behavior of the tires and they were then validated using a model provided by commercial ADAMS simulation software. In this paper, the tire friction model is added to the previously derived validated dynamics of SWMMs in Ref. [5]. The effect of various parameters such as the terrain traction on the system response in terms of its postural stability will then be investigated.

However, another issue that is very important in the control of mobile manipulators is the ability to maintain stability. Postural stability as a vital problem has been studied by both robotics and vehicle research communities. It is remarked that the danger of roll-over accidents is significant in passenger vehicles, as roll-overs result in more than 9000 fatalities and 200,000 non-fatal injuries merely in the US annually [6]. In addition to passenger vehicles, in legged robots (biped, monopod, quadruped, etc.) as well as wheeled mobile robots a loss of postural stability may have potentially serious consequences, and this calls for its thorough analysis to better predict and eliminate the possibility of falling.

Attempts to propose dynamic stability measures have provided successful results. The zero moment point (ZMP), first discussed by Elfman in 1938 for the study of human biomechanics, has only more recently been used in the context of legged machine control [7]. However, the ZMP for systems subjected to only weight force is not sensitive to the variation of the center of mass height [8, 9]. In the field of wheeled mobile robots this ground reference point was exploited for the first time by Sugano et al. [10]. Yet, the ZMP does not provide any specific indication about the system instability [11]. The foot rotation indicator (FRI) point extends the concept of ZMP and quantifies the severity of foot rotational acceleration during the single support phase [11]. To include the mass moment of inertia the original formulation of ZMP has been modified [12]. In addition to the ZMP and FRI, another ground reference point has been introduced recently as the centroidal moment point [7] or zero rate of angular momentum point [13]. It was established based on recent biomechanical investigations that confirm that the total angular momentum is highly regulated throughout the walking cycle of a human.

Papadopoulos and Rey have reported another measure named the force–angle margin [14]. Shiller and Mann have suggested a velocity/acceleration-based dynamic stability measure for off-road vehicles [15]. More recently, a new postural stability metric has been suggested by Moosavian and Alipour, which is called the moment-height stability (MHS) measure [8, 9]. This measure is exploited for general three-dimensional (3-D) systems in this paper by which the effect of various parameters on the stability of the system is analyzed.

2. Benchmark System Description

The next generation of robot systems must be autonomous and dexterous [16]. Dexterity implies the mechanical ability to carry out various kinds of tasks in various
situations. To have higher dexterity, a robotic system should have two or more arms, rather than one [16]. Therefore, in order for dexterous object manipulation, a mobile robotic system should consist of multiple manipulators. To develop dynamics and postural stability analysis of such systems, in this paper, a SWMM equipped with two 3-d.o.f. manipulators is considered and referred as a ‘benchmark’. The base is mounted on a suspension subsystem as depicted in Fig. 2.

The considered platform in Fig. 2 consists of a 6-d.o.f. sprung mobile base and four 3-d.o.f. unsprung wheels (whose vertical vibrations are modeled separately as rigid masses, $m_{tB}$ and $m_{tF}$, mounted on linear springs). The attached manipulators have the configuration of a PUMA 560 as shown in Fig. 1 where each has 3 d.o.f. (with no wrist).

3. Kinematics and Dynamics Modeling

The system can be divided into two subsystems. One is the base, which provides the mobility of the system (vehicle), and the other is the manipulator arms. The motion of the platform center of mass ($C_0$) or possibly another convenient reference point of the base is used to describe the base translation with respect to an inertial frame of reference (XYZ), which is consistent with the direct path kinematics approach developed by Moosavian and Papadopoulos [17]. The movement of platform body is modeled as a 3-d.o.f. linear motion along $X$, $Y$- and $Z$-inertial axes, and three rotational motions about three base body frame axes (roll, pitch and yaw), which are the $x_0$-$y_0$-$z_0$ Euler angles $\phi$, $\theta$, and $\psi$. The origin of the body frame $x_0$-$y_0$-$z_0$ coincides with center of mass of the base and travels/rotates with the platform. The inertial and base body frames are depicted in Fig. 2.

In order to present a complete model for such systems, the wheels are modeled separately as rigid masses mounted on linear springs. Angular velocities of the base
body frame are $\omega_x$, $\omega_y$, and $\omega_z$, which are about the $x$-, $y$- and $z$-axes, respectively. Note that, in our study, the roll and pitch angles are small and less than $6^\circ$ while the yaw angle can be any arbitrary value. This is assumed as it is tacitly supposed that the robot is traveling over a gentle rough terrain, and hence the platform does not rotate about the roll/pitch axes more than $6^\circ$. However, the yaw angle ($\psi$) represents the heading angle of the platform. Subsequently, it can adopt any arbitrary value to let the robot start its motion from any initial heading angle to an arbitrary final one. In order to derive the mutual dynamic interaction wrenches between the manipulators and the base, first consider the platform as link number ‘0’ and assume that the absolute acceleration of point ‘$C_0$’ is specified in the inertial coordinate as:

$$a_{C_0} = \ddot{X}I + \ddot{Y}J + \ddot{Z}K,$$

(1)

where $I$, $J$ and $K$ are the unit basis vectors of the inertial frame.

Next, either by employing inward/outward iterations or an explicit formulation [5], the interaction forces/torques between the mobile base and manipulators can be calculated in terms of generalized variables describing the system pose and their derivatives.

Applying Newton’s equations in the inertial frame, the equations of motion of the base are obtained as:

$$\sum F_X = m_0a_{C_0}X$$

(2a)

$$\sum F_Tc(\psi) - F_{\text{fric}c}(\psi) + (F_{\text{arms}})_X = m_0\ddot{X}$$

(2b)

$$\sum F_Y = m_0a_{C_0}Y$$

(2c)

$$\sum F_Ts(\psi) + F_{\text{fric}s}(\psi) + (F_{\text{arms}})_Y = m_0\ddot{Y}$$

(2d)

$$\sum F_Z = m_0a_{C_0}Z$$

(2e)

$$F_{FR} + F_{FL} + F_{BR} + F_{BL} - m_0g + (F_{\text{arms}})_Z = m_0\ddot{Z},$$

(2f)

where $F_{FR}$, $F_{FL}$, $F_{BR}$ and $F_{BL}$ are the suspension forces exerted on the base body (Fig. 3). Also, $(F_{\text{arms}})_X$, $(F_{\text{arms}})_Y$ and $(F_{\text{arms}})_Z$ indicate overall forces due to interaction between the base and arms along axes of the inertial reference frame. In addition, $s(\cdot)$ and $c(\cdot)$ stand for $\sin(\cdot)$ and $\cos(\cdot)$, respectively.

Also, Euler’s equations in the base body fixed frame can be written as follows:

$$\sum N_{x0}^{C_0} = I_{x0}x_0\dot{\omega}_{x0} - (I_{y0}y_0 - I_{z0}z_0)\omega_{y0}\omega_{z0}$$

(3a)

$$\sum N_{y0}^{C_0} = I_{y0}y_0\dot{\omega}_{y0} - (I_{z0}z_0 - I_{x0}x_0)\omega_{z0}\omega_{x0}$$

(3b)

$$\sum N_{z0}^{C_0} = I_{z0}z_0\dot{\omega}_{z0} - (I_{x0}x_0 - I_{y0}y_0)\omega_{x0}\omega_{y0},$$

(3c)
where $\sum N_{C0}^x$, $\sum N_{C0}^y$ and $\sum N_{C0}^z$ represent the overall resultant moments exerted on the base body about various axes of its frame $x_0y_0z_0$. The equation of motion for the wheels (unsprung masses) can be written as:

$$K_{ij}(U_{ji} - (Z)_{ji} + (Z_0)_{ji}) - m_{ij}g = m_{ij}(\ddot{Z})_{ji}; \quad \begin{cases} j = F, B \\ i = R, L, \end{cases}$$

(4)

where $U_{FR}$, $U_{FL}$, $U_{BR}$ and $U_{BL}$ are displacements of front right, front left, rear right and rear left wheels, which are all due to excitation from the uneven ground surface profile, respectively.

4. Tire Model

The model developed in the former section does not include the wheel characteristics. This helps maintain the generality of the algorithm developed in this paper for any kind of tire dynamics. In addition, it was determined that the use of a nonholonomic model must be limited to lightweight vehicles that operate under very low speeds, low accelerations and lightly loaded conditions [18]. This fact motivates the dynamics modeling of the mobile manipulators subjected to wheel slip as it is done in this study.

In general, mobile manipulators may have rigid or flexible rubber (pneumatic) wheels. The robot wheel can be assumed to be rigid, such as Sojourner and the currently planned Mars rovers. In contrast to a rigid wheel on firm terrain, if it is assumed that there is a rubber tire that has a contact patch with the ground, then any kind of tire friction model including the Dugoff, the semi-empirical tire model called magic formula, and the LuGre model can be utilized for tire response investigation. Here, the Dugoff tire friction model is utilized for the description of
interaction between the tires and the ground [19]. It is recalled that Dugoff’s pneumatic tire friction model is a proper model to describe the longitudinal and lateral forces acting on the wheels of a wheeled mobile robot for high-load and high-speed applications.

Figure 4 shows the top view of the mobile base model and the nomenclature to be used in this section. The side slip angle of a wheel is defined to be the angle between the tire’s velocity vector and wheel plane. Hence, knowing the \( i \)th wheel’s velocity in the vehicle lateral direction, \( v_{iy} \), and longitudinal direction, \( v_{ix} \), the side slip angle of the wheel is simply defined as:

\[
\beta_i = \tan^{-1}\left( \frac{v_{iy}}{v_{ix}} \right) - \delta_i; \quad i = 1, \ldots, 4, 
\]

(5)

where \( \delta_i \) denotes the steering angle of the \( i \)th wheel. If one substitutes the components of the \( i \)th wheel’s velocity vector (in the vehicle coordinate system) by the velocity components of the reference point \( C_0 \) then side slip angles \( \beta_i \) \( (i = 1, \ldots, 4) \) for each wheel can be calculated as:

\[
\beta_1 = \tan^{-1}\left( \frac{-\dot{X}s\psi + \dot{Y}c\psi + L_F\dot{\psi}}{Xc\psi + \dot{Y}s\psi + W_R\dot{\psi}} \right) - \delta_1 \tag{6a}
\]

\[
\beta_2 = \tan^{-1}\left( \frac{-\dot{X}s\psi + \dot{Y}c\psi + L_F\dot{\psi}}{Xc\psi + \dot{Y}s\psi - W_L\dot{\psi}} \right) - \delta_2 \tag{6b}
\]

\[
\beta_3 = \tan^{-1}\left( \frac{-\dot{X}s\psi + \dot{Y}c\psi - L_B\dot{\psi}}{Xc\psi + \dot{Y}s\psi + W_R\dot{\psi}} \right) - \delta_3 \tag{6c}
\]

\[
\beta_4 = \tan^{-1}\left( \frac{-\dot{X}s\psi + \dot{Y}c\psi - L_B\dot{\psi}}{Xc\psi + \dot{Y}s\psi - W_L\dot{\psi}} \right) - \delta_4, \tag{6d}
\]

where \( L_F \) and \( L_B \) denote the longitudinal distance of the platform center of mass \( (C_0) \) relative to the front and rear wheels centers, respectively (Fig. 4).
The longitudinal wheel slip ratio $s_i$ is defined as:

$$s_i = \frac{V_i - R_w \omega_i}{V_i}, \quad R_w \omega_i < V_i$$

(7a)

$$s_i = \frac{V_i - R_w \omega_i}{R_w \omega_i}, \quad R_w \omega_i \geq V_i,$$

(7b)

where $R_w$ is the effective tire radius, $\omega_i$ and $V_i$ are the angular velocity and the linear velocity component of the wheel center, that is parallel to the $i$th wheel plane, respectively. The tire forces are determined using the simplified model written as:

$$F_{x_i} = -f_i C_{x_i} s_i$$

(8a)

$$F_{y_i} = -f_i C_{y_i} \beta_i,$$

(8b)

where:

$$f_i = \begin{cases} 
1; & F_{R_i} \leq \frac{\mu_i F_{z_i}}{2} \\
2 - \frac{\mu_i F_{z_i}}{2 F_{R_i}} \frac{\mu_i F_{z_i}}{2 F_{R_i}}; & F_{R_i} > \frac{\mu_i F_{z_i}}{2} 
\end{cases}$$

(9)

$$F_{R_i} = \sqrt{(C_{x_i} s_i)^2 + (C_{y_i} \beta_i)^2} \quad (i = 1, \ldots, 4),$$

(10)

where $F_{z_i}$ is the vertical load on each wheel. Also, $C_{x_i}$ and $C_{y_i}$ represent longitudinal and lateral tire stiffness, respectively. Note that by considering the following equations the overall longitudinal/lateral traction force $F_T/F_{\text{fric}}$ applied to the base (Fig. 3) can be written as:

$$F_T = X_1 + X_2 + X_3 + X_4$$

(11a)

$$F_{\text{fric}} = Y_1 + Y_2 + Y_3 + Y_4,$$

(11b)

where

$$X_i = F_{x_i} c \delta_i - F_{y_i} s \delta_i; \quad i = 1, \ldots, 4$$

(12a)

$$Y_i = F_{x_i} s \delta_i + F_{y_i} c \delta_i; \quad i = 1, \ldots, 4.$$  

(12b)

Also, based on Figs 3 and 4, $N_n$ that conducts the platform heading angle may be written as:

$$N_n = (X_1 + X_3) W_R - (X_2 + X_4) W_L + (Y_1 + Y_2) L_F - (Y_3 + Y_4) L_B.$$  

(13)

Finally, the wheel dynamics is given as:

$$\dot{\omega}_i = \frac{1}{I_w} (-R_w F_{x_i} + T_i) \quad (i = 1, \ldots, 4),$$

(14)

where $T_i$ represents the input actuating torque applied to the $i$th wheel.
5. MHS Measure

The MHS criterion has been utilized for stability investigation of wheeled mobile robotic manipulators with rigid elements [8, 9]. This measure is based on stabilizing/destabilizing moments exerted on the platform that direct the rotational behavior. Here, in order to highlight the effect of suspension forces on the system stability, a virtual structure consisting of the tires and a variable-form body, connecting the positions where the suspension elements attach to the tires, is considered. It is worth mentioning that the whole robotic system has been mounted above this structure. Therefore, once this imaginary structure tends to turn over the complex system, consisting of a vehicle and manipulator arms, has a tendency to experience the instability.

For MHS computation, the applied forces/torques (except the forces exerted on the tires) on the virtual structure should be computed. Next, for each edge of the support polygon a unit vector is defined such that all the unit vectors make a closed-loop direction. In this study, we choose the clockwise direction when the support polygon is observed from above. If \( p_1, p_2, \ldots, p_n \) represent the coordinates of various vertices of the convex hull of the virtual structure support area, described in the base coordinate frame, then the unit vectors of the support boundary edges can be computed as:

\[
\hat{a}_i = \frac{p_{i+1} - p_i}{\|p_{i+1} - p_i\|} \quad i = \{1, 2, \ldots, n - 1\} \tag{15a}
\]

\[
\hat{a}_n = \frac{p_1 - p_n}{\|p_1 - p_n\|} \tag{15b}
\]

After that, the applied moments about different vertices of the support pattern will be calculated (and called \( M_1, M_2, \ldots, M_n \)) [8, 9]. Then, the moments about edges \((1, 2), (2, 3)\) and \((n, 1)\) (Fig. 5) are named as \( M_{v1}, M_{v2}, \ldots, M_{vn} \), respectively, which can be calculated as:

\[
M_{v_i} = M_i \hat{a}_i \quad i = \{1, 2, \ldots, n\}. \tag{16}
\]

![Figure 5](image-url)
The dynamic stability associated with the \( i \)th edge of the support boundary polygon is considered as:

\[
\alpha_i = M_{vi} \quad i = \{1, 2, \ldots, n\}.
\]  

(17)

Finally, by considering the most critical case, the overall dynamic MHS measure \( \alpha \) is computed as [8, 9]:

\[
\alpha = (h_{c.m.})^\lambda \cdot \min_i(\alpha_i) \quad i = \{1, 2, \ldots, n\},
\]  

(18)

where \( h_{c.m.} \) denotes the overall system center of mass height and:

\[
\lambda = \begin{cases} 
-1; & \text{if min}_i(\alpha_i) > 0 \\
+1; & \text{otherwise.}
\end{cases}
\]  

(19)

6. Motion Over Level Terrain

6.1. Effect of Suspension Characteristics on Postural Stability of SWMMs

In this subsection, the effect of suspension stiffness on the postural stability of the system is investigated once the system is traveling over a level terrain. To this end, a system as depicted in Fig. 1, with the specifications mentioned in Tables 1–3 has been considered. Note that in Table 3, \( m_{ij} \) and \( l_{ij} \) represent the mass and length of the \( i \)th link of the \( j \)th manipulator arm, respectively. It is pointed out that the stiffness and damping coefficients of the suspension subsystem provide a static deflection of approximately 5 cm with a response below critical damped.

In most of the next computer simulations, the concurrency between manipulation and locomotion will be taken into account. First, it is assumed that the system is moving with a constant velocity of \( v = 5 \) m/s and the end-effectors have grasped a 400-kg payload that is close to the base at the start point. Due to the gravitational loads the center of mass of the base has a displacement of approximately 8 cm while the base rotates about the front edge approximately 3° clockwise (the

<table>
<thead>
<tr>
<th>Table 1.</th>
<th>Specifications of the main body of the platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_0 ) (kg)</td>
<td>( L ) (wheelbase)</td>
</tr>
<tr>
<td>Value (unit)</td>
<td>600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2.</th>
<th>Specifications of the suspension subsystem of the wheeled mobile manipulator system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{FR} = K_{FL} = K_{BR} = K_{BL} )</td>
<td>( C_{FR} = C_{FL} = C_{BR} = C_{BL} )</td>
</tr>
<tr>
<td>Value (unit)</td>
<td>42e3 (N/m)</td>
</tr>
</tbody>
</table>
Table 3.
Specifications of the manipulator arms of the mobile robotic system

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{11}$ = $m_{12}$</td>
<td>85 (kg)</td>
</tr>
<tr>
<td>$m_{21}$ = $m_{22}$</td>
<td>32 (kg)</td>
</tr>
<tr>
<td>$m_{31}$ = $m_{32}$</td>
<td>8 (kg)</td>
</tr>
<tr>
<td>$l_{11}$ = $l_{12}$</td>
<td>1 (m)</td>
</tr>
<tr>
<td>$l_{21}$ = $l_{22}$</td>
<td>1.2 (m)</td>
</tr>
<tr>
<td>$l_{31}$ = $l_{32}$</td>
<td>0.7 (m)</td>
</tr>
</tbody>
</table>

Figure 6. Braking actuating torque exerted on each wheel.

approximate edges configuration can be seen in Fig. 4). Then, the base decelerates with a variable acceleration produced using the wheels actuating torques as depicted in Fig. 6. Concurrently, the second and third joints of the manipulators are moving such that these extend the end-effectors to the point farther from the base, as depicted in Fig. 7, while the first joints are locked in their home configurations. It should be added that in the present case the steering angles (i.e., $\delta_f$, $\delta_r$), are set equal to zero during this motion. To provide a smooth trajectory for manipulators, a quintic polynomial trajectory has been adopted between the two poses [20].

The moving links travel outside the support polygon such that the system is going unstable (Fig. 7). Variation of the MHS measure during such a maneuver, between $t = 0$ and $t = 5$ s, is shown in Fig. 8, which after about 4 s becomes negative, meaning that the system is going unstable. The effect of base suspension on the postural stability of the system is also explored. As can be observed, if the suspension stiffness decreases the mean value of the stabilizing MHS measure of the system increases. Yet, the system experiences instability a little sooner. Hence, for motion over even terrain, the assumption of rigid suspension a little overestimates the stability status of the system. Based on Fig. 9, it can be also shown that in this maneuver the system will pitch-over around the front edge of the support boundary polygon. The longitudinal and side position of the base center of mass are shown in Figs 10 and 11. As can be seen, the system does not slip sideways because the steering angels were set to zero.

In the presented simulation, the steering angles were set to zero. Herein, one more case is addressed in which the steering angles vary during the motion. It is
Figure 7. (a) Sequence of motion of manipulators with respect to the moving base. The front configuration shows the initial posture, while the back one displays the final pose. (b) Trajectory of motion for the second joint. (c) Trajectory of motion for the third joint.

Figure 8. Variation of MHS measure for different stiffness coefficients of the suspension element for motion over a flat surface.
**Figure 9.** MHS associated with the various edges of the support boundary polygon. Levels 1 and 3 denote left and right sides of the support polygon. Also, levels 2 and 4 represent the front and rear edges of the support polygon.

**Figure 10.** Longitudinal position of the base center of mass.

**Figure 11.** Side position of the base center of mass.
supposed that the robot has a slight initial forward velocity $v = 0.1 \text{ m/s}$. Then, it accelerates via variable actuating torques exerted on the rear wheels as shown in Fig. 12. The front wheels are merely steering and not driving. Also, $\delta_r = 0$, whereas the front steering angles are:

$$\delta_f = \begin{cases} 
\frac{1}{36} \pi t; & \text{if } t \leq 2 \\
\frac{1}{18} \pi; & \text{otherwise}
\end{cases}$$

(20)

In this case, the whole joints of the holonomic arm are active and move as illustrated in Fig. 13. It should be noted that in this case, the second and third joints of the arms move as shown in Fig. 7b and c.

As can be observed in Fig. 14, if the suspension stiffness decreases, the system experiences instability sooner. Besides, it can be seen in Fig. 15 that in the current case the system is toppling down around a tip-over axis coincident with edge 34. Based on the developed comprehensive model, the obtained results show the merits of the proposed MHS measure for SWMMs, in terms of predicting the exact time of instability occurrence, for either pitch-over or roll-over modes of toppling down.

6.2. Effect of Terrain Traction Characteristics on Postural Stability

In this subsection, the effect of frictional forces on the postural stability of SWMMs is investigated. Here, it is assumed that the introduced robot is traveling on a flat firm terrain. Moreover, the wheels and arms are subject to the same actuation torques and joint kinematic trajectories shown in Figs 6 and 7, respectively. Note that in this case, the steering angles are again set as $\delta_f = \delta_r = 0$. Figure 16 shows the effect of different friction conditions on the dynamic stability of the system.

Based on Fig. 16, it can be seen that the system with higher friction experiences instability at $t \approx 4.5 \text{ s}$, whereas the system with lower friction keeps stable for the whole maneuver. Consequently, it can be concluded that once the wheels
are subjected to relatively high actuating torques, postural stability of the system diminishes as the friction between the base and the ground increases.

7. Motion Over Rough Terrain

7.1. Effect of Suspension Characteristics on Postural Stability

In this subsection, the effect of suspension stiffness on the postural stability of the system is investigated once the introduced system is traveling over a smooth rough terrain.
Figure 15. MHS associated with the various edges of the support boundary polygon.

Figure 16. MHS measure for different friction coefficient values: higher friction $\mu = 0.9$; lower friction $\mu = 0.1$ (motion over flat terrain).

terrain. A smooth rough terrain is considered as:

$$Z = 2.5 \left( \frac{3}{2} Y - \frac{1}{2} Y^3 \right) X \sin \left( \frac{2\pi}{\lambda} X \right) \text{ (mm)}, \quad (21)$$

where $X$ and $Y$ are measured in m. This terrain topography has been illustrated in Fig. 17. Again, the wheels and arms are assumed to be subjected to the same actuation torques and joint kinematic trajectories shown in Figs 6 and 7, respectively. Note that in this case, the steering angles are also set as $\delta_f = \delta_r = 0$.

Longitudinal and side position of the base center of mass are shown in Figs 18 and 19. As it can be seen in this case, the system slips sideways. This confirms that although the steering angles are zero, the robot with a fixed axle slips over rough terrain.
Comparing Figs 8 and 20: (i) it can be observed that terrain roughness has affected the system stability for the system with higher stiffness, especially when the amounts of irregularities are larger, and (ii) it can be seen that the system with higher stiffness experiences instability at $t \approx 4.1$ s, whereas the system with lower
stiffness becomes unstable at $t \approx 4.3$ s. Consequently, it can be concluded that once the system is traveling over a rough terrain, as the stiffness of the suspension elements decreases the stability of the system improves. Hence, adding the flexible suspension to a wheeled platform not only enhances the mobility of the system, but also improves the postural stability of them for movement on rough and gentle rugged terrains.

7.2. Effect of Terrain Traction Characteristics on Postural Stability

In this subsection, a SWMM is considered with the similar specifications mentioned in the previous subsection. Figure 21 explains the consequence of different friction conditions on the dynamic stability of the system. Comparing Figs 16 and 21: (i) it can be observed that terrain roughness has effectively affected the system stability, and (ii) it can be seen that the system with higher friction experiences instability.

Figure 20. Variation of MHS measure for different stiffness coefficients of the suspension element for motion over a rough terrain.

Figure 21. Variation of MHS measure for different friction coefficient values: higher friction $\mu = 0.9$; lower friction $\mu = 0.1$ (motion over rough terrain).
a little sooner. Consequently, again it can be concluded that once the wheels are subjected to relatively high actuating torques, the postural stability of the system weakens as the friction between the base and the ground increases.

Based on the obtained results, a look-up table can be formed and if a robot has an active suspension subsystem then in dangerous situations the stability of the system can be changed by varying the stiffness coefficient. In addition, this study investigates the effect of friction on system stability. Specifically, as mentioned before, it was shown that once the system is in motion over a terrain with high friction the system has less tip-over stability margin (in comparison to the terrain with lower friction).

8. Effect of Grasping Posture on the Tip-Over Stability of the System

In this section, the effect of the grasping posture of a heavy object and the relevant robot’s configuration on the postural stability of the system is examined. To this end, it is assumed that the desired task is to manipulate a 400-kg payload along a circular path, as depicted in Fig. 22, with the yaw angle trajectory as depicted in Fig. 23. To this end, the MIC algorithm [21] is adopted to control the overall robotic system to accomplish the desired mission.

It is assumed that the object can be pivotally grasped from various positions. Herein, three possible poses for grasping the object are considered. In the first grasping posture, the right and left end-effectors’ positions with respect to the object center of mass in the payload attached frame are as $E_1 G_O = (-0.25, 0, 0)^T$ and $E_2 G_O = (0.25, 0, 0)^T$, respectively (Fig. 24, in which the point $G_O$ is the origin of the object attached frame). In the second grasping posture, the right and left end-effectors’ positions with respect to the object center of mass in the payload attached frame are as $E_1 G_O = (0.3, 0, 0)^T$ and $E_2 G_O = (0.8, 0, 0)^T$, respectively. Finally, in the third grasping posture, the right and left end-effectors’ positions with respect to the object center of mass in the payload attached frame are as $E_1 G_O = (0, 0, 0)^T$ and $E_2 G_O = (0.5, 0, 0)^T$, respectively. These different initial configurations for

![Figure 22. Projection of the object path in the XY plane during maneuvers.](image-url)
grasping of the object are due to the different configurations of the moving base and arms. In order to use the MIC control law, the dynamics of the mobile manipulator system should first be written in the task space level. The vector of output variables of the mobile manipulator is considered as 

\[ \tilde{X} = [X_C, Y_C, \psi, X_E^{(1)}, X_E^{(2)}]^{T} \times 9 \times 1, \]

where 

\[ X_C, X_E^{(1)}, X_E^{(2)} \]

denote the position vectors of first and second end-effectors, respectively.

Dynamics of the system in the generalized coordinate space can be written as

\[ H_{us} \ddot{q}_{us} + V_{us}(q, \dot{q}) + G_{us}(q) = 0_{4 \times 1} \] (22a)

\[ H_a \ddot{q}_a + V_a(q, \dot{q}) + G_a(q) = Q_a - J_a^{(1)^T} F_e^{(1)} - J_a^{(2)^T} F_e^{(2)}, \] (22b)

where:

\[ q = [q_{us}, q_{sp}, q_{arms}]^{T}_{16 \times 1} \quad \text{and} \]

\[ q_a = [q_{sp}, q_{arms}]^{T}_{12 \times 1} \]

\[ = [X_C, Y_C, Z_C, \varphi, \theta, \psi, \theta_{11}, \theta_{21}, \theta_{31}, \theta_{12}, \theta_{22}, \theta_{32}]^{T}. \]
Note that in the above equations $\mathbf{q}_{us}$ denotes the vector containing the vertical displacement of the tires. Also, (22a) renders the equations of motion of unsprung masses, while (22b) describes the equations of motion of the sprung mass and the robot manipulator arms. Consequently, $\mathbf{H}_{us}$ and $\mathbf{H}_{a}$ denote the mass/inertia matrix associated with (22a) and (22b), respectively. Similarly, $\mathbf{V}_{us}$ and $\mathbf{V}_{a}$ denote non-linear velocity dependent terms associated with (22a) and (22b). Also, $\mathbf{G}_{us}$ and $\mathbf{G}_{a}$ denote gravity dependent terms associated with (22a) and (22b). $\mathbf{Q}_{a}$, furthermore, indicates the vector of generalized forces associated with $\mathbf{q}_{a}$ coordinates. In addition, the Jacobian matrix $\mathbf{J}_{a}^{(i)}$ is a $6 \times 12$ matrix defined as:

$$
\begin{bmatrix}
\dot{\mathbf{X}}_{E}^{(i)} \\
\omega_{E}^{(i)}
\end{bmatrix} = \mathbf{J}_{a}^{(i)} \mathbf{q}_{a}, \quad i = 1, 2.
$$

(23)

The relationship between the output speeds vector and the speed vector $\mathbf{q}_{a}$ can be written as:

$$
\dot{\mathbf{X}} = \mathbf{J}_{c} \mathbf{q}_{a}.
$$

(24)

To apply the MIC law, a desired impedance relationship at the object level is written as:

$$
\mathbf{M}_{\text{des}} \ddot{\mathbf{e}} + \mathbf{k}_{d} \dot{\mathbf{e}} + \mathbf{k}_{p} \mathbf{e} + \mathbf{F}_{c} = \mathbf{0},
$$

(25)

where $\mathbf{M}_{\text{des}}$, $\mathbf{k}_{d}$ and $\mathbf{k}_{p}$ are the desired mass, damping and stiffness matrices, $\mathbf{F}_{c}$ is the contact force (in the contact phase), and $\mathbf{e} = \mathbf{X}_{\text{des}} - \mathbf{X}$ is the object tracking error. However, as described in Ref. [21], the object equation of motion can be obtained as:

$$
\mathbf{M} \ddot{\mathbf{x}} + \mathbf{F}_{\omega} = \mathbf{F}_{c} + \mathbf{F}_{o} + \mathbf{G} \mathbf{F}_{c},
$$

(26)

where $\mathbf{M}$, $\mathbf{F}_{\omega}$, $\mathbf{F}_{o}$, $\mathbf{G}$ and $\mathbf{F}_{c}$ are mass/inertia matrix, vector of nonlinear velocity dependent terms, external forces/torques, grasp matrix and finally forces/torques exerted on the object by the manipulator end-effectors, respectively. The MIC law enforces the same impedance on various parts of the system. Therefore, we can write the same impedance law for the system as:

$$
\mathbf{\tilde{M}}_{\text{des}} \ddot{\mathbf{e}} + \mathbf{\tilde{k}}_{d} \dot{\mathbf{e}} + \mathbf{\tilde{k}}_{p} \mathbf{e} + \mathbf{U}_{fc} \mathbf{F}_{c} = \mathbf{0},
$$

(27)

where $\mathbf{\tilde{e}} = \mathbf{\tilde{X}}_{\text{des}} - \mathbf{\tilde{X}}$ is the tracking error of the system controlled variables. According to the MIC law the applied forces/torques in the operational space could be divided in two parts as:

$$
\mathbf{\tilde{Q}}_{\text{app}} = \mathbf{\tilde{Q}}_{m} + \mathbf{\tilde{Q}}_{f},
$$

(28)

where $\mathbf{\tilde{Q}}_{m}$ is the applied control force causing motion of the end-effectors, while $\mathbf{\tilde{Q}}_{f}$ is the required force to be applied on the manipulated object by the end-effectors. The motion-concerned part of the applying force can be obtained based on a feed-
back linearization as [21]:

\[
\begin{align*}
\tilde{Q}_{m} &= \tilde{H}(q)M^{-1}_{\text{des}}[M_{\text{des}}\ddot{X}_{\text{des}} + K_d\dot{e} + K_p\varepsilon + U_{fc}F_c] + \tilde{C}(q, \dot{q}) \\
\tilde{H}(q) &= (J_c^\#)^T H_a J_c^\# \\
\tilde{C} &= (J_c^\#)^T C_a - \tilde{H} J_c \dot{q}_a \\
U_{fc} &= [U_{fc}^{(0)} U_{fc}^{(1)} U_{fc}^{(2)}]^T \\
U_{fc}^{(0)} &= \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 4} \end{bmatrix} \\
U_{fc}^{(1)} &= \begin{bmatrix} 1_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} \\
U_{fc}^{(2)} &= \begin{bmatrix} 1_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}.
\end{align*}
\] (29a) (29b) (29c) (29d) (29e) (29f) (29g)

To obtain \(\tilde{Q}_f\), based on the object dynamics as described in (26), and considering the target impedance, the required end-effectors forces/torques, \(F_{\text{ereq}}\), on the object are obtained as [21]:

\[
F_{\text{ereq}} = G^\# \{MM^{-1}_{\text{des}}(M_{\text{des}}\ddot{X}_{\text{des}} + K_d\dot{e} + K_p\varepsilon + F_c) + F_\omega - (F_c + F_o)\}. \quad (30)
\]

Therefore, the controlled forces \(\tilde{Q}_f\) in (28) required to be applied on the manipulated object by the end-effectors are:

\[
\tilde{Q}_f = \begin{bmatrix} 0_{3 \times 1} \\ F_{\text{ereq}} \end{bmatrix}. \quad (31)
\]

The actual paths of the platform center of mass and end-effectors in the \(XY\) plane are illustrated in Fig. 25 once the system has the first grasping posture. However, the actual paths of the platform center of mass and the end-effectors with the second and third grasping posture are shown in Figs 26 and 27, respectively.

Variation of system tip-over stability during performing this heavy object manipulation with three configurations is shown in Fig. 28. It can be observed that the response of the system in terms of its stability is different for the three kinds of grasping posture and their relevant system configurations.

![Figure 25](image_url)
Figure 26. Paths of the base center of mass and end-effectors during the second posture of grasp.

Figure 27. Paths of the base center of mass and end-effectors during the third posture of grasp.

Figure 28. MHS measure for three distinct robot configurations during the same object manipulation task.

The obtained results reveal the significance of the manner in which the robot approaches a heavy payload for its grasping and manipulation. They also motivate dynamically stable motion planning for autonomous heavy object manipulation tasks to avoid damaging the robot and to utilize its maximum allowable potential [22].
9. Conclusions

In this study, dynamics modeling, postural stability and control of SWMMs manipulating heavy objects were taken into account while considering the tire nonlinear behavior. Considering pneumatic tires, the semi-empirical Dugoff tire friction model was employed to illustrate the longitudinal and lateral forces created at the contact patch of the wheels. Next, using the obtained dynamics and the MHS measure, the results of suspension stiffness and terrain traction attributes on postural stability of such systems were studied. It was particularly shown that once the wheels are subjected to fairly high actuating torques while moving over a flat terrain then its postural stability diminishes as the friction force between the base and the ground increases. Besides, when the system is moving over an even terrain then as the stiffness of suspension elements decreases the stability of the system endangers. On the contrary, once the system is traveling over a rough terrain, as the suspension stiffness decreases the stability of the system improves. Hence, adding the flexible suspension to wheeled platforms not only enhances the mobility of such systems, but also improves the postural stability for movement on rough terrains. Finally, the effect of grasping posture on the tip-over stability of the system was examined. To this end, the MIC law was utilized to control the system for the same cooperative object manipulation task. It was shown that when a system is carrying a heavy payload along a prescribed trajectory the system stability drastically depends on the overall configuration of the system.

References


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