Distributed Double-Differential Orthogonal Space-Time Coding for Cooperative Networks

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Abstract—In this paper, distributed double-differential coding is proposed to avoid the problem of carrier offsets in amplify and forward protocol based cooperative network with two relays. A double-differential orthogonal-space-time block code is transmitted by the relays in a distributed manner without channel knowledge. We derive a low complexity linear decoder which does not require the channel and carrier offset knowledge. A pairwise error probability (PEP) analysis is also conducted and we found an upper bound of the PEP for the proposed system. In addition, an optimized power allocation is proposed to improve the SER performance of the system.

I. INTRODUCTION
Cooperation among the users can avoid the difficulties of implementing actual antenna array and convert the single-input single-output (SISO) system into a virtual multiple-input multiple-output (MIMO) system [1], [2]. Advantages of the MIMO system like the diversity gain can be achieved at a SISO wireless network through cooperative communication.

It is normally assumed that the destination and relay have perfect knowledge of the channel. However, in practice, it is difficult for the relays and the users to estimate the channel gains accurately. This problem becomes worse for fast fading channels. In order to avoid channel estimation, differential modulation can be implemented in cooperative systems [3]–[6]. In [3]–[6], a simple cooperative system consisting single source destination pair with one relay is considered. Distributed differential space-time coding is proposed in [7] for any number of relays. The differential system requires the channel to be constant over at least two symbol durations. However, the presence of carrier offset due to the mismatch between the transmit and receive oscillators or relative motion of the receiver and transmitter, makes the block fading channel behave as time-varying channel, which does not remain constant over two consecutive time-periods. Hence, the performance of the differential scheme degrades substantially.

Double-differential (DD) coding [8], [9] is a key technique which could be used to avoid the need of carrier offset and channel estimation. It reflects its utility specially for those channels which remain constant for a small number of symbol durations and are perturbed with carrier offsets. DD coding enables the receiver to make decisions based on three consecutively received data blocks/matrices without any carrier offset or channel knowledge. DD coding for MIMO system was proposed in [10], however, it can only be applied to a specific class of space-time block code (STBC), which belongs to the diagonal unitary group. A time-division multiplexed (TDM) distributed DD coding based on [10] for cooperative systems is proposed in [11]. This scheme uses regenerative relays for transmission of a diagonal double-differentially encoded matrix. It requires previous source discoveries and a complicated ordering protocol such that each relay can know its transmission time accurately. A simultaneous transmission strategy is also proposed in [11] in order to avoid the ordering of the relays. However, this scheme requires that the channel remains constant over more than $10^4$ time-intervals.

In this paper, our main contributions are as follows: 1) We propose a distributed double-differential coding, which is applicable to any square orthogonal space-time block code (OSTBC) using $M$-PSK constellation and amplify and forward protocol based cooperative system with two relays. 2) A low-complexity linear decoder of the proposed distributed DD code is derived. 3) We found an upper bound of the PEP of the proposed distributed coding over flat fading Rayleigh channels with carrier offsets. 4) Based on this PEP bound an optimized power allocation is proposed.

The rest of the paper is organized as follows: In Section II, the system model is discussed. Distributed double-differential encoding is explained in Section III. Decoding of the proposed distributed DD code is presented in Section IV. Section V performs the theoretical performance analysis of the proposed scheme. Simulation results and comparisons are discussed in Section VI. Finally, some conclusions are drawn in Section VII.

Notation: $X$ denotes matrix, $x$, $X$ are used for variables, $(\cdot)^H$ provides the Hermitian of a matrix or vector, complex conjugation of a matrix or vector is represented by $(\cdot)^*$, $(\cdot)^T$ gives the transpose of a matrix or vector, $E[\cdot]$ represents expectation, $Pr\{\cdot\}$ stands for probability, $\|\cdot\|^2$ denotes Frobenius norm of a matrix and Euclidean norm of a vector, $I_p$ is used for the $p \times p$ identity matrix, and $Re\{\cdot\}$ and $Im\{\cdot\}$ represent the real and imaginary operators, respectively, which return the real and imaginary part, respectively, of the matrix/vector/scalar they are applied to.

II. SYSTEM MODEL
We consider a cooperative communication system with one source, one destination, and $N$ relays, all with single antenna, as shown in Fig. 1, with no direct channel between source and
destination. It is assumed that the source, which could be base-
station (BS) or access point (AP), can select \( N \) best relays
which are static and their oscillators are perfectly matched
with the oscillator of the source. Hence, the channels between
the source and relays have no carrier offsets. However, the
destination (mobile user) is far from the source (BS) and
also moving or its oscillator might not be matched with the
oscillators of the relays. Consequently, the channel between
the relays and the destination is perturbed by the carrier offsets.
It is also assumed that the destination is sufficiently far away
from the relays such that all relays share a common angle of
arrival. Under this assumption all channels between relays and
the destination can be assumed to be perturbed by a single
carrier offset. This point is well initialized and elaborated
in [10] for MIMO systems.

The channels between the source and the relays, and
between the relays and the destination are assumed to be
Rayleigh distributed. The time-varying channel between the
\( i \)-th relay and the destination, i.e., \( h_{i,1}[t] \) is modeled as
\( h_{i,1}[t] = h_{i,1} e^{j\omega t} \), where \( h_{i,1} \) is a flat fading channel
coefficient which remains constant over at least three
consecutive block transmissions, and \( \omega \) is the random (normalized) carrier
offset which remains constant over the same three blocks for
which the channel stays constant. However, due to the phase
term \( e^{j\omega t} \), \( h_{i,1}[t] \) varies with time \( t \) even though \( h_{i,1} \) and \( \omega \)
are constant.

### III. DISTRIBUTED DOUBLE-DIFFERENTIAL ENCODING

For simplicity of understanding, we will discuss the pro-
posed distributed double-differential coding for a cooperative
system with two relays. However, the generalization to the
higher number of relays is possible but much more involved.
Let \( s_k = [s_{k,1}, s_{k,2}] \), \( s_{k,i} \in M \)-PSK, be the data to be trans-
mitted by the source in \( k \)-th block. \( s_k \) is first mapped to
the Alamouti code [12] matrix \( S_k \in \mathbb{C}^{2\times2} \). It is assumed that
\( \| s_k \|^2 = 1 \) to avoid the power fluctuation due to the differen-
tial encoding. Before transmission, \( S_k \) is double-differentially
encoded as follows: For \( k \geq 2 \), \( S_k \) is single-differentially
modulated as

\[
C_k = C_{k-1} S_k. \tag{1}
\]

where \( C_k \) is a \( 2 \times 2 \) initialization matrix which could be an
OSTBC or identity matrix. Let \( d_0 \in \mathbb{C}^{1\times2} \) be a row vector
such that \( d_0 d_0^* = 1 \). Then the double-differentially encoded
row vector \( d_k = [d_k^1, d_k^2] \in \mathbb{C}^{1\times2} \), for \( k \geq 1 \), is obtained as follows:

\[
d_k = d_{k-1} C_k = d_{k-1} C_{k-1} S_k = d_{k-2} C_{k-2} \ldots C_2 S_2. \tag{2}
\]

In general, for \( k \geq 2 \), we transmit the following double-
differentially encoded row vector:

\[
d_k = d_0 C_1 \ldots C_{k-2} C_{k-1} S_k. \tag{3}
\]

Let the data received at the relays corresponding to \( d_k \), \( k \geq 0 \)
be expressed by the two row vectors \( y_{0,i,k} \) and \( y_{0,2,k} \), where
\( \{0,i,k\} \) represents data received at relay \( i \), \( i \in \{1,2\} \), from
the source in block \( k \). The data received at \( i \)-th relay will be

\[
y_{0,i,k} = [y_{0,1,i,k} y_{0,2,i,k}] = \sqrt{2P_1 h_0,i d_k} + e_{0,i,k}, \tag{4}
\]

where \( P_1 \) is the average power per transmission, \( h_{0,i} \) is the
channel between the source and \( i \)-th relay and \( e_{0,i,k} \in \mathbb{C}^{1\times2} \) is
complex additive white Gaussian noise (AWGN) vector with
\( \sigma^2/2 \) variance per real dimension.

It can be seen from (4) that each relay has a noisy version
of \( d_k \). The transmission of the double-differential OSTBC is
performed in a distributed manner as follows: The \( i \)-th relay
computes the following row vector:

\[
\tilde{d}_{i,k} = y_{0,i,k} U_i + y_{0,2,k}^* V_i, \tag{5}
\]

where \( U_i \) and \( V_i \) are real unitary matrices of size \( 2 \times 2 \). For
the Alamouti code \( U_i \) and \( V_i \) are given by

\[
U_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, V_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, U_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{6}
\]

**Proposition 1:** Let \( U \in \mathbb{C}^{2\times2} \) be an OSTBC matrix, then
\( U U_i = U_i \) and \( U V_i = V_i U \), where \((\cdot)^* \) means complex
conjugation. Proposition 1 can be proved by matrix multiplication.

The data received at the destination in the next two con-
secutive time-intervals can be written as

\[
y_{1,k} = \begin{bmatrix} y_{1,k}^1 \\ y_{1,k}^2 \end{bmatrix} = \sqrt{\frac{P_2}{1 + P_1 \sigma_0^2}} e^{j2\omega k} h_1 \begin{bmatrix} \tilde{d}_{1,k}^1 \\ \tilde{d}_{2,k}^1 \end{bmatrix} \Omega + e_{1,k}, \tag{7}
\]

where \( P_2 \) is the average power of the effective channel from
the relays to the destination, \( \sigma_0^2 \) is the variance of \( h_0 = [h_{0,1}, h_{0,2}] \), \( \omega \) is
the random carrier offset, \( h_1 = [h_{1,1}, h_{2,1}] \in \mathbb{C}^{1\times2} \) is a row
vector consisting of the channel coefficients from the relays to
the destination, \( \Omega = \text{diag} \{1,e^{j\omega}\} \) has size of \( 2 \times 2 \), and \( e_{1,k} \)
is complex AWGN noise. From (4), (5), and (7), we have

\[
y_{1,k} = \sqrt{\frac{2P_1 P_2}{1 + P_1 \sigma_0^2}} e^{j2\omega k} h_1 \begin{bmatrix} \tilde{d}_{1,k}^1 \\ -\tilde{d}_{2,k}^1 \end{bmatrix} \Omega + e_{1,k}, \tag{8}
\]

where \( h = [h_{0,1}, h_{0,2}, h_{2,1}] \) is the effective channel
between the source and the destination and \( e_k \) is the total
effective noise given by

\[
e_k = \sqrt{\frac{P_2}{1 + P_1 \sigma_0^2}} e^{j2\omega k} h_1 \begin{bmatrix} e_{0,1,k}^1 \\ -e_{0,2,k}^1 \end{bmatrix} \Omega + e_{0,1,k}, \tag{9}
\]
It can be shown after some manipulations, that if \( h_1 \) is known, then \( e_k \) is circularly symmetric complex Gaussian distributed noise with zero mean and covariance matrix \( \Psi \) given by
\[
\Psi = \left( 1 + \frac{P_2}{1 + P_1 \sigma_0^2} \|h_1\|^2 \right) \sigma^2 I_2. \tag{10}
\]

IV. DECODING OF THE DISTRIBUTED DD CODE

From (8), it can be seen that that if \( \omega, h, \) and \( d_k \) are known for all \( k \), the p.d.f. of \( y_{1,k} \) is
\[
\begin{align*}
f(y_{1,k}|\omega, h, d_k) = & \frac{1}{\pi^2 \det(\Psi)} \exp \left( - \left[ y_{1,k} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega k} h D_k \Omega \right]^* \Psi^{-1} \\
& \times \left[ y_{1,k} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega k} h D_k \Omega \right] \right),
\end{align*}
\]
where
\[
D_k = \begin{bmatrix} d_k U_1 \\ d_k^* V_2 \end{bmatrix} = \begin{bmatrix} d_k^1 & d_k^2 \\ - (d_k^2)^* & (d_k^1)^* \end{bmatrix}. \tag{12}
\]
Let \( y_k = [y_{1,k-2} y_{1,k-1} y_{1,k}] \) be the vector consisting of the data samples received in three consecutive blocks at the destination. When \( \omega, h, d_{k-2}, C_{k-1}, \) and \( S_k \) are known,
\[
\begin{align*}
f(y_k|\omega, h, d_{k-2}, C_{k-1}, S_k) = & \frac{1}{(\pi v^2)^3} \exp \left( - \frac{1}{v^2} \right) \\
\times \prod_{l=k-2}^k \left\| y_{1,l} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega l} h D_l \Omega \right\|^2, \tag{13}
\end{align*}
\]
where \( v^2 = \left( 1 + \frac{P_2}{1 + P_1 \sigma_0^2} \|h_1\|^2 \right) \sigma^2 \). In order to find a maximum-likelihood (ML) estimate of the unknown data \( S_k \) we need to maximize (13). Apparently, minimization of the following metric with respect to (w.r.t) all unknown quantities \( \omega, h, d_{k-2}, \) and \( C_{k-1}, \) and, subsequently, over \( S_k \):
\[
\Gamma_k = \sum_{l=k-2}^k \left\| y_{1,l} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega l} h D_l \Omega \right\|^2, \tag{14}
\]
is equivalent to maximize (13). Minimization of (14) w.r.t. \( h, \omega, D_{k-2}, \) and \( C_{k-1}, \) and then \( S_k \) is very complicated, therefore, we focus on a low complexity linear decoder which makes the decision independent of the channel and carrier offset knowledge.

A. Linear Decoder of Distributed DD Code

To simplify the decision process, we consider a degenerated decision metric from (14) as follows:
\[
D_k = \sum_{l=k-2}^{k-1} \left\| y_{1,l} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega l} h D_l \Omega \right\|^2, \tag{15}
\]
i.e., out of the three data vectors received at block \( k-2, k-1, \) and \( k, \) we only consider the first two data vectors received at \( k-2 \) and \( k-1. \) From (2), (12), and Proposition 1, it can be shown that
\[
D_{k-1} = \begin{bmatrix} d_{k-1} U_1 \\ d_{k-2} U_1 \end{bmatrix} = \begin{bmatrix} d_{k-2}^1 U_1 \\ d_{k-2}^2 V_2 \end{bmatrix} C_{k-1} = D_{k-2} C_{k-1}. \tag{16}
\]

From (15) and (16), \( D_k \) can be expressed as
\[
D_k = \left\| y_{1,k-2} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega(k-2)} h D_{k-2} \Omega \right\|^2 + \left\| y_{1,k-1} - \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega(k-1)} h D_{k-2} C_{k-1} \Omega \right\|^2. \tag{17}
\]

By means of [13], (17) is minimized w.r.t. \( g_k = \frac{2P_1P_2}{1 + P_1 \sigma_0^2} e^{2j\omega(k-2)} h D_{k-2} \Omega \) as
\[
\hat{g}_k = \frac{1}{2} \left( \hat{\epsilon}_{\omega}^* y_{1,k-1} \hat{C}_{k-1} + y_{1,k-2} \right), \tag{18}
\]
where \( \hat{\epsilon}_{\omega} = e^{2j\omega} \) and \( \hat{C}_{k-1} = \Omega^H C_{k-1} \Omega. \) By substituting (18) into (17) and using the unitary property of OSTBC [14], (17) reduces into
\[
D_k = \left\| y_{1,k-1} - \hat{\epsilon}_{\omega} y_{1,k-2} \hat{C}_{k-1} \right\|^2, \tag{19}
\]
From (19), it can be seen that if \( \hat{C}_{k-1} \) is unknown, it is impossible to find the estimate of \( \hat{\epsilon}_{\omega}. \)

**Lemma 1:** If \( C_{k-1} = I_2, \) the estimator of \( \hat{\epsilon}_{\omega} \) is given by
\[
\hat{\epsilon}_{\omega,m,k-1} = \exp \left( j \arg \{ y_{1,k-1} y_{1,k-2}^H \} \right). \tag{20}
\]

Lemma 1 can be proved by differentiating (19) w.r.t. \( \omega. \)

However, we cannot take the liberty of assuming \( C_{k-1} \) as the identity matrix as from (1) it can be seen that \( C_{k-1} \) depends upon the OSTBC data matrix \( S_{k-1} \) which is different from the identity matrix. Nevertheless, we may assume that the initialization matrix \( C_1 \) is the identity matrix. Then, the estimate of \( \hat{\epsilon}_{\omega} \) can be found from (20) at \( k = 2, \) such that \( \hat{\epsilon}_{\omega,1} \) can be used in the place of \( \hat{\epsilon}_{\omega} \) in the further analysis. From Proposition 1 and (12), it can be shown that
\[
D_k = D_{k-2} C_{k-1}^2 S_k. \tag{21}
\]

From (8) and (21), the data received in the \( k \)-th block at the destination can be written in the terms of \( \hat{\epsilon}_{\omega}, g_k, \) and \( C_{k-1} \) as
\[
y_{1,k} = \hat{\epsilon}_{\omega}^2 g_k \Omega^H C_{k-1}^2 S_k \Omega + e_k. \tag{22}
\]

Hence, we can minimize the following decision metric to find the estimate of \( S_k, k \geq 2:
\[
\hat{S}_k = \arg \min_{S_k \in \Xi} \left\| y_{1,k} - \hat{\epsilon}_{\omega}^2 g_k \Omega^H C_{k-1}^2 \hat{\Omega} \hat{S}_k \right\|^2, \tag{23}
\]
where \( \Xi \) is the set of all OSTBC matrices consisting of symbols from the \( M \)-PSK constellation, \( \hat{\Omega} \) is obtained from \( \hat{\epsilon}_{\omega,1}, C_{k-1} \) is the estimate of \( C_{k-1} \) obtained using (1), and
\[ \hat{S}_k = \hat{\Omega}^\ast \hat{S}_k \hat{\Omega}. \] Let \( \hat{h}_k \triangleq \hat{E}_{\omega,1} \hat{g}_k \hat{\Omega} \hat{C}_{k-1}^\ast \hat{\Omega} \), then (23) can be written as
\[ \hat{S}_k = \arg \min_{\hat{S}_k \in \mathbb{C}} \left\| y_{1,k} - \hat{h}_k \hat{S}_k \right\|^2. \quad (24) \]

**Lemma 2:** If \( S_k \) is a square OSTBC matrix, then \( \hat{S}_k = \Omega^\ast \hat{S}_k \Omega \), is also a square OSTBC matrix.

As \( S_k \in \mathbb{C}^{2 \times 2} \) is OSTBC it can be expressed as [14, Eq. (7.11)]
\[ S_k = \sum_{n=1}^{2} \left( \text{Re} \{ s_n \} A_n + j \text{Im} \{ s_n \} B_n \right), \quad (25) \]
where \( A_n \) and \( B_n \) are fixed (in general complex-valued) code matrices of size \( 2 \times 2 \), which satisfy the properties given by [14, Eq. (7.4.4)]. It can be shown after few manipulations that we can express \( \hat{S}_k \) in the terms of \( \hat{A}_n = \Omega^\ast \hat{A}_n \Omega \) and \( \hat{B}_n = \Omega^\ast \hat{B}_n \Omega \) as
\[ \hat{S}_k = \sum_{n=1}^{2} \left( \text{Re} \{ s_n \} \hat{A}_n + j \text{Im} \{ s_n \} \hat{B}_n \right). \quad (26) \]

It can be verified that \( \hat{A}_n \) and \( \hat{B}_n \) satisfy [14, Eq. (7.4.4)].

After some manipulations it can be shown that decoding of (24) reduces into
\[ \hat{s}_k = \arg \min_{s_k \in \mathcal{X}} \left\{ s_k - \frac{\Delta_1}{\left\| \hat{h}_k \right\|^2} \right\|^2 + \arg \min_{s_k \in \mathcal{X}} \left\{ s_k - \frac{\Delta_2}{\left\| \hat{h}_k \right\|^2} \right\}, \quad (27) \]

where \( \Delta_m = \text{Re} \{ \hat{h}_k \hat{A}_m y_{1,k}^\ast \} - j \text{Im} \{ \hat{h}_k \hat{B}_m y_{1,k}^\ast \}, m \in \{1,2\}, \) and \( \mathcal{X} \) is the M-PSK alphabet.

**V. PERFORMANCE ANALYSIS OF DISTRIBUTED DD CODING**

Here, we analyze the pairwise error probability (PEP) of distributed DD code over flat fading Rayleigh channels with carrier offset. From (23), the probability of detecting \( S_k \) in place of \( S_k^0 \), where \( S_k \neq S_k^0 \), can be written as
\[ \Pr \{ S_k^0 \rightarrow S_k \} = \Pr \left\{ \left\| y_{1,k}^0 - \hat{E}_{\omega,1} \hat{g}_k \hat{\Omega} \hat{C}_{k-1}^\ast \hat{S}_k \hat{\Omega} \right\|^2 < \left\| y_{1,k}^0 - \hat{E}_{\omega,1} \hat{g}_k \hat{\Omega} \hat{C}_{k-1}^\ast \hat{S}_k^0 \hat{\Omega} \right\|^2 \right\}, \quad (28) \]
where \( y_{1,k}^0 = \sqrt{\frac{P_1 P_2}{1 + P_1 \sigma_0^2 + P_2}} e^{j \omega \mu_k} h D_{k-2} C_{k-1}^\ast S_k^0 \Omega + e_k \) and \( \hat{C}_{k-1} \) is the estimate of \( C_{k-1} \).

**Theorem 1:** The probability of detecting \( S_k \) in place of \( S_k^0 \) can be upper bounded as
\[ \Pr \{ S_k^0 \rightarrow S_k \} \leq \exp \left\{ -\frac{\lambda_{\min}^2(\mathcal{X}) \left\| v_k \hat{S}_k \right\|^2}{6 \left( 1 + \frac{P_2}{1 + P_1 \sigma_0^2} \right) \left\| h_1 \right\|^2 \sigma^2} \right\}, \quad (29) \]

where \( \hat{S}_k = S_k^0 - S_k, W_k = \hat{E}_{\omega,1} \hat{g}_k \hat{\Omega} \hat{C}_{k-1}^\ast \hat{\Omega}, v_k = g_k \hat{\Theta}_k, V_k = \hat{S}_k \hat{\Theta}_k, \)
\[ \Theta_k = \frac{\hat{E}_{\omega,1} \hat{g}_k \hat{\Omega} \hat{C}_{k-1}^\ast \hat{\Omega} \hat{C}_{k-1}^\ast + I_2}{2} \times \hat{\Omega} \hat{C}_{k-1}^\ast, \]
\[ \mathcal{X}_k = \hat{\Omega}^\ast (S_k^0 \hat{\Omega} W_k (V_k^\ast)^{-1} \hat{\Omega} + \hat{\Omega}^\ast V_k^{-1} W_k S_k^0 \hat{\Omega}), \]
and \( \lambda_{\min}(\mathcal{X}_k) \) is the minimum eigenvalue of \( \mathcal{X}_k \).

Assuming the worst carrier offset \( (\omega = \pi/2) \) and its worst estimate \( (\hat{\omega} = \pi/4) \), and averaging over channel \( h_0 \sim \mathcal{CN}(0, \sigma_0^2 I_2) \) we have
\[ \mathbb{E}_{h_0, i} \left[ \Pr \{ S_k^0 \rightarrow S_k \} \right] \leq \left\| I_2 + \frac{2 P_1 P_2 \sigma_0^2}{12(1 + P_1 \sigma_0^2 + P_2 \left\| h_1 \right\|^2)} \times (S_k^0 - S_k) H_1 (S_k^0 - S_k) \right\|^{-1}, \quad (30) \]
where \( H_1 = \text{diag} \left\{ \left\| h_{1,1} \right\|^2, \left\| h_{2,1} \right\|^2 \right\} \) and \( \mathcal{X}_k \) is the \( \mathcal{X}_k \) with \( \omega = \pi/2, \hat{\omega} = \pi/4 \).

Theorem 1 can be proved after many manipulations, using [14, Theorem 4.2], the Chernoff bound [15, Eq. (2.1.172)], and Fischer’s matrix inequality [16, Eq. (11.8.1)].

As \( h_1 \sim \mathcal{CN}(0, \sigma_1^2 I_2) \), then \( H_1 \) can be expressed as \( H_1 = \sigma_1^2 \text{diag} \left\{ \left\| h_{1,1} \right\|^2, \left\| h_{2,1} \right\|^2 \right\} \) where \( h_{i,1} \sim \mathcal{CN}(0, 1) \), \( i \in \{1,2\} \). It can be seen from (30) that for minimizing the error probability we need to maximize the following term:
\[ \tau = \frac{2 P_1 P_2 \lambda_{\min}^2(\mathcal{X}_k) \sigma_0^2}{12(1 + P_1 \sigma_0^2 + P_2 \left\| h_1 \right\|^2)}, \quad (31) \]

Since it is very difficult to obtain exact solution for PEP because of the expectation over \( h_1 \), we replace \( \left\| h_1 \right\|^2 \) by \( 2 \sigma_1^2 \) in (31) and form the following maximization problem:
\[ \max_{P_1, P_2 \geq 0} \tau = \frac{2 P_1 P_2 \lambda_{\min}^2(\mathcal{X}_k) \sigma_0^2}{12(1 + P_1 \sigma_0^2 + P_2 \left\| h_1 \right\|^2)} \]
such that \( P = P_1 + P_2 \),
\[ \left\{ \begin{array}{l} P_1(\sigma_0^2 + \sigma_1^2) = (2 \sigma_1^2 P_1 + 1) (P \sigma_0^2 + 1) \sqrt{2 \sigma_1^2 - \sigma_0^2} \sigma_0^2 + 1, \\ P_2 = \frac{(2 \sigma_1^2 P_1 + 1) (P \sigma_0^2 + 1) - (P \sigma_0^2 + 1)}{2 \sigma_1^2 - \sigma_0^2} \sigma_0^2 + 1. \end{array} \right. \quad (32) \]

**VI. SIMULATION RESULTS**

The simulations are performed with Alamouti STBC [12], \( \sigma_1^2 = 1, M = 4 \), random \( \omega \in [\pi, \pi] \), \( D_0 = C_1 = I_2 \), and it is assumed that the destination perfectly knows \( D_0 \) and \( C_1 \). The channel is assumed constant over three consecutive blocks. The simulation results are obtained from \( 10^3 \) channel realizations.
A. Proposed Distributed DD Code versus Direct DD Scheme

In Fig. 2, we have plotted the simulation results for the proposed scheme using $\sigma_0^2=1$ and $\sigma_1^2=1,10$. The SER versus SNR plots are drawn under uniform and optimized power allocations. The optimized power allocation is calculated at each SNR from (33). It can be seen form Fig. 2 that for $\sigma_1^2=1$ there is no improvement with optimized power allocation. However, at $\sigma_1^2=10$ there is an improvement of approximately 1 dB at SER $=10^{-3}$. We have also shown the performance of direct transmission DD scheme, and the proposed scheme outperforms the direct transmission DD scheme for SNR $>12.5$ dB.


In Fig. 3, we have implemented the proposed distributed DD code and the conventional distributed DD code [11] by assuming that the channel between the source and the relays is perfect and noise free. The distributed DD code of [11] uses the time-division multiplexing (TDM) transmission strategy suggested in [11, Section III]. It can be seen from Fig. 3 that the proposed distributed DD coding outperforms the same rate conventional scheme [11] at all SNRs. For example, a gain of 5 dB is achieved at SER $=10^{-4}$. The reason for this gain is that the distributed DD code of [11] utilizes DD code of [10] which is based on diagonal unitary matrices. In DD coding of [10], a diagonal matrix is transmitted in place of an M-PSK symbol. Whereas, the proposed distributed DD code utilizes full square OSTBC matrix for DD encoding and, therefore, provides better coding gain as compared to the same rate distributed DD code of [11].

VII. CONCLUSIONS

We have proposed distributed double-differential coding for orthogonal space-time block codes over Rayleigh channels with carrier offset in a cooperative network. The proposed distributed code is able to decode the data without knowing the carrier offsets or channel coefficients, and it achieves higher performance gain as compared to the previously proposed distributed double-differential code based on diagonal unitary matrices.

REFERENCES


