Mixed Membership Stochastic Blockmodels

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Overview

1. Notation and motivation
2. Previous blockmodels
3. The Mixed Membership Stochastic Blockmodel
4. Conclusion and next steps

Overview

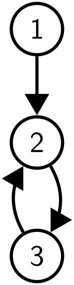
Network theory: notation

- Individuals $p, q \in \{1, \ldots, N\}$.
- We observe relations/interactions $R(p, q)$ on *pairs* of individuals.
- Here we assume $R(p, q) \in \{0, 1\}$, $R(p, p) = 0$, but do not assume $R(p, q) = R(q, p)$ (we deal with *directed* networks).

Network theory: data representations

p	q	R(p,q)
1	2	1
2	3	1
3	2	1

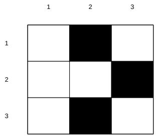
Table



Graph

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix
i, j element is
$R(i, j)$

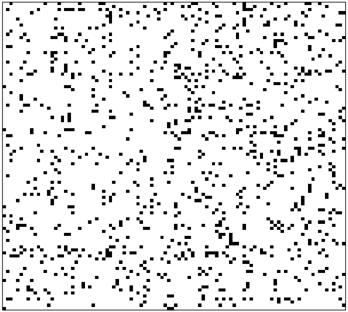


Adjacency matrix,
black=1, white=0

The Problem: Scientific Motivation

- We believe the relations are a function of unobserved groupings among the individuals.
- We want to recover the groups so we can a) predict new relations or b) interpret the existing network structure.
- Example: Monk network.

The Problem: Pictures



The Problem: Pictures

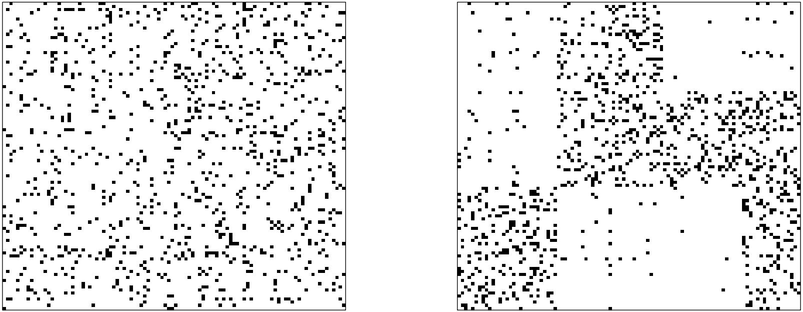


Figure: Two visualizations of the same binary adjacency matrix. Each filled-in square represents a directed edge. Left: ordered randomly. Right: ordered by group membership.

Overview

Brief blockmodel history

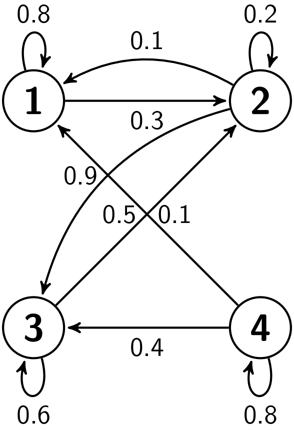
- 1975: CONCOR developed by Harrison White and colleagues
- 1983: Holland, Laskey & Leinhardt introduce *stochastic* blockmodel for blocks known *a priori*.
- 1987: Wasserman & Anderson extend to *a posteriori* estimation.
- 2004: Kemp, Griffiths & Tenenbaum allow unknown and unlimited number of blocks in the Infinite Relational Model.

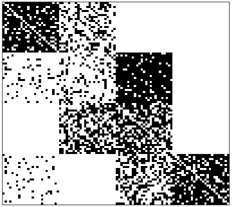
Infinite Relational Model

- Observe binary relations $R(p, q)$ between nodes $p, q \in \{1, \ldots, N\}$.
- Each node p is a member of exactly one block of K total blocks, $K \leq N$ unknown. Let z_p be an indicator vector of block membership for node p, i.e. $z_p = (0, 1, 0)$.
- B is a $K \times K$ matrix of block relationships. If p is in block g and q is in block h then the probability of observing an interaction from node p to node q is B_{gh}.
- $R(p, q) \sim \text{Bernoulli}(z_p^T B z_q)$.
- For example, if p is in block 3 and q is in block 2 then $P(R(p, q) = 1) = B_{32}$.

Block structure

$$\begin{pmatrix} .8 & .3 & 0 & 0 \\ .1 & .2 & .9 & 0 \\ 0 & .5 & .6 & 0 \\ .1 & 0 & .4 & .8 \end{pmatrix}$$



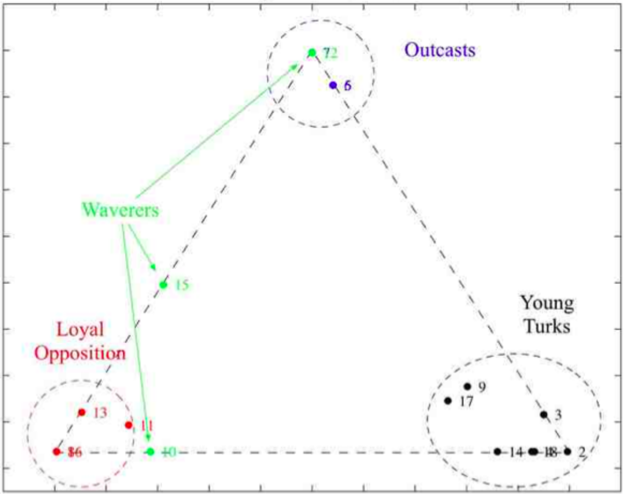


Overview

The Mixed Membership Stochastic Blockmodel

- Previous models assume each node is assumed to belong to exactly one latent block - e.g. $z_p = (0, 1, 0, 0)$.
- Instead, in the MMB we assume each node has a distribution π_p over the latent blocks.
- For each interaction from p to q, both p and q draw a particular block to be a part of for the interaction: $z_{p \to q} \sim \text{Discrete}(\pi_p)$, $z_{p \leftarrow q} \sim \text{Discrete}(\pi_q)$.
- Then $R(p, q) \sim \text{Bernoulli}(z_{p \to q}^T B z_{p \leftarrow q})$.
- K chosen by BIC.

Breiger et al. (1975)



1	Ambrose
2	Boniface
3	Mark
4	Winfrid
5	Elias
6	Basil
7	Simplicius
8	Berthold
9	John Bosco
10	Victor
11	Bonaventure
12	Amand
13	Louis
14	Albert
15	Ramuald
16	Peter
17	Gregory
18	Hugh

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Conclusion and next steps

- Blockmodels allow clustering nodes from observed network data.
- MMB extends blockmodels to let nodes be in different groups to different extents, but commit to one group during any given interaction.
- Next steps: estimation...