The massless gravitino and the AdS/CFT correspondence

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We solve the Dirichlet boundary value problem for the massless gravitino on AdS_{d+1} space and compute the two-point function of the dual CFT supersymmetry currents using the AdS/CFT correspondence principle. We find analogously to the spinor case that the boundary data for the massless $(d+1)$ dimensional bulk gravitino field consists of only a $(d-1)$ dimensional gravitino.

I. INTRODUCTION

Recently Maldacena [1] has conjectured that the large N limit of certain d dimensional conformal field theories is dual to supergravity or string theory on $d+1$ dimensional Anti-de Sitter (AdS) space times a compact manifold. A prescription for generating correlators of operators in the conformal field theory (CFT) from solutions of the supergravity equations of motion has been given in [2,3]. The prescription associates to each field ϕ_i in the supergravity action a corresponding local operator $\mathcal{O}^i$ in the CFT such that the following relation (in Euclidean space) holds:

$$e^{-S_{eff}(\phi_i)} = \left\langle e^{\int_\partial \phi_{i,0}\mathcal{O}^i} \right\rangle. \tag{1.1}$$

The effective action S_{eff} is evaluated on the solutions to the supergravity equations of motion subject to the boundary conditions $\phi_i|_\partial = \phi_{i,0}$ where ∂ denotes the boundary of AdS_{d+1} space. On the right-hand-side of (1.1) the expectation value of the given exponential is taken in the dual conformal field theory, with $\phi_{i,0}$ acting as a source for the CFT operator $\mathcal{O}^i$. Using this relation various two-point [2]- [5] and three and four-point [5]- [14] correlation functions have been computed, including detailed checks of various Ward identities.

One subtlety of the prescription (1.1) involves the manner in which $S_{eff}(\phi_i)$ is evaluated on the supergravity solutions. Specifically $S_{eff}(\phi_i)$ diverges and must be regularized. As a result ambiguities in the overall coefficient of CFT correlators obtained from (1.1) arise, and therefore the CFT Ward identities may not be satisfied. However a regularization procedure which produces correlators satisfying the Ward identities has been found in [5]. The procedure involves solving the Dirichlet boundary value problem for the supergravity fields for a deformed boundary of AdS such that S_{eff} is well-defined and only after obtaining the CFT correlators is the limit back to the true boundary of AdS taken.

In this paper we consider the AdS/CFT correspondence for the massless gravitino whose dual CFT operator is the supersymmetry current [15]. In section 2 we solve the massless gravitino equations of motion on the AdS_{d+1} background following the techniques of [8]. We find, in analogy to the spinor case [4,8], that the boundary data for the massless $(d+1)$ dimensional bulk gravitino field consists of only a $(d-1)$ dimensional gravitino due to the first order nature of the equation of motion. In section 3 we use the correspondence (1.1) to compute the two-point function for the dual supersymmetry currents, taking care to evaluate the gravitino action in the manner discussed above, and find the expected result. In section 4 we make some concluding remarks.

II. CONSTRUCTING THE SOLUTION

In this section we solve the boundary value problem for the massless Rarita-Schwinger field. Our method for solving the equation of motion parallels that of [8]. We first find the most general solution to the Fourier transformed equation of motion. This solution contains an exponentially growing mode in k and therefore is not Fourier transformable. Demanding Fourier transformability we are forced to constrain the boundary data such as to remove the undesired mode. We then re-express the solution in terms of the desired boundary data and Fourier transform back to position space obtaining the bulk values of the Rarita-Schwinger field.

The action for the massive Rarita-Schwinger field in $d+1$ dimensions is given by

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$$S = \frac{1}{2}\int d^{d+1}x\, e\,(\bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho + m_1 \bar{\psi}_\mu \psi^\mu + m_2 \bar{\psi}_\mu \Gamma^{\mu\nu}\psi_\nu) \tag{2.1}$$

where

$$D_\nu \psi_\rho = (\partial_\nu + \frac{1}{4}\omega_\nu^{ab}\gamma_{ab})\psi_\rho \tag{2.2}$$

and m_1 and m_2 are related to the mass m and cosmological constant Λ (see [16] and [17] for the relation for supergravity on $AdS_5 \times S_5$ and $AdS_7 \times S_4$ respectively). Our notation is as follows: e_a^μ is the vielbein and e its' determinant, ω_μ^{ab} is the spin connection, Γ^μ are the curved space gamma matrices related to the flat space gamma matrices γ^a by $\Gamma^\mu = e_a^\mu \gamma^a$, the flat space gamma matrices satisfy the anticommutation relations $\{\gamma^a, \gamma^b\} = 2\delta^{ab}$, gamma matrices with more than one index are antisymmetrized as $\Gamma^{\mu_1 \cdots \mu_n} = \Gamma^{[\mu_1} \cdots \Gamma^{\mu_n]}$, coordinate indices are denoted by lower case Greek letters $\mu, \nu, ...$ running from 0 to d and lower case Latin letters $i, j, ...$ running from 1 to d, and lower case Latin letters $a, b, ...$ denote Lorentz indices. Varying the action (2.1) with respect to $\bar{\psi}_\mu$ results in the equation of motion

$$\Gamma^{\mu\nu\rho} D_\nu \psi_\rho + m_1 \psi^\mu + m_2 \Gamma^{\mu\nu}\psi_\nu = 0 \tag{2.3}$$

while varying with respect to ψ_ρ results in the equation of motion for the adjoint Rarita-Schwinger field

$$\bar{\psi}_\mu \overleftarrow{D}_\nu \Gamma^{\mu\nu\rho} - m_1 \bar{\psi}^\rho - m_2 \bar{\psi}_\mu \Gamma^{\mu\rho} = 0 \tag{2.4}$$

where $\overleftarrow{D}_\nu = \overleftarrow{\partial}_\nu - (1/4)\omega_\nu^{ab}\gamma_{ab}$.

We now specialize to the Euclidean AdS_{d+1} background geometry and choose coordinates such that the metric takes the form

$$ds^2 = \frac{1}{(x^0)^2}\big((dx^0)^2 + d\mathbf{x}\cdot d\mathbf{x}\big) \tag{2.5}$$

where we use boldface letters to denote the inner product of d-dimensional vectors in the flat Euclidean metric, $\boldsymbol{x}\cdot\boldsymbol{y} := x_i \delta^{ij} y_j$, and use the flat Euclidean metric to raise and lower indices, $x^i = \delta^{ij} x_j$. Choosing the corresponding vielbein to be

$$e_\mu^a = \frac{1}{x^0}\delta_\mu^a \tag{2.6}$$

it is straightforward to show that the spin connection is given by

$$\omega_\mu^{ab} = -\frac{1}{x^0}(\delta_\mu^a \delta_0^b - \delta_\mu^b \delta_0^a), \tag{2.7}$$

as is easily verified by substituting into $de^a + \omega^{ab} \wedge e_b = 0$.

To solve the equation of motion (2.3) it is first convenient to rewrite it in the form

$$\Gamma^\nu (D_\nu \psi_\rho - D_\rho \psi_\nu) + m_- \psi_\rho - \frac{m_+}{d-1}\Gamma_\rho \Gamma^\nu \psi_\nu = 0 \tag{2.8}$$

where $m_\pm = m_1 \pm m_2$. Expanding (2.8) and substituting (2.6) for the vielbein and (2.7) for the spin connection we find the $\rho = 0$ equation

$$\left(x^0\partial_0 + \frac{m_+}{d-1}\gamma_0\right)\boldsymbol{\gamma}\cdot\boldsymbol{\psi} = \left(x^0 \boldsymbol{\gamma}\cdot\boldsymbol{\partial} - \frac{d}{2}\gamma_0 + m_- - \frac{m_+}{d-1}\right)\psi_0 \tag{2.9}$$

and the $\rho = i$ equation

$$\left(x^0(\gamma^0\partial_0 + \boldsymbol{\gamma}\cdot\boldsymbol{\partial}) - \left(\frac{d}{2} - 1\right)\gamma_0 + m_-\right)\psi_i = \left(x^0\gamma^0\partial_i + \frac{1}{2}\gamma_i + \frac{m_+}{d-1}\gamma_i\gamma^0\right)\psi_0 + \left(x^0\partial_i - \frac{1}{2}\gamma_i\gamma_0 + \frac{m_+}{d-1}\gamma_i\right)\boldsymbol{\gamma}\cdot\boldsymbol{\psi} \tag{2.10}$$

To solve these equations we work in momentum space. Define the Fourier transform

$$\psi_\mu(x^0, \boldsymbol{x}) = \frac{1}{(2\pi)^{d/2}}\int d^d k\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\tilde{\psi}_\mu(x^0, \boldsymbol{k}) \tag{2.11}$$

and substitute into (2.9) and (2.10). We obtain after some rearranging

$$\left(x^0\partial_0 + \frac{m_+}{d-1}\gamma_0\right)\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}} = \left(ix^0\boldsymbol{k}\cdot\boldsymbol{\gamma} - \frac{d}{2}\gamma^0 + m_- - \frac{m_+}{d-1}\right)\tilde{\psi}_0 \tag{2.12}$$

$$\left(x^0\partial_0 - ix^0\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 - \left(\frac{d}{2}-1\right) + m_-\gamma^0\right)\tilde{\psi}_i = \left(ix^0k_i\gamma^0 + \frac{1}{2}\gamma_i - \frac{m_+}{d-1}\gamma_i\gamma^0\right)\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}} + \left(ix^0k_i - \frac{1}{2}\gamma_i\gamma^0 - \frac{m_+}{d-1}\gamma_i\right)\tilde{\psi}_0 \tag{2.13}$$

for the $\rho = 0$ and $\rho = i$ equations respectively.

Solving equations (2.12,2.13) is straightforward but tedious. The resulting solution for the generic case presented so far is complicated and therefore we consider the simpler massless case $m = 0$ in the remainder. This is equivalent to demanding $m_+ = -m_- = \Lambda/2$ (Λ here is related to the cosmological constant by a dimension dependent factor). To simplify the equations above we start by defining the projection operator

$$P_i^j := \delta_i^j - \frac{\hat{k}_i}{d-1}\left(d\hat{k}^j - \hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\gamma^j\right) - \frac{\gamma_i}{d-1}\left(\gamma^j - \hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\hat{k}^j\right) \tag{2.14}$$

which is orthogonal to both k^i and γ^i. Defining the tranverse components of $\tilde{\psi}_i$ to k^i and γ^i respectively as $\tilde{\psi}_i^T := P_i^j\tilde{\psi}_j$ it follows that the field may be decomposed as

$$\tilde{\psi}_i = \tilde{\psi}_i^T + \frac{1}{d-1}(\gamma_i - \hat{k}_i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}} + \frac{1}{d-1}(d\hat{k}_i - \gamma_i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma})\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}. \tag{2.15}$$

The equation of motion for $\tilde{\psi}_i^T$ is easily derived by applying P_i^j to (2.13) obtaining

$$\left(x^0\partial_0 - ix^0\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 - \left(\frac{d}{2}-1\right) - \frac{\Lambda}{2}\gamma^0\right)\tilde{\psi}_i^T = 0. \tag{2.16}$$

Finding the equation of motion for $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ is a little more involved. First solve (2.12) for $\tilde{\psi}_0$ and substitute into (2.13) contracted with γ^i. This results in the algebraic relation[1]

$$\boldsymbol{k}\cdot\tilde{\boldsymbol{\psi}} = \left(\boldsymbol{k}\cdot\boldsymbol{\gamma} + i\frac{(d-1)\gamma^0 - \Lambda}{2x^0}\right)\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}. \tag{2.17}$$

Substituting this relation for $\boldsymbol{k}\cdot\tilde{\boldsymbol{\psi}}$ into (2.13) contracted with k^i and comparing with (2.12) results in the second algebraic relation

$$\tilde{\psi}_0 = -\gamma^0\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}. \tag{2.18}$$

From this and (2.12) the equation of motion for $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ follows

$$\left(x^0\partial_0 + ix^0\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 - \frac{d}{2} - \frac{\Lambda}{2}\gamma^0\right)\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}} = 0 \tag{2.19}$$

The equations of motion (2.16,2.19) are easily integrated in the form of path ordered exponentials. The exponentials are straightforward to evaluate and we find

$$\begin{aligned}\tilde{\psi}_i^T(x^0,\boldsymbol{k}) = (x^0)^{(d-1)/2}\Big(&(1+\gamma^0)\left(I_{(\Lambda-1)/2}(kx^0) + i\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 I_{-(\Lambda-1)/2}(kx^0)\right)\\ &+ (1-\gamma^0)\left(I_{-(\Lambda+1)/2}(kx^0) + i\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 I_{(\Lambda+1)/2}(kx^0)\right)\Big)\lambda_i(\boldsymbol{k})\end{aligned} \tag{2.20}$$

[1]Here is the essential between the massless and massive cases. In the massive case $\boldsymbol{k}\cdot\tilde{\boldsymbol{\psi}}$ cannot be expressed algebraically in terms of $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$.

where $\lambda_i(\boldsymbol{k})$ is transverse to both k^i and γ^i and

$$\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0,\boldsymbol{k}) = (x^0)^{(d+1)/2}\Big((1-\gamma^0)\Big(I_{(\Lambda-1)/2}(kx^0) - i\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 I_{-(\Lambda-1)/2}(kx^0)\Big) + (1+\gamma^0)\Big(I_{-(\Lambda+1)/2}(kx^0) - i\boldsymbol{k}\cdot\boldsymbol{\gamma}\gamma^0 I_{(\Lambda+1)/2}(kx^0)\Big)\Big)\lambda_0(\boldsymbol{k}). \tag{2.21}$$

Using the algebraic relation expressing $\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}$ in terms of $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ we now have the complete momentum space solution for the massless Rarita-Schwinger field. This solution however is not Fourier transformable since the modified Bessel function $I_\nu(kx^0)$ diverges exponentially for large k. In order to obtain a Fourier transformable solution we must constrain the boundary fields $\lambda_i(\boldsymbol{k})$ and $\lambda_0(\boldsymbol{k})$ to remove the growing modes. The same problem occurs for the spinor field as discussed in [8] and more generally for any field satisfying a first order equation of motion. The simple reason is that for a first order equation only the boundary values of the field are freely specifiable, whereas for a second order equation of motion the boundary value and first derivative of the field are freely speciable or conversely the boundary value and asymptotic behavior. As emphasized in [8] the asymptotic behavior of the field is essential for the AdS/CFT correspondence and while specifying it for a second order equation of motion is not a problem, for a first order equation it must be done at the expense of specifying completely the boundary values of the field.

We may easily find the constraints necessary to remove the offending terms by rewriting the solutions (2.20, 2.21) in terms of the modified Bessel functions $I_{(\Lambda\pm1)/2}(kx^0)$ and $K_{(\Lambda\pm1)/2}(kx^0)$ where $K_{(\Lambda\pm1)/2}(kx^0)$ decay exponentially for large k. The conditions necessary for removing the exponentially growing modes are easily shown to be $(1+i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\gamma^0)\lambda_i(\boldsymbol{k}) = 0$ and $(1-i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\gamma^0)\lambda_0(\boldsymbol{k}) = 0$ which effectively removes half of the components of λ_i and λ_0 respectively. Defining $\lambda_i^\pm := (1/2)(1\pm\gamma^0)\lambda_i$ and similarly for λ_0 these conditions may be rewritten in the equivalent form

$$\lambda_i^+ = i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\lambda_i^- \tag{2.22}$$

$$\lambda_0^+ = -i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\lambda_0^-. \tag{2.23}$$

Substituting these relations into the solutions (2.20, 2.21) results in[2]

$$\tilde{\psi}_i^T(x^0,\boldsymbol{k}) = (x^0)^{(d-1)/2}\Big(K_{(\Lambda+1)/2}(kx^0) + i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(kx^0)\Big)\lambda_i^-(\boldsymbol{k}) \tag{2.24}$$

$$\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0,\boldsymbol{k}) = (x^0)^{(d+1)/2}\Big(K_{(\Lambda+1)/2}(kx^0) - i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(kx^0)\Big)\lambda_0^-(\boldsymbol{k}) \tag{2.25}$$

where constants have been absorbed into λ_i and λ_0 respectively.

The final step before inverse Fourier transforming back to position space is to re-express the solutions (2.24,2.25) in terms of the given boundary data $\tilde{\psi}_i(\epsilon,\boldsymbol{k})$. Clearly the components of the boundary Rarita-Schwinger field are not all independent as discussed above. To find the independent components evaluate (2.24) at the boundary $x^0=\epsilon$ (we consider only the $\tilde{\psi}_i^T$ case in detail here as the $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ case is similar) and apply the projection operators $(1/2)(1\pm\gamma^0)$ deriving the relations

$$\tilde{\psi}_i^{T,-}(\epsilon,\boldsymbol{k}) = \epsilon^{(d-1)/2}K_{(\Lambda+1)/2}(k\epsilon)\lambda_i^-(\boldsymbol{k}) \tag{2.26}$$

$$\tilde{\psi}_i^{T,+}(\epsilon,\boldsymbol{k}) = \epsilon^{(d-1)/2}i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(k\epsilon)\lambda_i^-(\boldsymbol{k}). \tag{2.27}$$

Solving one relation for λ_i and substituting into the other yields

$$\tilde{\psi}_i^{T,+}(\epsilon,\boldsymbol{k}) = \frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon)}i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\,\tilde{\psi}_i^{T,-}(\epsilon,\boldsymbol{k}). \tag{2.28}$$

Using the small z expansion of $K_\nu(z)$

$$K_\nu(z) = \frac{1}{2}\left(\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}(1+\mathcal{O}(z^2)) + \Gamma(-\nu)\left(\frac{z}{2}\right)^{\nu}(1+\mathcal{O}(z^2))\right) \tag{2.29}$$

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[2] These solutions may equivalently be expressed in terms of λ_i^+ and λ_0^+.

we find that for regular boundary data in the $\epsilon \to 0$ limit we must demand that $\tilde{\psi}_i^{T,+}(\epsilon, \boldsymbol{k}) = 0$ and therefore that the appropriate boundary data is given by $\tilde{\psi}_i^{T,-}(\epsilon, \boldsymbol{k})$. Eliminating $\lambda_i(\boldsymbol{k})$ for $\tilde{\psi}_i^{T,-}(\epsilon, \boldsymbol{k})$ in (2.26) yields the momentum space solution for $\tilde{\psi}_i(x^0, \boldsymbol{k})$

$$\tilde{\psi}_i^T(x^0, \boldsymbol{k}) = \left(\frac{x^0}{\epsilon}\right)^{(d-1)/2} \frac{K_{(\Lambda+1)/2}(kx^0) + i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma} K_{(\Lambda-1)/2}(kx^0)}{K_{(\Lambda+1)/2}(k\epsilon)} \tilde{\psi}_i^{T,-}(\epsilon, \boldsymbol{k}) \tag{2.30}$$

expressed in terms of the desired boundary data. Similar manipulations with the $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ solution (2.25) yields the relation between the chiral components of $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(\epsilon, \boldsymbol{k})$

$$\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon, \boldsymbol{k}) = -i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon)}\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k}) \tag{2.31}$$

and the resulting solution expressed in terms of the boundary data $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k})$

$$\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0, \boldsymbol{k}) = \left(\frac{x^0}{\epsilon}\right)^{(d+1)/2} \frac{K_{(\Lambda+1)/2}(kx^0) - i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma} K_{(\Lambda-1)/2}(kx^0)}{K_{(\Lambda+1)/2}(k\epsilon)} \boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k}). \tag{2.32}$$

This is not quite the desired form for $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0, \boldsymbol{k})$ though. Recall that $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}$ and $\boldsymbol{k}\cdot\tilde{\boldsymbol{\psi}}$ are related as in (2.17). Evaluating this relation at $x^0 = \epsilon$ and using (2.31) to solve for $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+$ yields

$$\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k}) = 2\epsilon\frac{1 + i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\big(K_{(\Lambda-1)/2}(k\epsilon)/K_{(\Lambda+1)/2}(k\epsilon)\big)}{1 + \big(K_{(\Lambda-1)/2}(k\epsilon)/K_{(\Lambda+1)/2}(k\epsilon)\big)^2}\frac{2\epsilon\boldsymbol{k}\cdot\boldsymbol{\gamma} + i\big((d-1)\gamma^0 + \Lambda\big)}{(2\epsilon k)^2 - (d-1)^2 + \Lambda^2}\boldsymbol{k}\cdot\tilde{\boldsymbol{\psi}}(\epsilon, \boldsymbol{k}) \tag{2.33}$$

From the expansion (2.29) it follows that for regular boundary data in the $\epsilon \to 0$ limit we must demand that $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k}) = 0$, and so is not the correct boundary data. Applying $(1/2)(1+\gamma^0)$ to (2.33) (which annihilates the left-hand-side) one arrives at

$$\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\left(2k\epsilon - \Big((d-1)\gamma^0 + \Lambda\Big)\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon))}\right)\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon, \boldsymbol{k}) = -i\left(\Big((d-1)\gamma^0 + \Lambda\Big) + 2k\epsilon\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon))}\right)\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^+(\epsilon, \boldsymbol{k}). \tag{2.34}$$

Using the expansion (2.29) once again we see that the correct boundary data is $\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon, \boldsymbol{k})$. We may now express $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0, \boldsymbol{k})$ in terms of this boundary data by substituting (2.33) in (2.32) with $\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}(\epsilon, \boldsymbol{k})$ given in terms of $\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon, \boldsymbol{k})$ after using (2.34). Because the result is quite complicated we do not give the expression explicitly.

Finally we may inverse Fourier transform to find the bulk value of the field in position space. This is not as formidable as it might first appear because we need only work to leading order in ϵ. Substituting the solutions for $\tilde{\psi}_i^T(x^0, \boldsymbol{k})$ (2.30) and $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(x^0, \boldsymbol{k})$ (2.32, 2.33,2.34) into the decomposition (2.15), expanding the ϵ dependent terms and simplifying results in

$$\begin{aligned}\tilde{\psi}_i(x^0, \boldsymbol{k}) = {}& \frac{2^{-(\Lambda-1)/2}}{\Gamma\big((1+\Lambda)/2\big)}\epsilon^{(\Lambda-d)/2+1}(x^0)^{(d-1)/2}k^{(\Lambda+1)/2}\Bigg(\big(K_{(\Lambda+1)/2}(kx^0) + i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(kx^0)\big)\delta_i^j \\ &+ \frac{2}{d-1+\Lambda}\Big(\big(K_{(\Lambda-1)/2}(kx^0) + i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda+1)/2}(kx^0)\big)kx^0\hat{k}_i\hat{k}^j \\ &- K_{(\Lambda-1)/2}(kx^0)\big(i\gamma_i + (\Lambda-1)i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\hat{k}_i\big)\hat{k}^j\Big)\Bigg)\tilde{\psi}_j^-(\epsilon, \boldsymbol{k}).\end{aligned} \tag{2.35}$$

The Fourier transform of this expression is tedious to carry out, but straightforward, and we arrive at

$$\psi_i(x^0, \boldsymbol{x}) = -c(x^0)^{(d+\Lambda)/2-1}\int d^dy\,\frac{x^0\gamma^0 + (\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{\gamma}}{\big((x^0)^2 + |\boldsymbol{x}-\boldsymbol{y}|^2\big)^{(d+\Lambda+1)/2}}\left(\delta_i^j - 2\frac{(x-y)_i(x-y)^j}{(x^0)^2 + |\boldsymbol{x}-\boldsymbol{y}|^2}\right)\psi_j^-(\epsilon, \boldsymbol{y}) \tag{2.36}$$

where

$$c=\frac{1}{\pi^{d/2}}\frac{\Gamma\big((d+1+\Lambda)/2\big)}{\Gamma\big((1+\Lambda)/2\big)}\frac{d+1+\Lambda}{d-1+\Lambda}, \tag{2.37}$$

a factor of $\epsilon^{(\Lambda-d)/2+1}$ has been absorbed into $\psi_j^-(\epsilon,\boldsymbol{k})$, and $\psi_j^-(\epsilon,\boldsymbol{k})$ is transverse to γ^j as follows from (2.31) in the $\epsilon\to 0$ limit. The remaining field component is given by

$$\psi_0(x^0,\boldsymbol{x})=-\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\psi}(x^0,\boldsymbol{x}). \tag{2.38}$$

For the adjoint Rarita-Schwinger field the analysis of solving the equation of motion (2.4) exactly parallels the above discussion, therefore we only quote the results. The momentum space adjoint field may be decomposed as

$$\tilde{\bar{\psi}}_i=\tilde{\bar{\psi}}_i^T+\tilde{\bar{\boldsymbol{\psi}}}\cdot\hat{\boldsymbol{k}}\frac{d\hat{k}_i-\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\gamma_i}{d-1}+\tilde{\bar{\boldsymbol{\psi}}}\cdot\boldsymbol{\gamma}\frac{\gamma_i-\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\hat{k}_i}{d-1} \tag{2.39}$$

where $\tilde{\bar{\psi}}_i^T:=\tilde{\bar{\psi}}_jP^j_{\ i}$ and

$$\tilde{\bar{\boldsymbol{\psi}}}\cdot\boldsymbol{k}=\tilde{\bar{\boldsymbol{\psi}}}\cdot\boldsymbol{\gamma}\left(\boldsymbol{k}\cdot\boldsymbol{\gamma}+i\frac{(d-1)\gamma^0+\Lambda}{2x^0}\right). \tag{2.40}$$

The momentum space equations of motion for the the fields $\tilde{\bar{\psi}}_i^T$ and $\tilde{\bar{\boldsymbol{\psi}}}\cdot\boldsymbol{\gamma}$ may be derived and solved in strict analogy to the unbarred case and we find for the Fourier transformable part

$$\tilde{\bar{\psi}}_i^T(x^0,\boldsymbol{k})=\tilde{\bar{\psi}}_i^{T,+}(\epsilon,\boldsymbol{k})\frac{K_{(\Lambda+1)/2}(kx^0)-i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(kx^0)}{K_{(\Lambda+1)/2}(k\epsilon)}\left(\frac{x^0}{\epsilon}\right)^{(d-1)/2} \tag{2.41}$$

and

$$\tilde{\bar{\boldsymbol{\psi}}}(x^0,\boldsymbol{k})\cdot\boldsymbol{\gamma}=\tilde{\bar{\boldsymbol{\psi}}}^-(\epsilon,\boldsymbol{k})\cdot\boldsymbol{\gamma}\frac{K_{(\Lambda+1)/2}(kx^0)+i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}K_{(\Lambda-1)/2}(kx^0)}{K_{(\Lambda+1)/2}(k\epsilon)}\left(\frac{x^0}{\epsilon}\right)^{(d+1)/2}. \tag{2.42}$$

$\tilde{\bar{\boldsymbol{\psi}}}^-(\epsilon,\boldsymbol{k})\cdot\boldsymbol{\gamma}$ is not the desired boundary data to express $\tilde{\bar{\boldsymbol{\psi}}}(x^0,\boldsymbol{k})\cdot\boldsymbol{\gamma}$ in terms of as it goes to zero in the limit $\epsilon\to 0$. Rather we must express $\tilde{\bar{\boldsymbol{\psi}}}(x^0,\boldsymbol{k})\cdot\boldsymbol{\gamma}$ in terms of $\tilde{\bar{\boldsymbol{\psi}}}^+(\epsilon,\boldsymbol{k})\cdot\hat{\boldsymbol{k}}$ which can be done after using the relations

$$\tilde{\bar{\boldsymbol{\psi}}}^-(\epsilon,\boldsymbol{k})\cdot\boldsymbol{\gamma}=\tilde{\bar{\boldsymbol{\psi}}}(\epsilon,\boldsymbol{k})\cdot\boldsymbol{k}\,2\epsilon\frac{2\epsilon\boldsymbol{k}\cdot\boldsymbol{\gamma}+i\big((d-1)\gamma^0-\Lambda\big)}{(2\epsilon k)^2-(d-1)^2+\Lambda^2}\frac{1-i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\big(K_{(\Lambda-1)/2}(k\epsilon)/K_{(\Lambda+1)/2}(k\epsilon)\big)}{1+\big(K_{(\Lambda-1)/2}(k\epsilon)/K_{(\Lambda+1)/2}(k\epsilon)\big)^2} \tag{2.43}$$

and

$$\tilde{\bar{\boldsymbol{\psi}}}^+(\epsilon,\boldsymbol{k})\cdot\hat{\boldsymbol{k}}\,\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\left(2k\epsilon-\Big((d-1)\gamma^0+\Lambda\Big)\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon))}\right)=-i\tilde{\bar{\boldsymbol{\psi}}}^-(\epsilon,\boldsymbol{k})\cdot\hat{\boldsymbol{k}}\left(\Big((d-1)\gamma^0-\Lambda\Big)-2k\epsilon\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon))}\right). \tag{2.44}$$

Substituting these expressions into (2.42), keeping only the leading order terms in ϵ, and inverse Fourier transforming yields

$$\bar{\psi}_i(x^0,\boldsymbol{x})=c(x^0)^{(d+\Lambda)/2-1}\int d^dy\,\bar{\psi}_j^+(\epsilon,\boldsymbol{y})\frac{x^0\gamma^0+(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{\gamma}}{\big((x^0)^2+|\boldsymbol{x}-\boldsymbol{y}|^2\big)^{(d+\Lambda+1)/2}}\left(\delta_i^j-2\frac{(x-y)_i(x-y)^j}{(x^0)^2+|\boldsymbol{x}-\boldsymbol{y}|^2}\right) \tag{2.45}$$

where a factor of $\epsilon^{(\Lambda-d)/2+1}$ has been absorbed into $\bar{\psi}_j^+(\epsilon,\boldsymbol{y})$ and $\bar{\psi}_j^+(\epsilon,\boldsymbol{y})$ is transverse to γ^j. The remaining component of the adjoint field is given by

$$\tilde{\bar{\psi}}_0(x^0,\boldsymbol{x})=-\tilde{\bar{\boldsymbol{\psi}}}(x^0,\boldsymbol{x})\cdot\boldsymbol{\gamma}\gamma^0. \tag{2.46}$$

III. TWO-POINT FUNCTION

The AdS/CFT correspondence is prescribed by the relation

$$\exp(-S) = \left\langle \exp\Big(\int_\partial \bar{\boldsymbol{\xi}}^-(\boldsymbol{x})\cdot\boldsymbol{\psi}^-(\epsilon,\boldsymbol{x}) + \bar{\boldsymbol{\psi}}^+(\epsilon,\boldsymbol{x})\cdot\boldsymbol{\xi}^+(\boldsymbol{x})\Big)\right\rangle \tag{3.1}$$

where S is the action (2.1) evaluated on the solutions found in the previous section with prescribed boundary data $\psi_i^-(\epsilon,\boldsymbol{x})$ and $\bar{\psi}_i^+(\epsilon,\boldsymbol{x})$. The right-hand-side of (3.1) is the expectation value of the given exponential in the boundary conformal field theory with $\psi_i^-(\epsilon,\boldsymbol{x})$ and $\bar{\psi}_i^+(\epsilon,\boldsymbol{x})$ playing the role of source terms for the boundary conformal fields $\bar{\xi}_i^-(\boldsymbol{x})$ and $\xi_i^+(\boldsymbol{x})$ respectively. We expect these boundary conformal fields to be the supersymmetry currents of the boundary CFT [15] and indeed the two-point function we find below is in agreement with this expectation. The two-point function $\langle\xi_i^+(\boldsymbol{x})\bar{\xi}_j^-(\boldsymbol{y})\rangle$ is easily obtained from (3.1) by taking a pair of functional derivatives with respect to $\psi_j^-(\epsilon,\boldsymbol{y})$ and $\bar{\psi}_i^+(\epsilon,\boldsymbol{x})$. In our case the action (2.1) is first order in derivatives and therefore vanishes when evaluated on the solutions (2.36,2.38,2.45,2.46). This is also the case for spinors as first noted by [4]. To avoid this problem [4] added the most general Lorentz invariant and generally covariant boundary term quadratic in the spinor fields to the action[3]. Following this prescription we add to the action principle (2.1) the boundary term

$$S' = \int_\partial d^dx\sqrt{h}\,\Big(a\bar{\psi}_i(\epsilon,\boldsymbol{x})h^{ij}\psi_j(\epsilon,\boldsymbol{x}) + b\bar{\boldsymbol{\psi}}(\epsilon,\boldsymbol{x})\cdot\boldsymbol{\Gamma}\,\boldsymbol{\Gamma}\cdot\boldsymbol{\psi}(\epsilon,\boldsymbol{x})\Big) \tag{3.2}$$

where h_{ij} is the induced metric on the boundary, h its' determinant and a and b are undetermined coefficients.

To evaluate S' the solutions (2.36,2.45) could be substituted and the integral performed. However it is somewhat simpler to work in momentum space where the boundary action becomes

$$S' = \epsilon^{-(d-2)}\int d^dk\,\Big(a\tilde{\bar{\boldsymbol{\psi}}}(\epsilon,-\boldsymbol{k})\cdot\tilde{\boldsymbol{\psi}}(\epsilon,\boldsymbol{k}) + b\tilde{\bar{\boldsymbol{\psi}}}(\epsilon,-\boldsymbol{k})\cdot\boldsymbol{\gamma}\,\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(\epsilon,\boldsymbol{k})\Big) \tag{3.3}$$

where as before the dot product is defined using the Kronecker delta. Substituting the decompositions (2.15) and (2.39) for $\tilde{\psi}_i$ and $\tilde{\bar{\psi}}_i$ respectively in the boundary action (3.3) we rewrite it as $S' = S_1 + S_2$ where

$$S_1 = \frac{a}{\epsilon^{(d-2)}}\int d^dk\,\tilde{\bar{\psi}}_i^T(\epsilon,-\boldsymbol{k})\delta^{ij}\tilde{\psi}_j^T(\epsilon,\boldsymbol{k}) \tag{3.4}$$

$$S_2 = \frac{a}{\epsilon^{(d-2)}}\int d^dk\,\tilde{\bar{\boldsymbol{\psi}}}(\epsilon,-\boldsymbol{k})\cdot\boldsymbol{\gamma}\Big(1+\frac{b}{a}+i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\frac{(d-1)\gamma^0-\Lambda}{k\epsilon}+\frac{d}{d-1}\frac{(d-1)^2-\Lambda^2}{(2k\epsilon)^2}\Big)\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(\epsilon,\boldsymbol{k}). \tag{3.5}$$

Substituting the solutions (2.30, 2.41) evaluated at $x^0=\epsilon$ into S_1 and expanding the projection operator we find

$$S_1 = 2\frac{a}{\epsilon^{(d-2)}}\int d^dk\,\tilde{\bar{\psi}}_i^+(\epsilon,-\boldsymbol{k})\,i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon)}\Big(\delta^{ij}-\frac{d-2}{d-1}\hat{k}^i\hat{k}^j\Big)\tilde{\psi}_j(\epsilon,\boldsymbol{k}) \tag{3.6}$$

In the limit of small ϵ the ratio of modified Bessel functions may be expanded using (2.29) as

$$\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon)} = \frac{k\epsilon}{\Lambda-1}\Big(1+\sum_{i=1}^{\infty}c_ik^{2i}\Big)+\frac{\Gamma\big((1-\Lambda)/2\big)}{\Gamma\big((1+\Lambda)/2\big)}\Big(\frac{k\epsilon}{2}\Big)^{\Lambda}+\cdots \tag{3.7}$$

where the c_i are known coefficients which are not needed here. Substituting this into (3.6) we see that from the first term in the expansion only the leading order $k\epsilon$ term (multiplying $\hat{k}^i\hat{k}^j$) gives rise to a non-trivial contribution to S_1 while the remaining terms in parentheses give rise to contact terms. As we will see shortly though this term cancels against a contribution from S_2, therefore the leading order non-trivial contribution to S_1 actually comes from the $(k\epsilon)^\Lambda$ term in the expansion[4].

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[3] Justification for this term has recently been given in [18] from a Hamiltonian formulation and presumably extends to the gravitino case as well.

[4] We are assuming here that Λ is not equal to an odd integer and is >1. If Λ is an odd integer then the expansion contains log terms as well which give rise to the leading order non-trivial contributions to S_1 and S_2. Nevertheless the two-point function obtained below is valid for any $\Lambda > 0$.

Using (2.32,2.33,2.34) to express $\boldsymbol{\gamma}\cdot\tilde{\boldsymbol{\psi}}(\epsilon,\boldsymbol{k})$ in terms of $\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon,\boldsymbol{k})$ in S_2 and similarly using and (2.42,2.43, 2.44) to express $\tilde{\bar{\boldsymbol{\psi}}}(\epsilon,-\boldsymbol{k})\cdot\boldsymbol{\gamma}$ in terms of $\tilde{\bar{\boldsymbol{\psi}}}^+(\epsilon,-\boldsymbol{k})\cdot\hat{\boldsymbol{k}}$ we obtain after some algebra

$$S_2 = \frac{2a\epsilon^{-(d-2)}}{d-1+\Lambda}\int d^dk\,\tilde{\bar{\boldsymbol{\psi}}}^+(\epsilon,-\boldsymbol{k})\cdot\hat{\boldsymbol{k}}\,i\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}\left(2k\epsilon+\frac{d(d-1-\Lambda)}{d-1}\frac{K_{(\Lambda-1)/2}(k\epsilon)}{K_{(\Lambda+1)/2}(k\epsilon)}\right)\hat{\boldsymbol{k}}\cdot\tilde{\boldsymbol{\psi}}^-(\epsilon,\boldsymbol{k}) \tag{3.8}$$

where we have dropped higher order terms in ϵ and $K_{(\Lambda-1)/2}(k\epsilon)/K_{(\Lambda+1)/2}(k\epsilon)$ as they will either contribute only to contact terms or are simply higher order in ϵ than what we obtain below. Consequently the two-point function we obtain below is independent of the coefficient b. Substituting the expansion (3.7) we again see that there are non-trivial contributions to S_2 at order ϵ and at order ϵ^Λ. One may easily show that the order ϵ terms in S_1 and S_2 cancel so that the leading order contribution to S' comes at order ϵ^Λ. Collecting these terms and inverse Fourier transforming we find

$$S' = -2a\,c\int d^dy_1\,d^dy_2\bar{\psi}_i^+(\epsilon,\boldsymbol{y_1})\frac{\boldsymbol{\gamma}\cdot(\boldsymbol{y_1}-\boldsymbol{y_2})}{|\boldsymbol{y_1}-\boldsymbol{y_2}|^{d+\Lambda+1}}\left(\delta^{ij}-2\frac{(y_1-y_2)^i(y_1-y_2)^j}{|\boldsymbol{y_1}-\boldsymbol{y_2}|^2}\right)\psi_j^-(\epsilon,\boldsymbol{y_2}) \tag{3.9}$$

where we have absorbed a factor of $\epsilon^{(\Lambda-d)/2+1}$ in both $\psi_j^-(\epsilon,\boldsymbol{y_2})$ and $\bar{\psi}_i^+(\epsilon,\boldsymbol{y_1})$ respectively. We therefore obtain the two-point function

$$\langle\xi_i^+(\boldsymbol{y_1})\bar{\xi}_j^-(\boldsymbol{y_2})\rangle = 2a\,c\Pi_i^k\frac{\boldsymbol{\gamma}\cdot(\boldsymbol{y_1}-\boldsymbol{y_2})}{|\boldsymbol{y_1}-\boldsymbol{y_2}|^{d+\Lambda+1}}\left(\delta_{kj}-2\frac{(y_1-y_2)_k(y_1-y_2)_j}{|\boldsymbol{y_1}-\boldsymbol{y_2}|^2}\right) \tag{3.10}$$

where

$$\Pi_i^j = \delta_i^j - \frac{1}{d}\gamma_i\gamma^j \tag{3.11}$$

projects out the transverse components to γ^i, i.e., $\gamma^i\Pi_{ij}=0$. This projection operator must be present since the sources $\psi_i^-(\epsilon,\boldsymbol{x})$ and $\bar{\psi}_i^+(\epsilon,\boldsymbol{x})$ are transverse to γ^i and therefore the boundary conformal fields are as well.

This is the expected two-point function for a pair of Rarita-Schwinger fields of scaling dimensions $\eta=(d+\Lambda)/2$. This follows from the results of [19] where it is shown that the two-point function of fields $\mathcal{O}^I(\boldsymbol{x})$ and $\mathcal{O}^J(\boldsymbol{y})$ of equal scaling dimensions η is given by

$$\langle\mathcal{O}^I(\boldsymbol{x})\mathcal{O}^J(\boldsymbol{y})\rangle = \frac{D^I_K\big(I(\boldsymbol{x}-\boldsymbol{y})\big)g^{KJ}}{|\boldsymbol{x}-\boldsymbol{y}|^{2\eta}} \tag{3.12}$$

where $D^I_K(R)$ form a representation of $O(d)$ (we use upper case Latin letters I,J,K to denote an arbitrary representation) and the metric g^{IK} satisfies

$$D^{I_1}_{I_2}(R)D^{J_1}_{J_2}(R)g^{I_2J_2} = g^{I_1J_1}. \tag{3.13}$$

$I(x)$ is the inversion operator and is given by

$$I_{ij}(x) = \delta_{ij} - 2\frac{x_ix_j}{|\boldsymbol{x}|} \tag{3.14}$$

in the vector representation and

$$D(I(x)) = -\frac{\boldsymbol{x}\cdot\boldsymbol{\gamma}\gamma^0}{|\boldsymbol{x}|} \tag{3.15}$$

in the spinor representation. For the Rarita-Schwinger field that we are considering, the appropriate representation is given by the direct product of the vector and spinor representations with the transverse to γ^i components projected out. Specifically, for an $O(d)$ transformation $\Lambda^i_{\ j}$ the corresponding element of the representation for the Rarita-Schwinger field is $\Pi^i_{\ j}D(\Lambda)\Lambda^j_{\ k}$. Therefore for this representation and for $\eta=(d+\Lambda)/2$ we recover the two-point function in (3.10), up to an undetermined constant.

IV. DISCUSSION

We have solved the boundary value problem for the massless gravitino on the Euclidean AdS_{d+1} background and via the AdS/CFT correspondence [2,3] have computed the two-point function of supersymmetry currents in the dual boundary CFT. We have found, similarly to the spinor case [4,8], that the bulk solution for the massless gravitino is given in terms of a $(d-1)$ dimensional boundary gravitino. That is, for $\Lambda > 0(< 0)$, Fourier transformability of the bulk field $\psi_i(x^0, \boldsymbol{x})$ implies that we are only free to specify on the boundary the components of $\psi_i^{-(+)}(\epsilon, \boldsymbol{x})$ transverse to γ^i while $\psi_i^{+(-)}(\epsilon, \boldsymbol{x})$ necessarily vanishes in the $\epsilon \to 0$ limit. A similar statement holds for the adjoint gravitino with the boundary data now consisting of the components of $\bar{\psi}_i^{+(-)}(\epsilon, \boldsymbol{x})$ transverse to γ^i. For d odd this is exactly as it should be since the boundary Rarita-Schwinger field, the supersymmetry current, is transverse to γ^i and contains half as many spinor components. For d even however the boundary supersymmetry current contains $(d-1)\, 2^{d/2}$ components, i.e., effectively $(d-1)$ vector indices (the transverse to γ^i condition removes one vector index) but the number of spinor components appropriate to d dimensions. The boundary data however contains only half this number of components, and corresponds to one chirality of the supersymmetry current. For d even therefore the two-point function (3.10) is of chiral components of the supersymmetry current and therefore is only half of what we want. The other half, as noted in [8] for the spinor case, of the correlator for the other pair of chiral components of the supersymmetry currents can be obtained by considering another bulk gravitino field with Λ having the opposite sign.

The next natural step would be to include interactions for the massless gravitino allowing for further checks on the AdS/CFT correspondence via CFT Ward identities. Furthermore using the Hamiltonian techniques of [18] one could determine the coefficients a and b of the boundary action.

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