Abstract - A novel genetic algorithm application is proposed for adaptive power and subcarrier allocation in multi-user OFDM systems. The proposed method is compared with Eshan’s [2] power and subcarrier allocation algorithm and Wong’s modified genetic algorithm [19]. Our method has fast convergence and can handle large allocations of subcarriers to users without performance degradation. The simulation results show that our approach is a viable alternative to existing methods for optimum resource allocation.

Key words: Genetic Algorithm, fitness, subcarrier, OFDM, frequency, bit error rate

I. INTRODUCTION

The development of mobile communication has tremendously increased the growth of cellular users in recent years. However, the allocation of frequency spectrum for this purpose is limited. Consequently, efficient use of channel frequencies becomes more and more significant. Based on service requirements, the allocated spectrum is divided into a number of channels. To satisfy the large demand of the mobile services, channels need to be assigned and reused to minimize communication interference, and in turn, increase the traffic-carrying capacity of the system.

Orthogonal Frequency Division Multiplexing (OFDM) is a promising technology for high data rate transmission in wide band wireless systems for its ability to mitigate the effects of frequency selectively and combat inter-symbol interference. Adaptive modulation can greatly improve the system spectral efficiency by changing its modulation constellation and transmission power according to the frequency selective fading channel, the combination of OFDM and adaptive modulation can utilize the merits of both technologies and is attracting more and more interest.

The greedy algorithm (adaptive bit allocation algorithm) for single user OFDM system gives an optimal solution to minimize the overall bit power allocation [3, 7, 9]. The greedy algorithm may not be suitable in a multiuser environment because a subcarrier for one user will be best but not for other user or users. An optimum solution using greedy algorithm may be possible when the overall subcarriers are allocated to users by adaptive subcarrier allocation in the OFDM systems [19]. Rohling et al. [10] presented a simple greedy algorithm, and showed that it performs better than simple banded OFDM Access. Wahlqvist et al. [14] showed that dynamic resource allocation can improve quality of service (QoS).

The optimum solution for subcarrier and bit allocation can be categorized as static and dynamic allocation. The static subcarrier allocation uses fixed resource allocation schemes namely Time-Division Multiple Access (TDMA) and Frequency-Division Multiple Access (FDMA). In these schemes, the channel conditions are ignored and each user is allocated a predetermined time slot or frequency band to apply OFDM system with Amplitude Modulation (AM). Consequently, the allocated resources to users are underutilized due to AM and subcarriers that appear in deep fade for one user may not be in deep fade for other users [4, 5, 8, 15, 16, 21]. This problem is corrected by dynamically allocating the subcarrier, bit, power to different users based on the instantaneous channel information to minimize overall transmission power. Wong et al. [17] proposed iterative searching algorithm that applies Lagrangian relaxation for optimum multiuser subcarrier, bit, and power allocation. The algorithm is close to the lower bound with requirement of high and complex computation. Ehsan’s [2] algorithm over-simplifies the subcarrier allocation but could not fully utilize the multiuser diversity. Zhang [20] proposed water-filling algorithm (transmit more signal power in the better channels and less signal power in the poorer channels) similar to Wong’s [17, 18, 19] algorithm to avoid computational complexity.

In this research, we used a simple genetic algorithm [7, 11, 12] implemented in MatLab language. We proposed a novel allocation scheme for optimum allocation of subcarrier and power allocation to the users. Section 2 formulates the problem of dynamic resource allocation of resources in multiuser OFDM system and describes the system model of multiuser OFDM system. The genetic algorithm based description for this problem and parameters for genetic based system are discussed in Section 3. Numerical results are obtained and discussed in Section 4. Finally, conclusions are drawn in Section 5.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Let an OFDM system have K (k = 1, 2, . . . , K) users and N (n = 1, 2 . . . , N) subcarriers. The system assigns a subset of N subcarriers to a user and determnes the number of bits/symbol per each assigned subcarrier on downlink transmission.

Let $c_{k,n} \in \{0, 1, 2 . . . , M\}$ denote the number of bits of the $n^{th}$ subcarrier, which is assigned to the $k^{th}$ user. $R_k$ is the number of bits that need to be transmitted in an OFDM symbol. M is the maximum number of information bits per symbol that can be transmitted by each subcarrier. The parameter $c_{k,n}$ determines the adaptive modulation mode (BPSK, 16 QAM, or 64 QAM) for transmission for each
carrier. Let \( \alpha_{k,n} \) denote the channel gains over all \( N \) subcarriers for the \( k^{th} \) user. The required transmission power for the specified bit error rate (BER) at \( c_{k,n} \) bits per symbol is given by [17]

\[
P_{k,n} = \frac{f_{k}(c_{k,n})}{\alpha_{k,n}^2} \tag{1}
\]

For the multiuser OFDM systems under consideration, we do not allow more than one user to share a subcarrier. To formulate the allocation problem, we define

\[
\rho_{k,n} = \begin{cases} 
1, & \text{if } c_{k,n} \neq 0 \\
0, & \text{if } c_{k,n} = 0 
\end{cases} \tag{2}
\]

Variable \( \rho_{k,n} \) is either 1 or 0, and the sum of all \( \rho_{k,n} \) is equal to 1 for any particular \( n \). This implies that only one user can employ the \( n^{th} \) subcarrier. The required total transmission power (\( P^*_{k,n} \)) can be written as

\[
P^*_{k,n} = \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{f_{k}(c_{k,n})}{\alpha_{k,n}^2} \times \rho_{k,n} \tag{3}
\]

The subcarrier, bit and power allocation problem for minimizing the total transmission power can be formulated:

\[
\min_{c_{k,n}, \rho_{k,n}} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{f_{k}(c_{k,n})}{\alpha_{k,n}^2} \times \rho_{k,n} \tag{4}
\]

Subject to

\[
\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \text{for } n = 1, \ldots, N \tag{5}
\]

\[
\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} = N \quad \text{for } n = 1, \ldots, N \tag{6}
\]

and \( c_{k,n} \in \{0, 1, 2, \ldots, M\} \)

\[
R_k = \sum_{n=1}^{N} \alpha_{k,n} \quad \text{for } k = 1, \ldots, K \tag{7}
\]

and the required power for supporting \( c \) bits/symbol at a given BER (bit error rate) [13] is

\[
f(c_{k,n}) = N_0 \left[ Q^{-1}\left(\frac{BER}{4}\right)\right]^2(2^c - 1) \tag{8}
\]

where

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \tag{9}
\]

and

\( f_k(c_{k,n}) \) is convex and increasing in \( c \) and that \( f(0) = 0 \), means no bits transferred.

### III. GENETIC ALGORITHM BASED ALLOCATION

Genetic Algorithms (GAs) [6, 7, 12, 16] provide learning method motivated by an analogy to biological evolution. Rather than search from general-to-specific hypotheses, or simple-to-complex, GAs generate successor hypotheses by repeatedly mutating (mutation is a genetic operator used to maintain genetic diversity from one generation of a population of chromosomes to the next) and recombining parts of the best currently known hypotheses. At each step, a collection of hypotheses called the ‘current population’ is updated by replacing the fraction of population by offspring of the most-fit current hypotheses. The process forms a generate-and-test beam-search of hypotheses, in which variants of the best current hypotheses are most likely to be considered next. The genetic algorithms applications are inspired by many factors:

- A successful and robust method for adoption within biological systems.
- Can search spaces of hypotheses containing complex interacting parts, where the impact of each part on overall hypothesis fitness may be difficult to model.
- Easily parallelized and can take the advantage of the decreasing costs of powerful computer hardware.

The subcarrier allocation problem to multiple users has many different permutations, thereby making the solution space very large and a suboptimal allocation of subcarriers to users is acceptable. The GAs are most suitable where the solution space is very large and a suboptimal solution may be sufficient in many scenarios. The problem addressed by GAs is to search the space of candidate hypotheses to identify the best hypotheses identified by a fitness function. The typical GA operates by iteratively updating a pool of hypotheses, called a population. During each iteration, the members of the population are evaluated according to the fitness function. A new population is then generated by probabilistically selecting the most-fit individuals from the current population which is forwarded to next generation population. The iterative improvement generally leads to near optimal solutions. The above discussion shows that GAs are suitable for the optimization of the subcarrier and bit allocation problem in a multiuser OFDM system.

The optimization problem to be solved by GAs is given in Equation 4. The processing steps in GA based algorithm are as follows:

1. **Generate chromosome of N elements** (minimum length of chromosome is assumed as 50, thus there are 50 subcarriers) and total number of chromosomes (population) as 30 for the experiment. Each element in the chromosome is a subcarrier allocated to a user (one user may be allocated more than one subcarrier). Thus the population is a 2-D array, where the rows represent chromosome number and column of a row represents subcarriers.

2. **Evaluate** - use the water-filling method to allocate each user’s bits and subcarrier and calculate the overall transmission power.
3. **Generate** the new population using crossover and mutation (see Appendix A) probability.

4. **Repeat** step 2 and step 3 till the system converges.

In this paper, we calculated each user’s power requirement and the total transmission power required by all users. The subcarriers allocated as per the user’s request arrives. The fitness is equal to the power required for all users or required by all subcarriers allocated to users. The lower the value of power \( P_{k,n}^* \) is the higher fitness.

The genetic algorithms had built-in selection of stronger individuals to be the winners from the old generation to new generation. Each chromosome had the format shown in Figure 1. The value of each element in the array (chromosome) is confined to a user signal and randomly generated. The array represents a solution to the optimization problem.

<table>
<thead>
<tr>
<th>Subcarrier 1</th>
<th>Subcarrier 2</th>
<th>\ldots</th>
<th>Subcarrier n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome element -1</td>
<td>Chromosome element -2</td>
<td>\ldots</td>
<td>Chromosome element n</td>
</tr>
</tbody>
</table>

Figure 1. Coding of Genetic Algorithm

For the optimization of our subcarrier and bit allocation problem, the final optimal allocation is sure to have the following features:

- Equation (1) shows that the power gain \( p_{k,n} \) can be achieved by channel gain \( \alpha_{k,n} \) (larger the channel gain lower the power needed). Therefore, the subscriber with largest channel gain will find the lowest transmission power as in equation (4).

- From equation (7), the number of subcarriers that each user needs according to the rate \( R_k \). The channel gain for each user is

\[
R_k = \sum_{n=1}^{N} \alpha_{k,n} \quad \text{for } 1 \leq k \leq K \text{ subject to equation (2)}
\]

The number of subcarriers that a user \( k \) can take is given by \( m_k \):

\[
m_k = \frac{N \cdot R_k}{\sum_{k=1}^{K} R_k} \quad 1 \leq k \leq K \text{ when } \sum_{k=1}^{K} m_k \leq N
\]

Now generate \( k \) users so that the total users can take maximum of \( N \) subcarriers. Allocate the subcarrier to the user \( k \) that has largest channel gain at this subcarrier, i.e. max \( \alpha_{k,n}^2 \). If total bits allocated for user \( k \) is with one subcarrier is \( c_k \), then bits for allocated for user \( k \) with \( n \) subcarriers is \( \{n\} \cup c_k \).

We improved the GA processing by the following steps: (1) Add high fitness chromosome at the end of each generation or while forming the new generation. The searching time was reduced by adding the good genes to the population at the end of each generation because it converges quickly. (2) Vary the chromosome size to choose those sizes which result in faster convergence and generate better solution.

**IV. SIMULATION RESULTS**

In this section, we compare the results of the genetic algorithm model with the results of Ehsan[2] and Wang’s[23] algorithms. The important point to note in this problem is the chromosome length. The minimum chromosome length we selected was 50, because we selected a minimum bit rate \( R=256 \). We added a condition that the maximum bit rate \( R > 6 \times N \) (allocation vector takes 6 bits for 64 QAM), where \( N \) is maximum number of users or maximum number of subcarriers or chromosome length (minimum chromosome length must be 43 or above so we selected 50). The reason of adding this condition is as follows: if we take the chromosome length 30 then in the present case, data rate can not be more than 180 bits. We take absolute value of the frequency with absolute value between 0 \( \leq f \leq 1 \) and channel frequency was randomly generated. If two or more frequencies have the same values in a chromosome, it means user was assigned same number of subcarriers to meet the user needs to transfer the data. The simulations were done with chromosome length 50, 60, 70, 80, 90, and 100. The target bit error rate (BER) is set to \( 10^{-3} \). The bit allocation vector can take 0 bits (no modulation), 2 bits (QPSK), 4 bits (16 QAM), 6 bits (64 QAM). The other parameters are provided below:

- Each element of the chromosome represents a subcarrier (chromosome length 50, 60, 70, 80, 90, and 100)
- One or more subcarriers are assigned to each user.
- The total transmission power is considered (of a chromosome-50 users) instead of one user’s transmission power, so the balance among the users is kept.
- Subcarriers allocated according to need (in the present case randomly)
- Population: 30
- Generations: 10 to 200
- Crossover: 0.6
- Mutation: 0.03

The subcarrier allocation algorithm [1, 23] was used to calculate the power for the population in each generation. Figure 2 show that the system converges after 20 generations, where chromosome length is 50. After 20 generations the variation is minor, means no power gain after that. Figure 3 shows that the power allocated slowly stabilizes with chromosome length 60. The variation in power required decreased after reaching chromosome length 80. The results show that data transfer is efficient in power requirement with allocation of more subcarriers to users. Figure 4 shows the algorithm converged after 40 generations. In each case minimum 50 subcarriers are used. Please note that the number of users is \( \leq \) number of subcarriers or size of chromosome. Figure 5 shows that SNR converges after 30
generations. But after 20 generations it is close to convergence with a chromosome length of 50.

Eshan’s (see Fig. 2 of reference [2]) iterative and adaptive joint subcarrier and bit allocation algorithm requires 40 units of power with 10 users. In the present case, the Figure 3 converges at chromosome length 80 with low value, where maximum number of users is 80.

Wang’s [23] experimented with modified simple genetic algorithm. Modification is that a specified percentage of high fitness genes (chromosomes) replaces are added (replaced with low fitness genes) at the end of each generation for faster convergence. Wang’s modified GA model with 32 subcarriers, converges after 140 generations (see Fig 4d [2]) with 8 users. The present GA model with 50 subcarriers converges after 35 generations (maximum of 50 users).

V. CONCLUSIONS

We develop a solution using GAs for the adaptive bit and power allocation problem. Two modifications, increasing the chromosome length and adding the chromosomes with good genes increased the performance of our approach. The results in the our method with 50 subcarriers are better than Ehsan’s [2] and comparable with Wang’s [23] results and converge faster than Wang’s method. If we increase the number of subcarriers our method performs much better than Wang’s method.
applied, the offsprings are simply duplications of the parents.

**mutation**: Substitute one or more bits of an individual randomly by a new value (0 or 1)

10010010 10010101

10010010 00010101

**fitness function**: A fitness function must be devised for each problem; given a particular chromosome, the fitness function returns a single numerical fitness value, which is proportional to the ability, or utility, of the individual represented by that chromosome.

REFERENCES


