Intuition and Problem Solving
Jo M.C. Nelissen
Utrecht University

Abstract

The focus of this article is a theoretical discussion and analysis of the concept of intuition. The article investigates how intuition, in the psychological sense, is connected with concepts like problem solving, reflective thinking, automatized thinking activities and understanding ‘gestalt’ or structure and meaning.

Automatisms are based on insights and knowledge stored in (long term) memory. Intuitive thinking, however, is connected with experimenting, exploring new strategies and searching for new, non-conventional ways of thinking. Through intuition we become more aware of structures and through reflective analyses these structures may grow into explicit structures. Reflections on elaboration of thinking rules and strategies may be internalized and transformed in intuitions.

When we look at the process of intuition we are confronted with quick, tacit, spontaneous, more or less subconscious processes evolving mostly independent of reasoning (logic). If, however, we consider the structure (origin) of intuition we can characterize intuition as being connected with all the knowledge the subject ever learned. Now, intuition is not operating independent of learning (logic), it is associated especially with meaningful coherence.

Once a problem is solved on the basis of exploring intuitions, it is useful to reconsider intuitions critically. If the intuitions were successful, they may be integrated in the existing body of knowledge. So, intuition and the construction of knowledge can be conceived as complementary processes.

Keywords: intuition, meaning, Gestalt, internalization, understanding structure, reflection

He that trusteth in his own heart is a fool’.  

‘Do not trust anybody who claims to possess a good intuition’, Hogarth (2001)
Introduction

In Latin the concepts ‘intueor’ and ‘intuéri’ stand for ‘attentive considering’ and refer to a mental activity. This phenomenon, intuition, is what we want to discuss in this article. We consider intuition as a cognitive activity (‘ways of knowing’, Claxton, 2000) that can be typified as a quick exploration of thinking strategies. In such an exploration people use and apply knowledge that they have acquired already. So, in essence, we consider intuition as the exploration of internalized, compacted knowledge. Although it is a swift activity, it is certainly not a ‘trial-and-error’ activity triggered by impulsivity. In a rough way the problem solver – using the available, compacted knowledge – is aware of the direction in which the thinking process should proceed and what kind of information and knowledge can be relevant. The German psychologist Selz (1881-1943) qualifies the regulation upon which such processes are based as ‘anticipating tendency’, like the psychologist Fischbein (1975) is speaking of ‘anticipating intuition’. So intuition is connected with (mathematical) knowledge, but the reverse could also be and in fact is claimed. For Sinha (1988, p.5) intuition is ‘...the first and fundamental basis of knowledge’.

We can find the notion of intuition already in the works of Descartes (1596–1650) who identified intuition as a gift of God (see Claxton, 2000). In ‘The Ethics’ of Spinoza (1632–1677) the third and highest level of knowledge is called ‘scientia intuitive’ (intuitive, scientific knowledge). This does not arise from the first level, the level of ‘sense-perception’ (Copleston; 1963). Knowledge on this level is, according to Spinoza, ‘inadequate’. Intuitive knowledge arises from and is connected with the second level (‘cognitio secundi generis’), and is called ‘ratio’, reason. This knowledge is ‘adequate’. Intuition is, according to Spinoza, the result of knowledge on the second level, the ‘ratio’. Maybe the distinction between ‘inadequate’ and ‘adequate’ knowledge is comparable with Davydov’s (1977) empirical and theoretical thinking. And with Plato’s ‘sense experience’ and ‘pre-natal intuition of ideas’, while Plato identified intuition as the key for true knowledge (Rodriguez & Yu, 2006).

The creativity needed during the process of learning mathematics may be supported by intuition. After all, problem solving demands exploring new strategies, experimenting and searching for new, non-conventional ways of thinking. Such processes should be distinguished from standard procedures and algorithmic processes because they evolve more or less intuitively. Intuition may enhance the courage to avoid rigidity and the (uncritical) use of routines, as well as stimulate looking for original and elegant problem solving activities. That is what the mathematician Wilder (in Burton, 2004, p.74) notes: ‘Without in-
tuition, there is no creativity in mathematics (but) the intuitive component is dependent for its growth on the knowledge component'.

Intuitive decisions arise mostly without explicit reasoning. Many mathematicians come to act before all the possible thinking steps are fully considered or assessed. An intuitive thinking process is relatively independent from coercion by fixed rules, or as Sfard (2002) states: intuition progresses as reason without (explicit) rules.

At the same time, Babai (2010), however, analyses the remarkable phenomenon that students are affected by a small number of implicit (or intuitive) rules when solving tasks in science and mathematics. To give one example: ‘the free fall task’ with matchboxes; one is full and the other is empty. Both are released from the same height above the ground. Now the question is whether the full box will hit the ground before/at the same time/after the empty box. Many students (including first year college physics students), following the implicit/intuitive rule ‘more A – more B’ (weight – speed) claim that the heavier box falls faster.

**Intuitive, planned and automated approaches**

When confronted with a problem, mathematicians may prefer different solving strategies. A first strategy is that the problem is approached intuitively and in this case strategies are spontaneously explored without the preparation and use of a standard plan of action. One tries to construct a problem representation (Heller & Reif, 1984) and this is elaborated. This intuitive process is largely based on compacted mathematical knowledge and operates mostly a verbally, pre-conscious, quick and, moreover, is linked to emotion and affect.

In a second approach the problem solver prefers to start with the construction of a plan of action and to elaborate that plan in a systematic way. Such an approach cannot be characterized as intuitive. Polya (1962) is inclined to use the term ‘successive approximation’ in this case because the problem solver is systematically exploring heuristic ‘trial and error’ processes. This process is analytic, intentional and mostly verbal.

Mathematical thinking and problem solving however, may also evolve automatically. In this third approach a familiar (already acquired) procedure is executed as a routine. Although routines are applied implicitly, quickly and spontaneously, we don’t consider them as intuitive activities. After all, the subject is not confronted with a real problem to be solved and is aware of – without appreciable reflection and systematic effort – how to solve the problem or (better) perform the task. Hence, there is not any need to ferret out new strategies of thinking.
Hogarth (2010) discerns two cognitive systems: 1. a tacit system and 2. a deliberate system. Automatisms and intuitions belong to the tacit system, as Hogarth states. This classification alludes to ‘thinking fast’, which Kahneman (2011) distinguishes from ‘thinking slow’, systematic and statistical thinking. It seems, however, that Hogarth is inclined to consider automatisms and intuitions more or less as comparable functioning activities. So, if judgments – for instance whether or not two objects are similar – are executed quickly, he claims that these judgments can be typified as intuitive judgments. Hogarth: ‘Judgments of similarity are made quickly and, it would seem, intuitively’ (p.94). This claim, however, can be contended. Let us consider the simple task: ‘How much is 5 + 5?’ The reaction that most people immediately and automatized produce will be: ‘10’. This answer, or judgment, is generally made very quickly, without any doubt, deliberation or reflection. However, obviously there is no reason for the use of intuition, and hence for experimenting because the answer ‘10’ is fully based on knowledge, stored and at our disposal in the long term memory. Glöckner and Witteman (2010) identify ‘intuitive-automatic processes’ in the same way as Hogarth’. We should suggest, however, to make a distinction between automatic and intuitive processes, at least if we are dealing with mathematical problem solving.

In this article we discuss the concept of intuition, especially in the context of problem solving, in which processes of intuition are mostly elicited. Intuition is considered to be closely connected with (mathematical) knowledge and reflection (Myers, 2002; Bowers et al., 1990) as well as with emotion.

We will discuss several questions, such as how do intuitions emerge? Are people aware of their ongoing intuitions? Or are we discussing a phenomenon that passes off at an unconscious level? What can be considered – in a psychological sense – as core process of intuition? Do intuitions always support – or maybe sometimes inhibit or obscure – problem solving processes? In problem solving, intuition has typical functions and we discuss what these functions can be.

Intuitive thinking, we believe, should be stimulated in interactive education. This is valuable not only for the smartest students but for all, even low achieving, students.

**Intuition: conscious or unconscious?**

It is not very reasonable, to all appearances, to view intuition as some kind of alternative sense. And so the proverb of Pascal (1669), ‘*Le coeur a ses raisons que la raison ne connaît pas*’ (‘The heart has its reasons, that the reason does not know’), is a bit controversial. Neverthe-
less, when a problem is to be solved, subjects do not always follow a systematic plan of action. Mathematicians, as well, may be inclined to approach a problem immediately and without much pondering. Of course, insight and knowledge facilitate such an approach that can be considered as a rapid, spontaneous, curtailed, automatic and (at least partly) conscious process. As many human thinking activities are passing off rapid and smoothly, there is a tendency to denote them as intuitive activities. ‘In this sense’ Gregory asserts (1987, p. 389) ‘almost all judgments and behavior are intuitive’. This, however, is perhaps a risky claim and Gregory – like Hogarth – seems to subsume automated processes to intuition. Nevertheless one can typify seemingly simple acts of speaking to be more automatic and more non-conscious than controlled and conscious (see Sadler-Smith, 2008).

Moreover, Sinclair (2010) advocates to distinguish levels of ‘awareness’ of intuiting. The first level is non-conscious, for instance when intuition catches us by surprise. But at the actively conscious level the subject is aware of a clear intention and desired outcome. The process as such, however, remains non-conscious.

The Russian psychologist Krutetskii (1976) discusses the views of two leading French mathematicians, Poincaré and Hadamard, with respect to the nature of mathematical thinking. Krutetskii is a proponent of Hadamard’s view that creative processes, generally speaking, do not always occur consciously. Some ideas evolve as unconscious, some transform in ‘subconscious’ processes and partly they emerge and evolve as conscious processes. While Poincaré claims that creative processes are based on ‘esthetic instinct’, Hadamard wonders how the term ‘unconscious’ should be explained. In reality, the transition from unconscious to conscious evolves quickly and spontaneously. So it is difficult to observe and establish whether processes are conscious or not. Moreover, intuitive and discursive thinking, in the terms of Krutetskii, are constantly interwoven processes. Olive and Steffe (2002) propose a comparable distinction in intuition and reflection. Through intuition we become aware of structures, though not yet fully ordered and understood. Through reflection these structures (maps, schemas, networks) grow into more explicit and more powerful structures.

Reflection as such is stimulated in dialogue and interactive teaching, as research suggests (Nelissen, 1987). The student anticipates the comments the opponents made and these comments will be integrated in future thinking activities.

So, reflection is internalized dialogue, as intuition is linked with and stimulated by internalized reflection.
Intuition versus knowledge?

One can discern two modes of intuitions. To begin with, there are intuitions that evolve spontaneously out of life experiences or ‘practical wisdom’ (Furlong, 2000). All kinds of children’s daily experiences, for instance in the field of numbers and quantity, contribute to the growth of intuitive thinking. The development of intuitions is stimulated by the use of these experiences to construct images of (mathematical) ‘objects’. These intuitions stem from implicit learning and daily experiences. Hence, intuition is linked with implicit acquisition of knowledge and with tacit knowledge (Shirley & Langan-Fox, 1996). Fischbein (1975) characterizes these intuitions as ‘primary intuitions’. Intuitions, however, arising from systematic education and explicit (intentional) learning are called ‘secondary intuitions’ and these are the higher order intuitions connected with problem solving.

Sometimes, intuition is conceived as an independent and high sensible operating human ability that generates a spectrum of – more or less concealed – insights. However, it seems that we may be confronting some misunderstandings here. The first would be that intuition is something totally different compared with rationality and cognition. That is why it is suggested that you should trust your sensible intuition rather than your rational mind. It is believed, moreover, that intuition is capable of tracing and exploring hidden sources of knowledge and human energy.

A second misunderstanding alludes to the idea that intuitive thinking is better and more powerful than rational thinking. It is striking, however, that at the same time, the opposite view is defended, namely that rational thinking is superior to intuition. Claxton (2000) challenges this last view: ‘Measurement, justification and accountability is, in this case, all that rests’.

The idea that intuition is something quite different and even more powerful than rational thinking stems from the idea that there is a rupture between both modes of thinking. We encounter here, however, a false contradiction, as can be found in a publication of Gladwell (2007). He claims that not knowledge but ‘open-mindedness’ is the main source for intuition. Intuition is functioning optimal without the lumber of rational knowledge, Gladwell believes. Who would contend, however, with the significance of ‘open-mindedness’? Obviously it will make more sense to bridge (false) contradictions, as Atkinson and Claxton (2000, p.1) note, such as for instance the relation between ‘articulate /rational/ explicit and inarticulate/intuitive/implicit ways of knowing and learning’. 
Intuitive thinking is not the reverse of rational and analytical thinking. The opposite seems true, for intuition is integrated with rational thinking, in the sense in which Hamlon (2011) states that decision making involves a combination of rational and intuitive modes.

Unlike Hamlon, Sinclair (2010) asserts that intuiting is approximate and not so accurate when dealing especially with exact mathematical tasks. This statement, however, is not illustrated in the discussion that Burton (2004; 76) executed with a number of mathematicians. Most of them considered intuition an important feature of the way they analyzed mathematical problems.

Sinclair and Ashkanasy (2005) suggest that in the context of decision making processes ‘... an integrated model of analytical and intuitive decision making both approaches are used in a complementary and iterative fashion’ (p.3).

Not only the connection between rational/ explicit and intuitive/implicit ways of knowing should be discussed. What can be analyzed as well is the transformation of procedures and strategies (that we have previously learned) into intuitive thinking.

Procedures in grammar and strategies in mathematics, for instance, are mostly learned explicitly before they are conveyed in tacit, quick and implicit processes (compare Dörfler and Ackermann, 2012). Students are stimulated to reflect on elaborating these rules and strategies and this reflection is internalized. This process we consider as an important source of intuition: intuition is related to internalization of already acquired, compacted knowledge and reflections on that knowledge (rules, strategies). Simon therefore states (in Dörfler et al., p. 553): ‘Intuition and judgment are simply analyses frozen into habit and into the capacity for rapid response through recognition’.

What can be analyzed moreover is what can be distinguished and conceived as the process of intuition and the content or structure of intuition. When we focus on the process of intuition we are dealing with the question of how intuition is performed. Intuition is performed quickly, tacitly, spontaneously, meaningfully, more or less subconsciously, and so mostly independent of the principles and rules of reasoning (logic). However, if we focus on the origin, the structure of intuition – hence the question of what is the content of intuition – then we can characterize intuition as the composition of compacted rules, internalized reflections and knowledge the subject ever executed, elaborated and learned. Dennett (2013) even states that intuition is knowledge: ‘...the thought experiments I have dubbed intuition pumps’. For
instance, says Dennett, in the form of reduction ad absurdum ‘in which one takes one’s opponents’ premises and derives a formal contradiction (an absurd result), showing that they can’t be right’ (p. 5). So it is clear that under this aspect, intuition does not operate independent of knowledge.

To end this section we briefly summarize and discuss the systems of thinking constructed by Kahneman (2011). He distinguishes system 1, thinking fast, and system 2, thinking slow. System 1 encompasses intuitive heuristics, i.e. feelings of liking and disliking; the second set is intuition based on experience or relevant expertise. The third set of fast activities that Kahneman discerns are automatic activities. System two, the thinking slow system contains systematic, rational, deliberating, statistical and controlled activities. System 2, states Kahneman, ‘…is likely to reject the intuitive answer suggested by system 1’ (p. 65). The distinction in these two systems is valuable, although the contradiction between the two systems Kahneman suggests, may be too sharp, if we interpret Kahneman well. ‘System 1 is gullible and biased to believe, System 2 is in charge of doubting and unbelieving, but System 2 is sometimes busy, and often lazy’ (p. 81).

Kahneman is inclined to state that system 1 acts (more or less) independent of system 2. Moreover, system 1 ignores knowledge in system 2. And it seems that system 1 does not take optimal advantage of the useful knowledge that is available in system 2.

Last but not least, system 2 does not benefit from the creativity of system 1. These interpretations may provide us with enough stuff for an interesting discussion.

Intuition without knowledge?

People are urged, for instance when requested by their therapist, to trust their intuition. Intuition, it is suggested, functions independent of knowledge and rationality. It is sometimes recommended to ‘liberate’ our intuition and to consolidate the contact with our ‘inner’ intuitive sources. ‘Mediation’ and using our ‘energy’ can boost the growth of intuition. Moreover we should learn to listen to our intuitive ‘spirit’ and intuitive ‘voice’. Intuition is called ‘a new reality’, a ‘sixth sense’ (Naparstek, 1997) we can trust as an infallible and magic prompting or brain-wave.

For all we know, therapies based on such ideas may well be effective. In this article, however, we advocate another approach and so we just refer to Claxton’s comments on the sixth sense approach by (2000, p. 49): ‘… claims for its validity that seems grandiose or mystical’.
In what sense we could explore this 'sixth sense' during the processes of problem solving, is a question that is difficult to answer. As Paulos (1989) demonstrated, this question, in any case, should be answered negatively when subjects are confronted with mathematical problems, especially problems of probability and chance. Paulos illustrates how subjects are strongly inclined to attribute the cause of events to secret and mystical powers. Chance, as such, is distrusted, and in fact chance, it is believed, does scarcely exist. Probability and chance, however, as Paulos makes clear, are very difficult to grasp intuitively. So, this view does not support the idea that novices might act intuitively because they lack knowledge that interferes with the ability to generate novel insights (see Sinclair & Ashkanasy, 2005).

Young children's intuition
Intuitive processes emerge mostly spontaneously, quickly, directly, without reflection and sometimes subconsciously. These processes differ, as we saw, from other thinking activities that are based on systematic planning and reflection. Gopnik et al. (2009) observed the behavior of young children and she analyzed remarkable, cognitive achievements, for instance in causal thinking, in statistics and in counterfactual reasoning. The behavior of these children, however, is mostly spontaneous and non-reflective. So, are these young children acting intuitively? It is not easy to answer this question, because horizontal intuitions are just starting to emerge and vertical intuitions could not yet have evolved. It is true that young children are very alert observers and their most powerful mode of learning is imitation, as Vygotsky frequently has stressed. After all, that is how they acquire, for instance, their mother language. But imitation as such is not learned by imitation, children do not imitate imitation, they imitate behavior. So, it seems that all we may conjecture is that imitation is somehow interwoven with intuition. Of course, this intuition does not stem from knowledge and experience, but from innate, spontaneous and 'natural' abilities. Intuitions in this sense, as Pinker (2002) has pointed out, emerge early in life and they are related with networks in the brain and consist of different sets of genes. Pinker discerns several, what he calls, core intuitions, for instance for space, number, language and biology.

Intuition: understanding structure
Intuition is related with knowledge as a rich repository of experience (Fischbein, 1975; Myers, 2002) and this is a source for the intuition an expert is using. Myers: ‘Through experience we gain practical intuition – subtle, complex, ineffable knowledge that aids our problem solving’ (p.51). Experts, however, are much more experienced than novices, as Myers notes. He refers to an experiment by the psychologist Simon to illustrate that statement. Simon observed that amateur chess players
have stored in their mind only a limited amount of patterns, while more experienced players used a thousand and chess masters recognized several ten thousand patterns. Knowledge of structures seems crucial in this process of recognition.

In mathematical thinking, Fishbein states, intuition is closely related to cognitive activities, images and conceptual knowledge. A comparable view is suggested by van Hiele (1997). He wrote that subjects often do think non-verbally, especially in situations that demand swift actions. This non-verbal thinking is, according to Van Hiele, in essence intuitive thinking. Even expert intuition is mostly non-verbal, it is internalized and so linked to tacit knowledge (Sinclair & Ashkanasy, 2005). We immediately recognize a (familiar) situation, a friend (Simon in Rodriguez & Yu, 2006), a historical building and so on. However, when requested to express precisely in words and language a situation, a friend, this is a difficult task. But you recognize your friend immediately, because you grasp meaning immediately. And a friend is a meaningful person.

Intuitive thinking is characterized by insight in coherence or structure. That’s why students should learn to focus on coherence, visualizing and imagining to stimulate intuitive thinking. The chess master recognizes (mostly, but not always) in a short time if not immediately, a pattern in the configuration of the chess-set on the board. This intuitive recognition is based on knowledge and experience, ‘intuition as expert knowledge’, as Dörfler and Ackermann (2012) ascertain. Experts, like for instance senior firefighters, facing high-risk situations use intuition to make decisions under considerable time pressure. Pretz and Totz (2007) classify this intuition as ‘holistic intuition’ and this is ‘a Gestalt understanding of intuition’.

In many vocational situations it is important to react as quickly as possible to an unexpected problem. According to Dolk (1997) an essential feature of the behavior of teachers is to recognize immediately a ‘Gestalt’ in the teaching situation, because a teacher is supposed to react with alertness. Teachers constantly make ‘…..rapid, subtle decisions about how to respond to the dynamic, complex environment that classrooms constitute’, declare Brown and Coles (2000, p. 165). They describe the teacher as an ‘intuitive practitioner’, who reacts intuitively in problematic situations (see also Eraut, 2002). Teachers know how to act, because they analyze problems based on an identified Gestalt and that is a meaningful structure. They learn to analyze Gestalts on an always higher level of proficiency. This is like ‘making connections’ that Burton (2004) considers the essence of intuition. As one can observe in everyday practice, teachers are not constantly re-
flecting on their teaching activities in order to justify in a rational way their instruction and education. What they are doing frequently, however, is to react spontaneously to dynamic and ever transforming situations (or Gestalts).

Bowers (et al., 1990) consider as the essence of intuitive thinking the construction of structure, and they illuminated that viewpoint in experiments. They characterized intuition as 'a preliminary perception of coherence, meaning, structure' (p. 74). The researchers found that accurate, intuitive promptings were connected with 'coherencies' or structures. These structures had the features of 'tacit knowledge' and implicit perception of a Gestalt's coherence, evolving and transforming gradually in explicit representations. That happened, for instance, when people were asked to find a hypothesis for the solution of a problem.

The processing, in such cases, is often of a holistic nature, as Sinclair (2010) explained, focused on a new combination of disparate patterns. This process is slower than intuitive expertise. The creation does indeed emerge in an instant, but it is not always immediate. There seems to be a certain period of 'incubation' before an intuition is complete.

To summarize, intuitive thinking processes, aimed at the solution of problems, are based on the recognition and/or the construction of meaningful structures (or Gestalts).

**The powers and perils of intuition**

Generally speaking, intuition can support as well as inhibit thinking activities. Myers (2002) discusses research which reveals that the 'powers or perils' of intuition are connected with the subject and discipline (see also Sadler-Smith, 2008).

Students are asked to draw a line that represents the level of water we observe in the glass during the pouring of the water. They are shown a drawing of a glass of water, and they are told and then see in the drawing how the glass of water is poured into a bowl. In this ‘water-level task’ up to 40 percent of the population 'incorrectly intuits that the water would deviate from horizontal', (Myers, 2002, p. 108). These students therefore do not draw a horizontal line, but a sloping line going down.

Myers concludes that in science many people's intuitions may be wrong, while in other fields or disciplines intuition may emerge as reliable support. Moreover, according to Hogarth (2010), errors that subjects make are likely to vary in relation to the type of situation considered. Myers identifies what kind of processes is a threat (‘deadly sins’).
for the formation of reliable intuitions. For instance: always seeking confirmation of an idea and stressing personal insights too much. A more fruitful tuition for intuition is to provoke the emergence of interactive discussion in the classroom and to realize the notion that it sometimes makes sense not to follow familiar routines (‘slow thinking’).

**Intuition and problem solving**

It largely depends on children’s (daily) experiences whether intuitions will rise spontaneously or maybe not at all. If their experiences are poor or absent the intuitions – that may support problem solving – will not arise. We illustrate this point with a well-known experiment performed by Piaget. This experiment involves the so-called conservation of volume. The researcher pours water from a broad glass into a narrow (high) glass (or vice versa). The children are asked whether there is now more, less or the same amount of water in the other glass. Most of the participating children – around four years or younger – obviously were deceived perceptually and thought that now there was more water in the (narrow) glass. When, however, this experiment was repeated with children of potters (ceramists’ families on islands in the Pacific), the researchers noticed that these children were not deceived at all. The explanation is that their daily experience with pots of all kinds of sizes and shapes amounted to the growth of their (horizontal) intuition. So, they solved the problems.

Horizontal and/or vertical intuitions, however, may be too weak or even deficient to give support to the process of problem solving. Remember for instance *fractions*. For many students operations with fractions are very difficult to understand, because these students conceive fractions simple as whole numbers appearing on both sides of a slash. So, they think, $1/2 + 1/4 = 2/6$ and $3/9$ is more than $1/3$.

Also in the field of statistics (chance, accident) the horizontal intuitions of many young students are underdeveloped. So, these students often think that the teacher is much more skilled - when throwing a dice – to get a high number (the 6) than a child.

Another example is a well-known task in the field of geometry that looks as follows. ‘Suppose a rope is tied around the earth. We cut this rope and we insert a little rope of one meter length. So, the rope around the earth is now one meter longer. If this rope is now again tightened around the earth, what will then be the distance to the earth?’

Children – as well as a lot of adults – answer that the distance equals to almost nothing or perhaps a fraction of a millimetre. They can’t imagine that in reality it is more than 16 centimetre.
Children’s horizontal intuitions and experiences may of course, not only hinder but also sustain the problem solving (Myers, 2002). For instance, when they have to fairly divide a number of candies among friends.

**Stages of problem solving and intuition**

Let us start this section with an interesting question from Hogarth (2010, p. 142): ‘Is the subconscious wise in the sense of being better able to solve problems than the conscious self?’ So, is intuition wiser than the deliberate system? Hogarth holds that problem solving is not so much the result of the subconscious, intuitive mind that always keeps working; problem solving often requires seeing the problem from a new perspective and making novel connections. So, problem solving is often a combination of ‘subconscious and conscious processing’. Exploring, intuitive thinking and deliberate, systematic thinking are strongly interwoven in the practice of problem solving, according to Hogarth.

Processes of problem solving often pass through several stages. To begin with, the problem must be analyzed. In this stage the problem must be clearly identified and that provides the design for a plan of action. In the next stage approaches and strategies are considered and elaborated. The problem solver reflects on her/his activities. At the end of the process she/he evaluates the result of the whole process in order to assess whether in the future one should pass through the same ways of thinking and acting. With reference to such a general scheme of problem solving, we now analyze what is the significance of intuition during these processes.

When in a first stage a problem is identified, Polya (1962) wrote, a notion is generated and insight in the kind of problem mostly suggests an idea of how to approach the problem. This notion is highly intuitive and gives rise to the emergence of schemes, patterns, or models. This view of Polya endorses our opinion that intuition is strongly connected with insight in structure. Polya stresses in this context: ‘…the intuitive grasp of the situation, a little bit of a bright idea’ (p.24). However, at the same time, he warns: ‘Bright ideas are rare’. Anyhow, the significance of intuition in this stage of problem solving is that the thinking process is stimulated and mobilized. ‘The role of intuition is to prepare action,’ as Brown and Coles (2000, 173) assert. Intuition triggers taking initiatives and endorses unexpected ideas, and so, uncritical adaption of familiar routines is prevented (see Rumco & Sakamato 1999). Claxton (2000, 40) claims that intuition is related to ‘an emotional involvement on the part of the knower’ (cf. Sinclair, 2010).
In a second stage the problem solver explores strategies and experiments and tries out new approaches.

In a third stage the activities aimed at the final solution of the problem are executed. In this stage, but especially in the fourth stage, the problem solver critically reflects on the initial intuitions that emerged spontaneously. As several researchers stress, intuitions should be evaluated and approved at the end of the process by ‘discursive thinking’ (van Hiele, 1997; Krutetskii, 1976) and ‘reflection’ (Olive & Steffe, 2002; Nelissen, 1987). Processes of intuition take place abbreviated, and so, much quicker than processes of discursive thinking (Krutetskii, 1976). It is useful to reflect critically on intuitive processes in order to assess what is their usefulness and quality. If the assessment provides positive results, the intuition can be incorporated into ‘the mental toolbox’ of the problem solver. This stage is called ‘the integrative stage of intuition’ (Bowers in Shirley & Langan-Fox, 1996). Shirley & Langan-Fox (p. 574): ‘This occurs when adequate activation has gathered to cross he awareness threshold’. Intuitions are rendered as a part of rational systematic thinking, reflection and problem solving. So, there is no so-called antithetical relationship (Baylor, 2001) between intuitive and (meta-)cognitive thought processes.

Summary and conclusions

Intuition is closely connected with experience and knowledge. Intuitions are acquired spontaneously as well as systematically. Primary or horizontal intuition is based on spontaneous, implicit learning while secondary or vertical intuition is based on systematic education and explicit learning (Fischbein, 1975).

Intuition can be stimulated in education. Braw (2000) discussed the question ‘Tuition for intuition?’ with a number of useful suggestions. For instance: –students react to the intuitions that emerged during discussions – the students should be confronted with and get used to the idea that for one and the same problem more than one approach and solution may be valid.

As new problems are mostly elaborated and analyzed on the basis of intuition, intuition is indispensable for transfer. Transfer alludes to the exploration and application of knowledge, insights and strategies that are already acquired. That is why Brown and Coles (2000) note that intuitions should be flexible and adaptable in new situations.

In this article we claim that intuition stems from experience. It is a way of knowing that can be developed as a result of growing experience (Claxton). However, that experience should not be considered as just the rehearsal and application of always the same routines (Braw, 2000).
Knowledge and reflective thinking is necessary for the emergence of intuition and conversely, intuition is indispensable for the development of mathematical insights. Just as reflection can be conceived as internalized interaction (the subject anticipates the critics of the discussion partner and that results in self-criticism, hence reflection), intuition can be considered (at least partly) as internalized reflection. So, we agree with Sonenshein (2007, p. 1033) who states that as individuals develop experience – for instance reflective thinking – they can ‘internalize that experience into intuitions’.

Once a problem is solved on the basis of exploring and experimenting (hence: intuitively) it is useful to reconsider the integral intuitive process critically. If that process seemed to be successful, this intuition may be integrated in the existing body of knowledge. So, intuition and construction of knowledge can be viewed as complementary processes.

Intuition is not only related to rationality and cognition, but to creativity and emotion as well. That’s why Goldberg (1989) argues that an important function of intuition is creativity in dealing with alternatives, options and possibilities. After all, students should experience and learn that intuition emerges as fantasy, conjecture, hypothesis, impulse, image and so on. When exploring intuitions – and not just following standard procedures – students should dare to take certain risks and investigate new ways of thinking. As Wilder said: ‘Without intuition, there is no creativity.’

However, not only bright students should learn to take risks. Such an experience makes sense for all students. It allows the development of self-confidence, which is necessary for working with one’s own intuitions and avoiding the rigidity of prematurely falling back on standard routines.

As we saw, many scholars consider intuition as a cognitive process of grasping structure or Gestalt. Gestalt is affiliated with meaning and people mostly identify the significance of meaning immediately. Intuition is the act of grasping the meaning, as Bruner states (in Brown & Coles, 2000). We recognize here the (well known) ‘aha-Erlebnis’ that emerges spontaneously, sometimes even before any argumentation could take place.

References


Dolk, M. L. A. M. (1997) *Onmiddellij onderwijsgedrag over denken en handelen van leraren in onmiddellijke onderwijsituaties [Immediate Teaching Behavior on Teacher Knowledge and Behavior in Immediate Teaching Situations]*. Utrecht, the Netherlands: WCC.


http://dx.doi.org/10.1177/1350507605055351
http://dx.doi.org/10.5465/AMR.2007.26885677