Turbo Equalization with Cancelation of Nonlinear Distortion for CP-Assisted and Zero-Padded MC-CDM Schemes

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Abstract—In this paper, we consider MC-CDM schemes (Multicarrier Code Division Multiplexing) where clipping techniques are employed to reduce the envelope fluctuations of the transmitted signals. Both CP-assisted (Cyclic Prefix) and ZP (Zero-Padded) MC-CDM schemes are studied. We develop frequency-domain turbo equalizers combined with an iterative estimation and cancelation of nonlinear distortion effects, with relatively low complexity since they allow FFT-based (Fast Fourier Transform), frequency-domain implementations.

Our performance results show that the proposed receivers allow significant performance improvements at low and moderate SNR (Signal-to-Noise Ratio), even when strongly nonlinear transmitters are employed. The receiver for ZP MC-CDM is of special interest for systems where the duration of the channel impulse response is not a small fraction of the duration of the MC-CDM blocks, being suitable to MC-CDM systems with very large blocks (hundreds or even thousands of subcarriers), since they do not require the inversion nor the multiplication of large matrices.

Index Terms—MC-CDM, Zero-Padded MC-CDM, Turbo Equalization, Nonlinear Distortion

I. INTRODUCTION

MC-CDM schemes (Multicarrier Coded Division Multiplexing) [1] combine an OFDM modulation (Orthogonal Frequency Division Multiplexing) [2] with coded division multiplexing, typically employing orthogonal spreading codes. However, since the transmission over time-dispersive channels destroys the orthogonality between spreading codes, an FDE (Frequency-Domain Equalizer) is required before the despreading operation (as with MC-CDMA schemes [3]), usually optimized under an MMSE criterion (Minimum Mean-Squared Error). It was shown in [4] that the performance of MC-CDMA can be significantly improved if the linear FDE is replaced by an IB-DFE (Iterative Block Decision Feedback Equalizer) [5]. These techniques can also be employed with MC-CDM schemes.

As with conventional OFDM schemes, typically a CP (Cyclic Prefix) is inserted before each MC-CDM burst. This CP, which allows low-complexity FFT-based (Fast Fourier Transform) receiver implementations, should be longer than the maximum CIR length (Channel Impulse Response). The CP leads to a decrease on the power efficiency of the modulation, due to the “useless” power spent on the CP. For this reason, the length of the CP should be a small fraction of the length of the blocks. Typically the adopted CP length is an upper bound on the expected CIR length, which can be much higher than the true CIR length. In this case, ZP schemes (Zero Padded) can be a good alternative to CP-assisted schemes [6]. ZP schemes have better performance than CP-assisted schemes, but we need to employ complex receiver structures, involving the inversion and/or the multiplication of matrixes whose dimensions grow with the block length [7], which are not suitable when large blocks are employed. The receiver complexity with overlap-and-add techniques, is similar to the one of conventional CP-assisted schemes, but the performance is also identical [7]. A promising detection technique for ZP schemes is to employ an FDE operating on an extended version of the received block [8].

As with other multicarrier schemes, MC-CDM signals have strong envelope fluctuations and high PMEPR values (Peak-to-Mean Envelope Power Ratio), making them very prone to nonlinear effects. A promising approach to reduce the PMEPR of the transmitted signals while maintaining the spectral occupation of conventional schemes is to employ clipping techniques, combined with frequency-domain filtering [9], [10]. However, the nonlinear distortion effects can be severe when a low-PMEPR transmission is intended [9], [10]. To improve the performances, we can employ receivers where nonlinear distortion effects are iteratively estimated and compensated [11], [12].

In this paper, we consider MC-CDM schemes employing clipping techniques. Both CP-assisted MC-CDM and ZP MC-CDM schemes are considered. To improve the performances at low and moderate SNRs we consider the use of turbo equalization schemes, where the equalization and channel decoding operations are repeated iteratively, sharing information between them [13], [14]. We develop frequency-domain turbo equalizers for both CP-assisted and ZP MC-CDM schemes which combine an iterative estimation and cancelation of
nonlinear distortion effects. The proposed turbo receivers have relatively low complexity, since they allow FFT-based, frequency-domain implementations.

II. CP-ASSISTED MC-CDM AND ZP MC-CDM

A. Transmitted Signals

In this paper we consider point-to-point communications employing MC-CDM schemes. Fig. 1 shows the MC-CDM transmitter considered in this paper, as well as a conventional receiver. The frequency-domain block to be transmitted by the $n$th MC-CDM block is an interleaved version of the block $\{S_k^{(m)}; k = 0, 1, \ldots, N - 1\}$, where $N = KM$, with $K$ denoting the spreading factor and $M$ the number of data symbols for each spreading code. The frequency-domain symbols are given by $S_k^{(m)} = \sum_{p=1}^{M} \xi_p S_{k,p}^{(m)}$, where $P$ is the number of used spreading codes, $\xi_p$ is an appropriate weighting coefficient that accounts for the different powers assigned to different spreading codes (the power associated to the $p$th spreading code is proportional to $|\xi_p|^2$) and $S_{k,p}^{(m)} = C_{k,p}^{(m)} A_{k,p}^{(m)}$ is the $k$th chip for the $p$th spreading code ($x \bmod y$ is the reminder of division of $x$ by $y$), where $\{A_{k,p}^{(m)}; k = 0, 1, \ldots, M - 1\}$ is the block of data symbols associated to the $p$th spreading code and $\{C_{k,p}^{(m)}; k = 0, 1, \ldots, N - 1\}$ is the corresponding spreading sequence. An orthogonal spreading is assumed, with $|C_{k,p}^{(m)}| = 1$.

To reduce the envelope fluctuations on the transmitted signals we employ a clipping and filtering technique similar to the one of [9], [10]. Within that transmitter, $N' - N$ zeros are added to the original frequency-domain block (i.e., $N' - N$ idle subcarriers), leading to the augmented block $\{S_k'; k = 0, 1, \ldots, N'-1\}$, followed by an IDFT operation so as to generate a sampled version of the time-domain MC-CDM signal $\{S_k'; n = 0, 1, \ldots, N'-1\}$, with an oversampling factor $M_{Tx} = N'/N$. Each time-domain sample is submitted to a nonlinear device corresponding to an ideal envelope clipping, so as to reduce the envelope fluctuations on the transmitted signal. The clipped signal $\{S_k'^C; n = 0, 1, \ldots, N'-1\}$ is then submitted to a frequency-domain filtering procedure, through the set of multiplying coefficients $\{G_k; k = 0, 1, \ldots, N'-1\}$, to reduce the out-of-band radiation levels inherent to the nonlinear operation.

It is shown in [10] that the frequency-domain block to be transmitted $\{S_k^{T_x} = S_k'^C G_k; k = 0, 1, \ldots, N'-1\}$ can be decomposed into useful and nonlinear self-interference components: $S_k^{T_x} = \alpha_k S_k'^C G_k + D_k G_k$, with $\alpha$ defined in [9], [10] and $\{S_k'^C; k = 0, 1, \ldots, N'-1\} = \text{DFT} \{S_k'^C; n = 0, 1, \ldots, N'-1\}$. Throughout this paper we assume that $G_k = 1$ for the $N$ in-band subcarriers and 0 for the $N' - N$ out-of-band subcarriers. In this case, $S_k^{T_x} = \alpha_k S_k'^C + D_k$ for the in-band subcarriers and 0 for the out-of-band subcarriers.

It can also be shown that $D_k$ is approximately Gaussian-distributed, with zero mean; moreover, $E[D_k D_k^*]$ can be computed analytically, as described in [9], [10].

For CP-assisted MC-CDM schemes, the transmitted signal associated to a given data block is $s(t) = \sum_{n=-N_c}^{N_c} s_n^{T_x} h_{T_c} (t-n T_c)$, with $s_n^{T_x} = 0, 1, \ldots, N'-1$ = IDFT $\{s_k^{T_x}; k = 0, 1, \ldots, N'-1\}$, where $T_c$ denotes the sampling interval, $N_c$ denotes the number of samples at the cyclic prefix and $h_{T_c}$ is the impulse response of the reconstruction filter. It is assumed that the time-domain block is periodic, with period $N$, i.e., $s_n = s_{N-n}$. The first $N_c$ samples can be regarded as a CP for the MC-CDM block. Since the CP is usually not used for detection purposes, there is a degradation $\eta_G = N/(N + N_c)$, due to the "useless" power spent on it. To avoid this degradation we can replace the CP by $N_c$ zero-valued samples, which corresponds to employ ZP MC-CDM schemes.

B. Conventional Receivers

In the following we will present conventional MC-CDM receivers. For the sake of simplicity, we ignore nonlinear effects in this section (i.e., we will assume that $S_k^{T_x} = S_k$).

Let us consider first a CP-assisted MC-CDM scheme. If the CP is longer than the CIR the linear convolution associated to the channel can be regarded as a cyclic convolution relatively to the $N$-length, useful part of the received block, $\{y_n; n = 0, 1, \ldots, N-1\}$. This means that the corresponding frequency-domain block is $\{Y_k; k = 0, 1, \ldots, N - 1\} = \text{DFT} \{y_n; n = 0, 1, \ldots, N - 1\}$, where

$$Y_k = S_k H_k + N_k$$

with $H_k$ denoting the channel frequency response for the $k$th subcarrier and $N_k$ the corresponding channel noise. Clearly, the impact of a time-dispersive channel reduces to a scaling factor for each subcarrier. From (1), it is clear that the orthogonality between spreading codes is lost in frequency selective channels. For this reason, an FDE is required before the despreading operation [3]. The samples at the FDE output are $\hat{S}_k = Y_k F_k$ and the FDE coefficients are given by $F_k = H_k^{-1}([\beta + |H_k|^2]$, with $\beta = E[|N_k|^2]/E[|S_k|^2]$, which corresponds to the minimization of the MMSE in the frequency-domain samples $S_k$ (perfect channel estimation was assumed).

The data symbols associated to the $p$th spreading code can be estimated by despreading the samples at the FDE output $\hat{S}_k = Y_k F_k$, i.e., from $\hat{A}_{k,p} = \sum_{k'} E_{k'} \hat{S}_{k'} C_{k',p}$, with $\Psi$ denoting the set of frequencies employed to transmit the $k$th data symbol of each spreading code (for a $K \times M$ interleaving, the set $\Psi_k$ is given by $\Psi_k = \{k; k + M, \ldots, (K-1) M\}$).

Let us consider now ZP MC-CDM schemes. Fig. 1 shows the basic receiver for ZP MC-CDM schemes, which is based on the one proposed in [8] for ZP OFDM. Essentially this receiver employs an FDE operating on the augmented version of the received block $\{y_{n,J}; n = 0, 1, \ldots, JN-1\}$. Ideally, the signal component at the FDE output $\{\hat{S}_{n,J}; n = 0, 1, \ldots, JN-1\}$ is almost restricted to the first $N$ samples (it would be exactly restricted to these $N$ samples if the FDE coefficients

3Typically, the transmitted frequency-domain block is generated by submitting the block $\{S_k^{(m)}; k = 0, 1, \ldots, N - 1\}$ to a rectangular interleaver with dimensions $K \times M$, i.e., the different chips associated to a different data symbol are uniformly spread within the transmission band.

4It is assumed that all spreading codes are intended to a given user.
were optimized under the ZF criterion (Zero Forcing)) so the
remaining \((J-1)N\) samples can be removed. The detection is
based on the corresponding frequency-domain block \(\{S_k; k =
0, 1, \ldots, N-1\}\). DFT \(\{S_n; n = 0, 1, \ldots, N-1\}\).

If the duration of the "zero-padded region" is longer than
the duration of the overall channel impulse response, the \(N_G\)
zeros before the "MC-CDM block" can be regarded as a
CP for the "augmented ZP MC-CDM block" that includes the
subsequent \(N_G\) zeros. Therefore, a relation similar to (1)
could be derived for the augmented frequency-domain block:
\[Y_k^{(J)} = S_k^{(J)}H_k^{(J)} + N_k^{(J)},\]
where \(H_k^{(J)}\) and \(N_k^{(J)}\) replacing \(H_k\) and \(N_k\) of (1), respectively.
The equivalent transmitted frequency-domain block \(\{S_k^{(J)}; k = 0, 1, \ldots, JN-1\}\) is the
DFT of the block \(\{S_n; n = 0, 1, \ldots, JN-1\}\), where \(r_n = 1\)
for \(0 \leq n \leq N-1\) and 0 otherwise (once again, it is assumed
that the samples \(s_n\) are periodic with period \(N\)). It can be
shown that \(S_k^{(J)} = \sum_{k'=0}^{N-1} J S_{k'} R_{k' - J k^n}\), where \(\{R_k; k =
0, 1, \ldots, JN-1\}\) are the average values of \(\{s_k; k = 0, 1, \ldots, JN-1\}\).
the output of a linear FDE is the block \(\{S_k^{(J)}; k = 0, 1, \ldots, JN-1\}\),
with \(S_k^{(J)} = Y_k^{(J)} - \beta_{k} x_k^{(J)}\). The FDE coefficients are
\(F_k^{(J)} = H_k^{(J)}\beta/([\beta(\beta + |H_k^{(J)}|^2)]
\), with \(\beta(\beta + |H_k^{(J)}|^2)\). The \(\beta\) of (1), respectively. The equivalent transmitted
\(\rho_{k} = \max (\rho_{k,0}, \rho_{k,1})\) is a measure of the reliability of the decisions from the
previous iteration and \(\rho_{p} = 1/(2M) \sum_{m=0}^{M-1} \{\text{Re}\{\hat{A}_{k,p}\}\}^2 +
|\{\text{Im}\{\hat{A}_{k,p}\}\}| \). With respect to \(\eta_{k}\), for the first iteration \(\overline{D}_k = 0\).
and we can use the method of [9] to obtain
\(E[|D_k|^2]\). For the remaining iterations we can assume that
\(E[|D_k|^2] \approx 2(1-\rho^3)|\{\text{Re}\{\hat{A}_{k,p}\}\}\}
\) for \(\rho > 0.82\) (when \(\rho < 0.82\) the nonlinear
distortion estimates are not accurate enough, so we assume
\(\overline{D}_k = 0\).

For the first iteration, we do not have any information about
\(S_k\) and the correlation coefficient is zero. This means that the
receiver reduces to a linear FDE optimized under the MMSE
criterion [3]. After the first iteration, and if the residual BER is
not too high (at least for the spreading codes for which a higher
transmit power is associated), we have \(\overline{A}_{k,p} \approx A_{k,p}\) for most
of the data symbols, leading to \(\overline{S}_k \approx S_k\). Consequently, we
can use the feedback coefficients to eliminate a significant part
of the residual interference. The \(\overline{A}_{k,p}\) can also be effectively
employed for estimating the nonlinear distortion.

We can define a receiver that, as the turbo equalizers,
employs the channel decoder outputs instead of the uncoded
soft decisions in the feedback loop. The receiver structure,
that will be denoted as Turbo FDE, is similar to the IB-DFE
with soft decisions, but with SISO channel decoder outputs
(Soft-In, Soft-Out) employed in the feedback loop. The SISO
block, that can be implemented as defined in [15], provides
the LLRs of both the "information bits" and the "coded bits".
The input of the SISO block are the LLRs of the "coded bits"
and the deinterleaver output.

### III. RECEIVER DESIGN

The receiver for CP-assisted MC-CDM considered in this
paper is presented in Fig. 2 (the extension to ZP MC-CDM is
straightforward) and combines an IB-DFE with soft decisions
[13, 14] with estimation and compensation of nonlinear
distortion effects. For a given iteration, the signal at the
output of the FDE is \(\overline{S}_k = F_k (Y_k - H_k \overline{D}_k) - B_k \overline{S}_k\),
where \(\overline{S}_k = \{S_k; k = 0, 1, \ldots, N-1\}\) are the average values of \(\{s_k; k =
0, 1, \ldots, N-1\}\) associated to the previous iteration,
conditioned to the FDE output, and \(\{D_k; k = 0, 1, \ldots, N-1\}\)
is obtained by submitting \(\overline{S}_k\) to a replica of nonlinear device
at the transmitter (see Fig. 2). The optimum values of the
feedforward and feedback coefficients in the MMSE sense are
\[F_k = \frac{\kappa_{k} H_k^*}{\beta + |H_k|^2 + (1 - \rho^2)|\alpha H_k|^2}\]
\[B_k = \frac{F_k H_k}{\beta + |H_k|^2 + (1 - \rho^2)|\alpha H_k|^2}\]
and \(B_k = F_k H_k - 1\), respectively, where \(\kappa\) is selected to ensure
\[\sum_{k=0}^{N-1} F_k H_k/N = 1, \beta = E[|N_k|^2]/E[|S_k|^2]\]
\[\eta_{k} = E[|D_k - \overline{D}_k|^2]/E[|S_k|^2]\]. It can be shown that
the "overall averages" are \(\overline{S}_k = \sum_{p=1}^{P} \xi_{p}C_{k,p}\overline{A}_{k,p}\) 
\[\overline{A}_{k,p} = \frac{\{S_k^{(J)} - \alpha_{k} H_k^{(J)}\}}{|\alpha_{k} H_k^{(J)}|} \]
\[\alpha_{k} H_k^{(J)}\] denotes the despurred symbols and \(\alpha_{k} H_k^{(J)}\) denotes
\(\alpha_{k} H_k^{(J)}\) with \(\alpha_{k} H_k^{(J)}\) denoting the hard
decisions associated to \(A_{k,p}\). The coefficient \(\rho_{k,p}\) is a measure of the reliability of the decisions from the
previous iteration and \(\rho_{p} = 1/(2M) \sum_{m=0}^{M-1} \{\text{Re}\{\hat{A}_{k,p}\}\}^2 +
|\{\text{Im}\{\hat{A}_{k,p}\}\}| \). With respect to \(\eta_{k}\), for the first iteration \(\overline{D}_k = 0\).
and we can use the method of [9] to obtain
\(E[|D_k|^2]\). For the remaining iterations we can assume that
\(E[|D_k|^2] \approx 2(1-\rho^3)|\{\text{Re}\{\hat{A}_{k,p}\}\}|\)
\) for \(\rho > 0.82\) (when \(\rho < 0.82\) the nonlinear
distortion estimates are not accurate enough, so we assume
\(\overline{D}_k = 0\).
Let us consider now the impact of channel coding. We adopted a 64-state, rate-1/2 convolutional code with generators \(1 + D^2 + D^3 + D^5 + D^6\) and \(1 + D + D^2 + D^3 + D^6\). Since the performance of IB-DFE receivers is poor at low SNR (the typical working region when suitable channel coding schemes are employed), the coded performances of IB-DFE receivers with estimation and cancelation of nonlinear distortion effects do not perform very well. This is not the case of Turbo FDE receivers, as shown in Fig. 4 (due to lack of space, we just present results for CP-assisted MC-CDM; a similar behavior was observed for ZP MC-CDM). Clearly, the estimation and compensation of nonlinear distortion effects allows significant improvements relatively to turbo receivers without compensation, especially when we have strong nonlinear distortion effects and at low BER.

V. CONCLUSIONS AND FINAL REMARKS

In this paper, we considered CP-assisted MC-CDM and ZP MC-CDM schemes employing clipping techniques for reducing the PMEPR of the transmitted signals. We proposed frequency-domain turbo equalization receivers with estimation and cancelation of nonlinear distortion effects.

Our performance results showed that the proposed turbo receivers allow significant performance improvements at low and moderate SNR, even when a low-PMEPR MC-CDM transmission is intended. The receivers for ZP MC-CDM is of special interest for systems where the duration of the channel impulse response is not a small fraction of the duration of the MC-CDM block, being suitable to MC-CDM systems with very large blocks.

REFERENCES

Fig. 1. Transmission model and conventional receiver for low-PMEPR MC-CDM signals (shaded blocks are only used for ZP MC-CDM).

Fig. 2. Receiver with estimation and cancelation of nonlinear distortion effects for CP-assisted MC-CDM.

Fig. 3. Uncoded BER performances for IB-DFE receivers with or without compensation of nonlinear distortion effects.

Fig. 4. Coded BER performances for CP-assisted MC-CDM and Turbo FDE receivers with or without compensation of nonlinear distortion effects (for the sake of comparisons, we also include the performance for linear transmitters).