The Theory of Message Sequence Charts

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TIFR, Mumbai, 28 April, 2009
An ATM

Customer

ATM

Bank

passwd

authen

correct

OK

funds?

no

amount

sorry
An ATM

Customer

ATM

Bank

passwd

authen

correct

OK

funds?

yes

cash

amount

yes
Message Sequence Charts

Two clients and a server
Overview

Message Sequence Charts
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Message Sequence Charts

- Visual formalism for specifying scenarios.
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- Part of the UML Standard
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- Used quite extensively, for instance in the telecom industry.
- Formalize these pictures as ...
- Formulate natural (and relevant) questions ...
MSCs

Two clients and a server
MSCs as Labelled Partial Orders

Two clients and a server, and a partial order representation.
MSCs as Labelled Partial Orders

Two clients and a server, and a partial order representation

Assume that all the channels are FIFO.
MSCs as Labelled Partial Orders

Two clients and a server, and a partial order representation

Assume that all the channels are FIFO.
Linearizations of an MSC
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c₁ \rightarrow \text{s} \rightarrow \text{c₂}

\begin{align*}
\text{c₁} \xrightarrow{r₁} \text{s} \xrightarrow{r₂} \text{c₂} \\
\text{c₁} \xrightarrow{g₁} \text{s} \xrightarrow{x₂} \text{c₂}
\end{align*}

\begin{align*}
c₂!s(r₂) & \quad c₁!s(r₁) \\
c₁!s(r₁) & \quad s?c₁(r₁) \\
c₁!s(r₁) & \quad s?c₁(r₁) \\
c₁!s(r₁) & \quad s?c₂(r₂) \\
s?c₂(r₂) & \quad c₂!s(r₂) \\
s!c₂(g₂) & \quad c₂?s(g₂) \\
s?c₂(x₂) & \quad c₂!s(x₂) \\
s?c₂(x₂) & \quad c₂!s(x₂) \\
s!c₁(g₁) & \quad c₁?s(g₁) \\
c₁?s(g₁) & \quad s?c₁(r₁)
\end{align*}
Linearizations of an MSC

c_1!s(r_1) c_2!s(r_2) s?c_1(r_1) s?c_2(r_2) s!c_2(g_2) c_2?s(g_2) c_2!s(x_2) s?c_2(x_2) s!c_1(g_1) c_1!s(r_1) c_1?s(g_1) s?c_1(r_1)
Properties of Linearizations

Let \( w \) be a linearization of an MSC. Then,
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- A message is received only after it is sent.

\[ \forall x \leq w. \forall p, q. \quad \#_p!q x \geq \#_q?p x \]
Properties of Linearizations

Let $w$ be a linearization of an MSC. Then,

- A message is received only after it is sent.

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- All sent messages are received.

\[ \forall p, q. \quad \#_p!q^w = \#_q?p^w \]
Properties of Linearizations

Let \( w \) be a linearization of an MSC. Then,

- A message is received only after it is sent.

\[
\forall x \leq w. \forall p, q. \#_p!q x \geq \#_q?p x
\]

- All sent messages are received.

\[
\forall p, q. \#_p!q w = \#_q?p w
\]

As a matter of fact, under the FIFO assumption, any word satisfying these two properties is the linearization of a unique MSC.
Linearizations to MSCs

\[ c_2!s(r_2) \quad c_1!s(r_1) \quad c_1!s(r_1) \quad s?c_1(r_1) \quad s?c_2(r_2) \quad s!c_2(g_2) \quad c_2?s(g_2) \quad c_2!s(x_2) \quad s?c_2(x_2) \quad s!c_1(g_1) \quad c_1?s(g_1) \quad s?c_1(r_1) \]
Linearizations to MSCs

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s?c_2(x_2) \ s!c_1(g_1) \ c_1?s(g_1) \ s?c_1(r_1)
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Linearizations to MSCs

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Linearizations to MSCs

$c_2!s(r_2) \ c_1!s(r_1) \ c_1!s(r_1) \ \ ?c_1(r_1) \ \ ?c_2(r_2) \ \ !c_2(g_2) \ c_2?s(g_2) \ c_2!s(x_2) \ \ ?c_2(x_2) \ \ !c_1(g_1) \ c_1?s(g_1) \ \ ?c_1(r_1)$
Linearizations to MSCs

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Linearizations to MSCs

\[ c_2!s(r_2) \; c_1!s(r_1) \; c_1!s(r_1) \; s?c_1(r_1) \; s?c_2(r_2) \; s!c_2(g_2) \; c_2?s(g_2) \; c_2!s(x_2) \; s?c_2(x_2) \; s!c_1(g_1) \; c_1?s(g_1) \; s?c_1(r_1) \]
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\]

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Languages of MSCs

A language of MSCs is a set of MSCs (over a fixed set of processes and message alphabet).
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How do we describe MSC languages?
Concatenation of MSCs

$A_1 \circ A_2$

$p \quad q \quad r$

$A_1$

$A_2$

$A_1 \circ A_2$

$p \quad q \quad r$
Every event from $A_2$ in any process $p$ occurs after all the events from $A_1$ in the process $p$ have already occurred.
Concatenation of MSCs

Every event from $A_2$ in any process $p$ occurs after all the events from $A_1$ in the process $p$ have already occurred.

$$p!r \ p!q \ q?p \ q!r \ r?q \ p!q \ q?p \ r?p$$

is a linearization of $A_1 \circ A_2$. 
Message Sequence Graphs

- A finite state automaton
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- They are used to *specify* the desired (or undesired behaviours) of a system.
- They are global descriptions and do not give any guarantee about implementability.
Boundedness

A linearization of an MSC is $B$-bounded if no channel has more than $B$ messages at any point.
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The linearization

$$p!q \ q?q \ p!r \ p!q \ q?q \ p!q \ q?q \ r?q \ p!q \ q?q \ p!r \ q?q \ q?q \ p!r \ q?q \ r?q \ r?p$$

is 1-bounded while the linearization

$$p!q \ p!r \ p!q \ p!q \ q?p \ q?p \ p!r \ q?q \ q?p \ r?q \ r?p$$

is 3-bounded.
Boundedness ...

An MSC is existentially $B$-bounded if one of its linearizations is $B$-bounded.
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*The execution of the MSC can be scheduled in such a way that buffers are all $B$-bounded.*

An MSC is universally $B$-bounded if all of its linearizations are $B$-bounded.

An existentially 1-bounded and universally 3-bounded MSC.
An MSG is existentially $B$-bounded if every MSC it generates is existentially $B$-bounded.
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Boundedness ...

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![Diagram](image)

An MSG is **universally** $B$-bounded if every MSC it generates is universally $B$-bounded.
Boundedness ...

An MSG is **existentially bounded** if there exists a $B$ such that every MSC it generates is existentially $B$-bounded.

An MSG is **universally bounded** if there exists a $B$ such that every MSC it generates is $B$-bounded.
Deciding Boundedness

Can we check whether the language of an MSG is implementable with bounded buffers?
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- Checking whether an MSG is existentially $B$-bounded for a given $B$ is decidable.
Deciding Boundedness

Can we check whether the language of an MSG is implementable with bounded buffers?

- Every MSG is existentially $B$-bounded for some $B$.
- Checking whether an MSG is existentially $B$-bounded for a given $B$ is decidable.
- Checking whether an MSG is universally bounded ($B$-bounded for a given $B$) is decidable.
Communication graph of an MSC

Nodes are the processes. An edge from \( p \) to \( q \) if there is a message from \( p \) to \( q \).
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An MSG is **locally strongly connected** if and only if the MSC generated by any loop has a communication graph that is the disjoint union of SCCs.
Suppose \( p \) and \( q \) lie in the same SCC in the communication graph of \( M \) and further suppose that this SCC has \( k \) processes.
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In \( M^k \), the first \( q \) event guaranteed to be below the maximum event of \( p \).
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In \( M^{k+1} \), the first \( q?p \) event corresponding to the first \( p!q \) event (if any) is guaranteed to be below the maximum event of \( p \).

*The first message from \( p \) to \( q \) is acknowledged within \( M^{k+1} \).*
SCCs and boundedness

- Suppose $p$ and $q$ lie in the same SCC in the communication graph of $M$ and further suppose that this SCC has $k$ processes.

- In $M^k$, the first $q$ event guaranteed to be below the maximum event of $p$.

- In $M^{k+1}$, the first $q?p$ event corresponding to the first $p!q$ event (if any) is guaranteed to be below the maximum event of $p$.

  The first message from $p$ to $q$ is acknowledged within $M^{k+1}$.

- The channel from $p$ to $q$ is bounded by the size of $M^{k+1}$.
Regularity of linearizations

When do we say that a language of MSCs is regular?
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A language of MSCs is said to be regular if its set of linearizations forms a regular (word) language.
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Other definitions are possible ...
Every regular language of MSCs is universally bounded.
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Let $A$ be a finite automaton for the linearizations with $s$ as the initial state. If $w$ and $v$ lead to the same state $q$ from $s$ and $u$ is word that runs from $q$ to some final state then ...
Boundedness and Regularity

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What about the converse?
Iteration of concurrent events
The only constraint on relating the $p!q(m)$ events and the $r!s(m)$ is that they must be equal in number.
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Thus even bounded MSGs can exhibit nonregular behaviour.
Can we check whether a MSG describes a regular language or not?
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Answer: NO.
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The proof follows from a similar result for Mazurkiewicz Traces due to Sakarovich.
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What about sufficient conditions for regularity?
An MSC is said to be *locally synchronized* if its communication graph has a single nontrivial SCC.
Locally synchronized MSGs

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An MSG is said to be **locally synchronized** if every closed walk in the MSG generates a locally synchronized MSC.
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An MSG is said to be **locally synchronized** if every closed walk in the MSG generates a locally synchronized MSC.

**Theorem:** Every locally synchronized MSG generates a regular MSC language.
Every linearization of the MSCs generated must be exhibited.

\[ e_5(e_1e_2e_3e_4) * e_6 \]
Locally synchronized MSGs ...

- Every linearization of the MSCs generated must be exhibited.

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- How many and which nodes of the path in the MSG do we need to carry with us to generate these sequentializations?
Locally synchronized MSGs ...

- Every linearization of the MSCs generated must be exhibited.

\[ e_5(e_1e_2e_3e_4)^*e_6 \]

- How many and which nodes of the path in the MSG do we need to carry with us to generate these sequentializations?
- Nodes have to inserted in the middle, deleted from the middle.
Locally synchronized MSGs and regularity

There are regular MSC languages that are not generated by MSGs.
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- The language of an MSG is constructed from a finite set of basic building blocks (*atoms*).
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- There are regular MSC languages which need infinitely many basic blocks. (eg.)

\[ p!r \ p!q \ q?p \ (q!r \ r?q \ r!q \ q?q?)^* \ r?q \]
Locally synchronized MSGs and regularity

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- There are regular MSC languages which need infinitely many basic blocks. (eg.)

\[
p!r \ p!q \ q?p \ (q!r \ r?q \ r!q \ q?q)\ast \ r?p
\]

**Theorem:** Every finitely generated MSC regular language is the language of a locally synchronized MSG.
Message-passing Automata

1. An implementation model for MSCs. A bunch of finite automata communicating via buffered channels.
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2. A distributed model of computation.
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An Example: The producer–consumer system.
Message-passing Automata ...
Message-passing Automata ...

MPAs can accept languages that cannot be generated by MSGs.
When are MSG languages implementable using MPAs?
MSGs to MPAs

When are MSG languages implementable using MPAs?

- MPA languages are product languages. (i.e.) if $M$ is such that for all $p \in P$ there is an $M_p \in L$ such that $M \downarrow P = M_p \downarrow P$ then $M \in L$. 
When are MSG languages implementable using MPAs?

- MPA languages are product languages. (i.e.) if \( M \) is such that for all \( p \in P \) there is an \( M_p \in L \) such that \( M \downarrow P = M_p \downarrow P \) then \( M \in L \).

- An \( M \) of the form exhibited above, is said to be implied by \( L \).
When are MSG languages implementable using MPAs?

- MPA languages are **product languages**. (i.e.) if $M$ is such that for all $p \in P$ there is an $M_p \in L$ such that $M \downarrow P = M_p \downarrow P$ then $M \in L$.

- An $M$ of the form exhibited above, is said to be **implied** by $L$.

- An MSG language is an MPA language if and only if every MSC implied by its language also belongs to its language (it is **implied closed**).
Implied closure

$p \quad q \quad r \quad s \quad p \quad q \quad r \quad s$

$M_1 \quad \quad \quad M_2$

$m \quad m \quad m \quad m$
Implied closure

◮ p and q believe $M$ is $M_1$
◮ r and s believe $M$ is $M_2$
Implied closure

$p$ and $q$ believe $M$ is $M_1$

$r$ and $s$ believe $M$ is $M_2$

$M$ is in the implied closure of $\{M_1, M_2\}$. 
Implied closure
Implied closure
Implied closure

\[ \begin{array}{cccc}
p & q & r & s \\
\end{array} \]

\[ \begin{array}{cccc}
M \\
\end{array} \]

\[ \begin{array}{cccc}
p & q & r & s \\
\end{array} \]

\[ \begin{array}{cccc}
M' &  \\
\end{array} \]
Implied closure

\[ M \]

\[ M' \]

\[ p \quad q \quad r \quad s \]

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Implied closure
Implied closure

Confusing $M^{2k}M'^k$ and $M'^kM^{2k}$ generates up to $k$ messages in $p \rightarrow s$ channel.
Implied closure

Confusing $M^{2k}M'^k$ and $M'^kM^{2k}$ generates upto $k$ messages in $p \rightarrow s$ channel

**Theorem:** Checking whether the language of an MSG is implied closed is undecidable. It is undecidable even for locally synchronized MSGs.
Implied closure

Confusing $M^{2k}M'^k$ and $M'^kM^{2k}$ generates up to $k$ messages in $p \rightarrow s$ channel

Theorem: Checking whether the language of an MSG is implied closed is undecidable. It is undecidable even for locally synchronized MSGs.

A weaker notion of implementability yields interesting positive results.
Decision problems for MPAs

- Checking emptiness for MPAs is undecidable.
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- Checking whether the language of an MPA is $B$-bounded is undecidable.
Checking emptiness for MPAs is undecidable.

Checking whether the language of an MPA is $B$-bounded is undecidable.

Checking whether the language of an MPA is regular is undecidable.
The Model-checking problem

- **Positive Model-checking** Given a specification language $S$ and an implementation $L$ decide if $S \subseteq L$. 
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  *Are all the positive instances exhibited?*

- **Negative Model-checking** Given a specification language \( S \) and an implementation \( L \) decide whether \( S \cap L = \emptyset \).
The Model-checking problem

- **Positive Model-checking** Given a specification language $S$ and an implementation $L$ decide if $S \subseteq L$.
  
  Are all the positive instances exhibited?

- **Negative Model-checking** Given a specification language $S$ and an implementation $L$ decide whether $S \cap L = \emptyset$.
  
  Are all the negative instances avoided?
The Model-checking problem ...
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- If $S$ and $L$ are given as locally synchronized MSGs, both the model checking problems are decidable.
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- If $S$ and $L$ are given as locally synchronized MSGs, both the model checking problems are decidable.

- If $S$ is given by a locally synchronized MSG and $L$ is given by any MSG, both the model checking problems are decidable.
The Model-checking problem ...

- If $S$ and $L$ are given as locally synchronized MSGs, both the model checking problems are decidable.

- If $S$ is given by a locally synchronized MSG and $L$ is given by any MSG, both the model checking problems are decidable.
  1. Replace each node in the MSG with a linearization.
  2. Let $X$ be the regular language accepted by the resulting finite automaton.
  3. $L \subseteq S$ if and only if $X \subseteq S$ and $L \cap S = \emptyset$ if and only if $X \cap S = \emptyset$.

- These results can be generalized further ...
The Model-checking problem ...

- If $S$ and $L$ are given as locally synchronized MSGs, both the model checking problems are decidable.

- If $S$ is given by a locally synchronized MSG and $L$ is given by any MSG, both the model checking problems are decidable.
  1. Replace each node in the MSG with a linearization.
  2. Let $X$ be the regular language accepted by the resulting finite automaton.
  3. $L \subseteq S$ if and only if $X \subseteq S$ and $L \cap S = \emptyset$ if and only if $X \cap S = \emptyset$.

- These results can be generalized further ...
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  A regular language $R$ such that the set of MSCs generated by the words in $R$ is $L$.

- Given $B$, we can effectively construct $Lin^B(L)$ consisting of all the $B$ bounded linearizations of MSCs in $L$. 

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[One of the many results best proved via a translation to Mazurkiewicz traces.]
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Model checking MSGs w.r.t. GC-MSG specifications is decidable.
In the next lecture...

- A distributed synthesis theorem w.r.t. a weaker notion of implementability.
- Generalizing the decidability of model-checking beyond GC-MSGs.
- Generalizing the distributed synthesis theorem.
- Monadic Second order Logic (MSO) over MSCs and its relationship to regularity and MPAs.
- Extending MSCs with time.