Some Properties of Fuzzy Dot PS-Subalgebras of PS-Algebra

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Received 9 April 2014; accepted 20 April 2014

Abstract. In this paper, we introduce the notion of fuzzy dot PS-subalgebras in PS-algebras and establish its various properties.

Keywords: Fuzzy dot PS-subalgebras, PS-algebra

AMS Mathematics subject classification (2010): 06F35, 03G25, 03E72

1. Introduction
The concept of fuzzy set was initiated by L.A.Zadeh in 1965 [11]. Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. K.Iseki and S.Tanaka [1] introduced the concept of BCK-algebras in 1978 and K.Iseki [2] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. T.Priya and T.Ramachandran [5,6,7] introduced the class of PS-algebras, which is an another generalization of BCI / BCK / KU algebras. In this paper, we introduce the concept of fuzzy dot PS-subalgebras of PS-algebras as a generalization of a fuzzy PS-subalgebra of a PS-algebra and we investigate few basic properties related to fuzzy dot PS-subalgebra in detail.

2. Preliminaries
In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1 [1] A BCK-algebra is an algebra $(X, *, 0)$ of type (2,0) satisfying the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$
ii) $x * (x * y) \leq y$
iii) $x \leq x$
iv) $x \leq y$ and $y \leq x \Rightarrow x = y$
v) $0 \leq x \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$. 

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Definition 2.2. [2] A BCI-algebra is an algebra \((X, *, 0)\) of type \((2,0)\) satisfying the following conditions:

i) \((x * y) * (x * z) \leq (z * y)\)

ii) \(x * (x * y) \leq y\)

iii) \(x \leq x\)

iv) \(x \leq y\) and \(y \leq x \Rightarrow x = y\)

v) \(x \leq 0 \Rightarrow x = 0\), where \(x \leq y\) is defined by \(x * y = 0\), for all \(x, y, z \in X\).

Definition 2.3. A Q-algebra is an algebra \((X, *, 0)\) of type \((2,0)\) satisfying the following conditions:

i) \(x * x = 0\)

ii) \(x * 0 = x\)

iii) \((x * y) * z = (x * z) * y\), where \(x \leq y\) is defined by \(x * y = 0\), for all \(x, y, z \in X\).

Definition 2.4. [3] A d-algebra is an algebra \((X, *, 0)\) of type \((2,0)\) satisfying the following conditions:

i) \(x * x = 0\)

ii) \(0 * x = 0\)

iii) \(x * y = 0\) and \(y * x = 0\) imply \(x = y\), for all \(x, y \in X\).

Definition 2.5. [4,10] A KU-algebra is an algebra \((X, *, 0)\) of type \((2,0)\) satisfying the following conditions:

i) \((x * y) * ((y * z) * (x * z)) = 0\)

ii) \(x * 0 = 0\)

iii) \(0 * x = x\)

iv) \(x * y = 0\) and \(y * x = 0\) imply \(x = y\), for all \(x, y, z \in X\).

Definition 2.6. [5] A nonempty set \(X\) with a constant 0 and a binary operation ‘ \(*\) ‘ is called PS-algebra if it satisfies the following axioms.

1. \(x * x = 0\)
2. \(x * 0 = 0\)
3. \(x * y = 0\) and \(y * x = 0 \Rightarrow x = y\), \(\forall x, y \in X\).

Definition 2.7. [8] Let \(S\) be a non empty sub set of a PS-algebra \(X\). Then \(S\) is called a subalgebra of \(X\) if \(x*y \in S\), for all \(x, y \in S\).

Definition 2.8. [6,9] A map \(f : X \rightarrow Y\) is called a homomorphism if \(f(x * y) = f(x) * f(y)\), for all \(x, y \in X\), where \(X\) and \(Y\) are PS-algebras.

Definition 2.9. [11] Let \(X\) be a non-empty set. A fuzzy subset \(\mu\) of the set \(X\) is a mapping \(\mu : X \rightarrow [0,1]\).
Definition 2.10. [11] For any subsets $\lambda$ and $\mu$ of a set $X$, $(\lambda \cap \mu)(x) = \min \{\lambda(x), \mu(x)\}$.

Definition 2.11. [3,11] A fuzzy relation $\mu$ on a set $X$ is of a fuzzy subset of $X \times X$, that is, a map $\mu : X \times X \to [0,1]$.

Definition 2.12. [6] A fuzzy set $\mu$ in a PS-algebra $X$ is called a fuzzy PS-subalgebra of $X$ if $\mu(x \ast y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Remark:
(i) For any fuzzy subsets $\lambda$ and $\mu$ of a set $X$, we define $\lambda \subseteq \mu \iff \lambda(x) \leq \mu(x)$.
(ii) Let $f : X \to Y$ be a function from a set $X$ to a set $Y$ and let $\mu$ be a fuzzy subset of $X$. Then the fuzzy subset $\lambda$ of $Y$ is defined by

$$
\lambda(y) = \begin{cases} 
\sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset, \forall y \in Y \\
0 & \text{Otherwise}
\end{cases}
$$

is called the image of $\mu$ under $f$, denoted by $f(\mu)$. If $\lambda$ is a fuzzy subset of $Y$, then the fuzzy subset $\mu$ of $X$ is given by $\mu(x) = \lambda(f(x))$, for all $x \in X$, is called the pre-image of $\lambda$ under $f$ and is denoted by $f^{-1}(\lambda)$.

4. Fuzzy Dot PS-Subalgebras of PS-algebras

For brevity, here $X$ denotes PS-algebra, unless otherwise specified.

Definition 3.1. A fuzzy subset $\mu$ of $X$ is called a fuzzy dot PS-subalgebra of a PS-algebra $X$, if $\mu(x \ast y) \geq \mu(x)$, $\mu(y)$, for all $x, y \in X$.

Example 3.2. Consider a PS-algebra $X = \{0, a, b\}$ having the following Cayley table.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>b</td>
<td>0</td>
</tr>
</tbody>
</table>

Define a fuzzy set $\mu$ in $X$ by $\mu(0) = 0.8$, $\mu(a) = 0.7$. It is easy to verify that $\mu$ is a fuzzy dot PS-subalgebra of a PS-algebra $X$.

Example 3.3. Consider a PS-algebra $X = \{0,1,2,3\}$ having the following Cayley table.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
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<td>2</td>
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<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
Define a fuzzy set \( \mu \) in \( X \) by \( \mu(0) = 0.8 \), \( \mu(1) = 0.5 \), \( \mu(2) = 0.4 \) and \( \mu(3) = 0.7 \). It is easy to verify that \( \mu \) is a fuzzy dot PS-subalgebra of a PS-algebra \( X \).

**Remark**:
1. Every fuzzy PS-subalgebra is a fuzzy dot PS-subalgebra of a PS-algebra but the converse is not true.
2. From the above example 3.3, it is seen that, the fuzzy dot PS-subalgebra \( \mu \) is not a fuzzy PS-subalgebra, because \( \mu(1*3) = \mu(2) = 0.4 < \mu(1) = \min \{ \mu(1), \mu(3) \} \).

**Theorem 3.4.** If \( \lambda \) and \( \mu \) are fuzzy dot PS-subalgebras of a PS-algebra \( X \), then \( \lambda \cap \mu \) is also a fuzzy dot PS-subalgebra of \( X \).

**Proof:** Let \( x, y \in X \). Then
\[
(\lambda \cap \mu)(x * y) = \min \{ \lambda(x) \cdot \lambda(y), \mu(x), \mu(y) \}
\]
\[
\geq \min \{ \lambda(x), \lambda(y), \mu(x), \mu(y) \}
\]
\[
\geq (\min \{ \lambda(x), \mu(x) \}, \min \{ \lambda(y), \mu(y) \})
\]
\[
= (\min \{ \lambda \cap \mu \}(x), \min \{ \lambda \cap \mu \}(y))
\]
Thus \( (\lambda \cap \mu) \) is also a fuzzy dot PS-subalgebra of \( X \).

**Theorem 3.5.** If \( \mu \) is a fuzzy dot PS-subalgebra of a PS-algebra \( X \), then \( \mu(0) \geq (\mu(x))^3 \), \( \forall x \in X \).

**Proof:** For every \( x \in X \), we have \( \mu(0) = \mu(x * 0) \)
\[
\geq \mu(x), \mu(0)
\]
\[
= \mu(x), \mu(x * x)
\]
\[
\geq \mu(x), \mu(x), \mu(x)
\]
\[
= (\mu(x))^3, \text{ which completes the proof.}
\]

**Definition 3.6.** The characteristic function of a non-empty subset \( A \) of a PS-algebra \( X \), denoted by \( \chi_A \), is defined by
\[
\chi_A(x) = \begin{cases} 
1 & , x \in A \\
0 & , x \notin A 
\end{cases}
\]

**Theorem 3.7.** Let \( A \subseteq X \). Then \( A \) is a subalgebra of a PS-algebra \( X \) if and only if \( \chi_A \) is a fuzzy dot PS-subalgebra of a PS-algebra \( X \).

**Proof:** Let \( x, y \in A \). Then \( x*y \in A \). Hence we get \( \chi_A(x*y) = 1 \geq \chi_A(x), \chi_A(y) \).
If \( x \in A \) and \( y \notin A \) (or \( x \notin A \) and \( y \in A \)), then we get \( \chi_A(x*y) \geq \chi_A(x), \chi_A(y) = 1.0 = 0 \), since \( \chi_A(x) = 1 \) and \( \chi_A(y) = 0 \).
Thus \( \chi_A \) is a fuzzy dot PS-subalgebra of a PS-algebra \( X \).
Conversely, assume that \( \chi_A \) is a fuzzy dot PS-subalgebra of a PS-algebra \( X \).
Now let \( x, y \in A \). Then \( \chi_A(x*y) \geq \chi_A(x), \chi_A(y) = 1.1 = 1 \), hence by definition, we have \( x*y \in A \), which completes the proof.

**Remark:**
Some Properties of Fuzzy Dot PS-Subalgebras of PS-algebras

1. A fuzzy subset $\mu$ of $X$ is a fuzzy subalgebra of PS-algebra $X$ if the level subset $\mu^t = \{x \in X \mid \mu(x) \geq t\}$, the upper level subset of $\mu$, is a subalgebra of $X$, for every $t \in [0, 1]$.

2. But from example 3.2, it is clear that, if $\mu$ is the fuzzy dot PS-subalgebra of $X$, then there exists $t \in [0, 1]$ such that $\mu^t = \{x \in X \mid \mu(x) \geq t\}$ is not a subalgebra of $X$. $\mu$ is a fuzzy dot PS-subalgebra of $X$ in example 3.2, then consider $\mu^{0.5} = \{x \in X \mid \mu(x) \geq 0.5\} = \{0, 1\}$, is not a subalgebra of $X$, since $1 \ast 3 = 2 \notin \mu^{0.5}$.

**Theorem 3.8.** Let $f: X \rightarrow Y$ be a homomorphism of a PS-algebra $X$ into a PS-algebra $Y$. If $\mu$ is a fuzzy dot PS-subalgebra of $Y$, then the pre-image of $\mu$ denoted by $f^{-1}(\mu)$, defined as $\{f^{-1}(\mu)\}(x) = \mu(f(x))$, $\forall x \in X$, is a fuzzy dot PS-subalgebra of $Y$.

**Proof:** Let $\mu$ be a fuzzy dot PS-subalgebra of $Y$. Let $x, y \in X$.

Now, $\{f^{-1}(\mu)\}(x \ast y) = \mu(f(x \ast y))$

$\geq \mu(f(x)) \ast \mu(f(y))$

$= \{f^{-1}(\mu)\}(x) \ast \{f^{-1}(\mu)\}(y)$

Therefore, $f^{-1}(\mu)$ is a fuzzy dot PS-subalgebra of $X$.

**Theorem 3.9.** Let $f: X \rightarrow Y$ be an onto homomorphism of PS-algebras. If $\mu$ is a fuzzy dot PS-subalgebra of $X$, then the image $f(\mu)$ of $\mu$ under $f$ is a fuzzy dot PS-subalgebra of $Y$.

**Proof:** For any $y_1, y_2 \in Y$, let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1 \ast y_2)$.

Consider the set $A_1 \ast A_2 = \{x \in X \mid x = a_1 \ast a_2 \text{ for some } a_1 \in A_1 \text{ and } a_2 \in A_2\}$

If $x \in A_1 \ast A_2$, then $x = x_1 \ast x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$ so that

$f(x) = f(x_1 \ast x_2) = f(x_1) \ast f(x_2) = y_1 \ast y_2$.

that is, $x \in f^{-1}(y_1 \ast y_2) = A_{12}$. Hence $A_1 \ast A_2 \subseteq A_{12}$. It follows that

$f[\mu](y_1 \ast y_2) = \sup_{x \in f^{-1}(y_1 \ast y_2)} \sup_{x \in A_{12}} \mu(x)$

$= \sup_{x \in A_1 \ast A_2} \sup_{x \in A_1} \mu(x_1) \ast \mu(x_2)$

$\geq \sup_{x \in A_1 \ast A_2} \sup_{x \in A_1} \mu(x_1) \ast \sup_{x \in A_2} \mu(x_2)$

since $\sup \mu(x) = \sup_{x \in A_1} \mu(x_1)$ is continuous, for every $\delta > 0$ there exists $\delta > 0$ such that if

$x_1 \in A_1 \setminus A_2$, $x_2 \in A_2 \setminus A_1$, then

$
\mu(x_1 \ast x_2) \geq \sup_{x_1 \in A_1 \setminus A_2} \mu(x_1) \ast \mu(x_2) \geq \sup_{x_1 \in A_1} \sup_{x_2 \in A_2} \mu(x_1 \ast x_2) \geq \sup_{x_1 \in A_1} \sup_{x_2 \in A_2} \mu(x_1) \ast \mu(x_2) = \mu(x_1 \ast x_2).$

Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that

$\mu(a_1) \geq \sup_{x_1 \in A_1} \mu(x_1) \ast \mu(a_2) \geq \sup_{x_2 \in A_2} \mu(x_2)$

$\mu(a_1) \ast \mu(a_2) \geq \sup_{x_1 \in A_1} \mu(x_1) \ast \sup_{x_2 \in A_2} \mu(x_2) = \mu(a_1 \ast a_2)$.

Consequently, $f[\mu](y_1 \ast y_2) \geq \sup \mu(x_1) \ast \mu(x_2)$

Hence $f(\mu)$ is a fuzzy dot PS-subalgebra of $Y$. 

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Theorem 3.10. Let $f : X \to X$ be an endomorphism on a PS-algebra $X$. If $\mu$ be a fuzzy dot PS-subalgebra of $X$. Define a fuzzy set $\mu_f : X \to [0,1]$ by $\mu_f(x) = \mu(f(x))$, $\forall x \in X$. Then $\mu_f$ is a fuzzy dot PS-subalgebra of $X$.

Proof: Let $\mu$ be fuzzy dot PS-subalgebra of $X$. Let $x, y \in X$.

Now, $\mu_f(x * y) = \mu(f(x) * f(y))$

\[
\geq \mu_f(x) \cdot \mu_f(y)
\]

$\Rightarrow \mu_f$ is a fuzzy dot PS-subalgebra of $X$.

Definition 3.11. Let $\lambda$ and $\mu$ be two fuzzy sets in a set $X$. The Cartesian Product $\lambda \times \mu : X \times X \to [0,1]$ is defined by $(\lambda \times \mu)(x,y) = \lambda(x) \cdot \mu(y)$.

Theorem 3.12. If $\lambda$ and $\mu$ are fuzzy dot PS-subalgebras of a PS-algebra $X$, then $\lambda \times \mu$ is also a fuzzy dot PS-subalgebra of $X \times X$.

Proof: For any $x_1, x_2, y_1, y_2 \in X$.

Then $(\lambda \times \mu)((x_1, y_1) \times (x_2, y_2)) = (\lambda \times \mu)(x_1 \times x_2, y_1 \times y_2)$

\[
= \lambda(x_1 \times x_2) \cdot \mu(y_1 \times y_2)
\]

\[
\geq (\lambda(x_1) \cdot \lambda(x_2)) \cdot (\mu(y_1) \cdot \mu(y_2))
\]

\[
= (\lambda \times \mu)(x_1, y_1) \cdot (\lambda \times \mu)(x_2, y_2)
\]

This completes the proof.

Definition 3.13. Let $\beta$ be a fuzzy subset of $X$. The strongest fuzzy $\beta$-relation on PS-algebra $X$ is the fuzzy subset $\mu_\beta$ of $X \times X$ given by $\mu_\beta(x,y) = \beta(x) \cdot \beta(y)$, for all $x, y \in X$.

Theorem 3.14. Let $\mu_\beta$ be the strongest fuzzy $\beta$-relation on PS-algebra $X$, where $\beta$ is a fuzzy subset of a PS-algebra $X$. If $\beta$ is a fuzzy dot PS-subalgebra of a PS-algebra $X$, then $\mu_\beta$ is a fuzzy dot PS-subalgebra of $X \times X$.

Proof: Let $\beta$ be a fuzzy dot PS-subalgebra of a PS-algebra $X$ and let $x_1, x_2, y_1, y_2 \in X$.

Then $\mu_\beta((x_1, y_1) \times (x_2, y_2)) = \beta(x_1 \times x_2, y_1 \times y_2)$

\[
= \beta(x_1 \times x_2) \cdot \beta(y_1 \times y_2)
\]

\[
\geq (\beta(x_1) \cdot \beta(x_2)) \cdot (\beta(y_1) \cdot \beta(y_2))
\]

\[
= (\beta(x_1) \cdot \beta(y_1)) \cdot (\beta(x_2) \cdot \beta(y_2))
\]

\[
= \mu_\beta(x_1, y_1) \cdot \mu_\beta(x_2, y_2)
\]

This completes the proof.

Theorem 3.15. Let $\mu_\beta$ be the strongest fuzzy $\beta$-relation on PS-algebra $X$, where $\beta$ is a fuzzy subset of a PS-algebra $X$. If $\mu_\beta$ is a fuzzy dot PS-subalgebra of $X \times X$, then $\beta$ is a fuzzy dot PS-subalgebra of a PS-algebra $X$.

Proof: Let $x, y \in X$.

Now, $(\beta(x \times y))^2 = \beta(x \times y) \cdot \beta(x \times y)$

\[
= \mu_\beta(x \times y) \cdot (x \times y)
\]
Some Properties of Fuzzy Dot PS-Subalgebras of PS-algebras

\[ \beta(x * y) \geq \mu_\beta(x,y) \cdot \mu_\beta(x,y) \]

\[ \Rightarrow \beta(x * y) \geq \beta(x) \cdot \beta(y), \text{ which completes the proof.} \]

**Definition 3.16.** Let $\beta$ be a fuzzy subset of a PS-algebra $X$. A fuzzy relation $\mu$ on PS-algebra $X$ is called a fuzzy $\beta$-product relation if $\mu(x,y) \geq \beta(x) \cdot \beta(y)$, for all $x, y \in X$.

**Definition 3.17.** Let $\beta$ be a fuzzy subset of a PS-algebra $X$. A fuzzy relation $\mu$ on PS-algebra $X$ is called a right fuzzy relation on $\beta$-product if $\mu(x,y) = \beta(y)$, for all $x, y \in X$.

**Remark:**
1. Let $\beta$ be a fuzzy subset of a PS-algebra $X$. A fuzzy relation $\mu$ on PS-algebra $X$ is called a left fuzzy relation on $\beta$-product if $\mu(x,y) = \beta(x)$, for all $x, y \in X$.
2. Left (right) fuzzy relation on $\beta$ is a fuzzy $\beta$-product relation.

**Theorem 3.18.** Let $\mu$ be a right fuzzy relation on a fuzzy subset $\beta$ of a PS-algebra $X$. If $\mu$ is a fuzzy dot PS-subalgebra of $X \times X$, then $\beta$ is fuzzy dot PS-subalgebra of a PS algebra $X$.

**Proof:**

\[ \beta(y_1 * y_2) = \mu((x_1 * x_2, y_1 * y_2)) \]
\[ = \mu((x_1,y_1),(x_2,y_2)) \]
\[ \geq \mu(x_1,y_1) \cdot \mu(x_2,y_2) \]
\[ = \beta(y_1) \cdot \beta(y_2), \text{ for all } x_1, x_2, y_1, y_2 \in X. \]

Hence, $\beta$ is a fuzzy dot PS-subalgebra of a PS-algebra.

**Theorem 3.19.** Let $X$ and $Y$ be PS-algebras. Let $\mu$ be a fuzzy dot PS-subalgebra of $X \times X$. Define a fuzzy set $\beta_x(\mu)$ of $X$ such that $\beta_x(\mu)(x) = \mu(x,0)$, $\forall x \in X$. Then $\beta_x(\mu)$ is a fuzzy dot PS-subalgebra of $X$.

**Proof:**

\[ \beta_x(\mu)(x * y) = \mu((x * y),(0 * 0)) \]
\[ = \mu((x,y),(0,0)) \]
\[ \geq \mu(x,0) \cdot \mu(y,0) \]
\[ = \beta_x(\mu)(x) \cdot \beta_x(\mu)(y) \]
\[ \therefore \beta_x(\mu) \text{ is a fuzzy dot PS-subalgebra of } X. \]

**Theorem 3.20.** Let $X$ and $Y$ be PS-algebras. Let $\mu$ be a fuzzy dot PS-subalgebra of $X \times Y$. Define a fuzzy set $\beta_y(\mu)$ of $Y$ such that $\beta_y(\mu)(y) = \mu(0,y)$, $\forall y \in Y$. Then $\beta_y(\mu)$ is a fuzzy dot PS-subalgebra of $Y$.

**Proof:**

\[ \beta_y(\mu)(x * y) = \mu((0,x),(0,y)) \]
\[ = \mu((0,0),(x,y)) \]
\[ = \mu((0,x),(0,y)) \]
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\[ \geq \mu (0,x) \cdot \mu (0,y) = \beta_y (\mu) (x) \cdot \beta_y (\mu) (y) \]

\[ \therefore \beta_y (\mu) \text{ is a fuzzy dot PS-subalgebra of } Y. \]

4. Conclusion
In this article authors have been discussed fuzzy dot PS-subalgebra in fuzzy PS-algebra. The relationship between fuzzy dot PS-subalgebra and fuzzy subalgebra also established. It has been observed that PS-algebra as a generalization of BCK/BCI/Q/d/TM/KU-algebras. This concept can further be generalized to Intuitionistic fuzzy sets, interval valued fuzzy sets, Anti fuzzy sets for new results in our future work.

Acknowledgement
Authors wish to thank Dr. K.T.Nagalakshmi, Professor and Head, Department of Mathematics, K L N College of Information and Technology, Pottapalayam, Sivagangai District, Tamilnadu, India, Prof. P.M.Sithar Selvam, Department of Mathematics, PSNA College of Engineering and Technology, Dindigul, Tamilnadu, India, for their help to make this paper as successful one.

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