Message-Optimal Connected Dominating Sets in Mobile Ad Hoc network

Khaled M. Alzoubi
Peng-Jun Wan
Ophir Frieder

Presented By Yiwei Wu
Abstract

- It is NP-hard to find a minimum connected dominating set (MCDS).
- New distributed heuristics for constructing a CDS were developed, with constant approximation ratio of 8, but these heuristics are based on construction of a spanning tree, which make it very costly in terms of communication overhead to maintain the CDS in the case of mobility and topology changes.
- In this paper, the author propose the first distributed and fully localized approximation algorithm to construct a MCDS with a constant approximation ratio, and linear time and linear message complexity.
1. Introduction

- The author assume that each node has a unique ID, and each node know the IDs of all its neighbors.
- Scheduling of transmission is the responsibility of the MAC layer.
- The topology of a wireless ad hoc network can be modeled as a *unit-disk graph*. 
1. Introduction – cont.

- A **connected dominating set** (CDS) for a graph $G=(V,E)$ is a subset $V'$ of $V$ such that each node in $V-V'$ is adjacent to some node in $V'$, and $V'$ induces a connected subgraph.

- A **maximal independent set** (MIS) is an independent set such that adding any other node to the set makes the set no longer independent. -- is DS

- Virtual backbone can be formed by nodes in CDS
1. Introduction – cont.

- Construction of the CDS consists of two localized phases:
  1. Construction of the MIS. (MIS nodes are referred to as dominators)
  2. Each dominator is connected to all dominators within three-hop distance by connectors.
  3. Both of the dominator and connector nodes form the CDS
2. Virtual Backbone

- Virtual backbone $U$ is a union of two subsets of nodes $S$ and $C$
- The nodes in $S$ form a maximal independent set
- The nodes in $C$ are connectors that connect $S$. 
2. Virtual backbone – cont.

- Lemma 1 Let $S$ be any MIS of the unit-disk graph $G$ and $u$ be an arbitrary node in $S$.
  1. The number of nodes in $S$ that are exactly two hops away from $u$ is at most 23.
  2. The number of nodes in $S$ that are at most three hops away from $u$ is at most 47.
2. Virtual backbone – cont.

Figure 2: The disks of radius 0.5 centered at the nodes in $S$ that are within three hops away from $u$ all lie within the annulus centered at $u$ of radii 0.5 and 2.5 and are disjoint.
2. Virtual backbone – cont.

- **Lemma 2.** The size of any independent set in $G$ is at most $4^{*}\text{opt}-1$
- **Theorem 3.** The size of CDS $U$ is within a constant factor of opt.
  \[|U| \leq 48^{*}|S| \leq 192^{*}\text{opt}+48\]

Opt -- the size of a minimum CDS in $G$
3. Distributed construction

- Phase 1: a MIS $S$ is constructed. The nodes in $S$ are referred to as *dominators*, and the nodes not in $S$ are referred to as dominatees.

- Phase 2: each dominatee node identifies the dominators that are at most two hops away from itself and broadcasts this information. Then each dominator identifies a path of at most three hops to each dominator. The set $C$ then consists of all dominatee nodes in these paths, which are referred to as connectors.
3. Distributed construction – cont.

- Local Variables and Structures

![Diagram of local variables and structures](image-url)
3.1 Local variables and structures –cont.

- $x_1$ stores the number of current candidate neighbors, and is initially equal to the total number of neighbors.
- $x_2$ stores the number of current candidate neighbors with lower IDs, and is initially equal to the total number of neighbors with lower IDs.
- $y$ counts the number of neighboring dominatees that have reported their list of adjacent dominators.
- $z$ which counts the number of reports yet to be received from its neighbors on their lists of single-hop dominators and lists of two-hop dominators.
3.1 Local variables and structures – cont.

- *list1* stores the IDs of the neighboring dominators.
- *list2* (respectively, *list3*) stores the ID of the dominators with *larger* IDs that are two (respectively, three) hops away and the IDs of its neighbors to reach these dominators.
- *Rlist* contains two parameters. The first parameter is a pair of IDs of two dominators to which it maintains connectivity. The second parameter contains the ID of the associated connector that connects the two dominators in the first parameter, if the two dominators are three hop distance.
- *Clist* which is initially empty and stores the IDs of neighboring connectors.
3.2 Messages and Actions

- State diagram

Diagram:
- Candidate
- Dominator
- Dominatee
- Connector

Relationships:
- Candidate to Dominator
- Dominatee to Connector
3.2 Messages and Actions

Example:

Send dominator message

Send dominatee message
3.2 Messages and Actions

Example:

node 7 sends a LIST1 message, which includes the IDs of nodes 3 and 4;

node 5 sends a LIST2 message, which includes the IDs of nodes 1 and 2;

node 6 sends a LIST2 message, which includes the IDs of nodes 3 and 4;

node 7 sends a LIST2 message, with the empty list;
3.2 Messages and Actions - 5

Example:

- Node 5 declares itself as a connector then sends a CONNECTOR1 message.
- Node 6 declares itself as a connector then sends a CONNECTOR2 message.
- Node 7 declares itself as a connector then sends a CONNECTOR1 message.
3.3 Message and Time Complexity

Theorem 4. This distributed algorithm for constructing a CDS has an $O(n)$ time complexity, and $O(n)$ message complexity.
4 Mobile maintenance

- Dominator Node Movement
- Dominatee or Candidate Node Movement
- Connector Node Movement
Example: Dominator node movement with two broken links simultaneously

1. Send WARNING
2. Send Dominator Message
3. Send List2
4. Send List3 message, selecting node2 as a connector to reach dominator node4

Send CONNECTOR1 Message
5. Conclusion

- This is the first message-optimal distributed approximation algorithm for constructing a CDS with a constant approximation ratio in linear time and linear messages.

- The algorithm is fully localized, and it does not rely on the spanning tree construction.
6. Advantage and Disadvantage – My review

- This algorithm is based on the assumption that the topology of a wireless ad hoc network is modeled as *unit-disk graph*, but actually it is not.
Thank You