High Speed Quantile Based Histogram Equalization for Brightness Preservation and Contrast Enhancement

Mayank Tiwari, Bhupendra Gupta, Manish Shrivastava

Indian Institute of Information Technology, Design & Manufacturing Jabalpur, MP, India 482005.
E-mail: gupta.bhupendra@gmail.com, http://www.iiitdmj.ac.in/bhupen.php
Mob. No. +91 9425155554, Fax: +91-761-2632524

Abstract

In this paper we introduce a new histogram equalization based contrast enhancement method called High Speed Quantile Based Histogram Equalization (HSQHE) suitable for high contrast digital images. The proposed method is an effective tool to deal with the “mean-shift” problem, which is a usual problem with the histogram equalization based contrast enhancement methods. The main idea of HSQHE is to divide input image histogram into two or more sub-histograms, where segmentation is based on quantile values. Since the histogram segmentation is based on the quantile values, the entire spectrum of gray level will always play an important role in enhancement process. Also the proposed method does not require the recursive segmentation of the histogram as in many other methods, and hence proposed method requires less time for segmentation. The experimental results show that the performance of proposed HSQHE method is better as compared to other existing methods available in literature. Also this method preserves image brightness more accurately than the prevailing state of art and takes less time as compared to the other methods.

Keywords: Contrast Enhancement, Histogram Equalization, Quantiles, High Speed, Brightness Preservation.

1. Introduction

Large number of digital image contrast enhancement methods are available in order to optimize the visual quality of the image through grey-level or his-
togram modification. Histogram equalization is one of the simplest and widely
used methods for digital image contrast enhancement [1]. In histogram equaliza-
tion we reduce the number of gray levels by combining two or more less frequent
neighboring gray levels (having small probabilities) in one gray level; also we
stretch high frequent intensities over high range of gray levels. This process
of combining less frequent gray levels and stretching high frequent gray levels
leads to a more flat histogram of gray levels of a given image, i.e., the proba-
bility distribution of gray levels converges to uniform probability distribution.
As we know that the entropy of an image will be maximum if the probability
distribution of gray level is uniform. Although Histogram Equalization provides
an overall contrast enhancement, it may also cause significant shift in average
luminance of input image. This phenomenon is known as “mean-shift”, some-
times this may even cause the degradation of input image and annoying visual
effects, like over-enhancement etc.

There are many variants of HE methods, available in the literature to over-
come the “mean-shift” problem and preserving the original brightness of the
image. The most common approach for brightness preservation is by dividing
histogram of input image into sub-histogram, and then equalizing each sub-
histogram independently. This approach was initiated by Yeong-Taeg, in [2]
and known as Brightness Preserving Bi-Histogram Equalization (BBHE) where
the image X is divided into two sub images $X_L$ and $X_U$ based on mean $X_M$
of brightness of input image and then equalize these two sub-histograms sepa-
rately. Later Zhang in [3] proposed Equal Area Dualistic Sub-Image Histogram
Equalization (DSIHE), through this approach also separates an input image
into two sub-sections, the only difference between BBHE and DSIHE is, in later
method the separation is based on the median value.

An extension of BBHE has been proposed by Chen and Ramli in [4], known
as Minimum Mean Brightness Error Bi Histogram Equalization (MMBEBHE).
This method provides maximum brightness preservation. The MMBEBHE proposes to perform the separation based on the threshold level, which would yield minimum Absolute Mean Brightness Error [4]. The MMBEBHE is a useful tool to control the brightness difference between input and output image. In [5] Chen and Ramli proposed another interesting method called Recursive Mean-Separate Histogram Equalization (RMSHE), here authors suggested recursive division of histograms, based on the local mean. In each recursive step existing sub-histogram is divided into two sub-histograms. After \( r \)th recursion the number of sub-histogram is \( 2^r \), where number of recursion depends on choice of user. Also authors proved mathematically that as \( r \) increases, the mean brightness of processed image approaches towards the mean brightness of input image. K. S. Sim and other in [6], improved Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE) into Recursive Sub-Image Histogram Equalization based contrast enhancement (RSHE), by introducing recursive segmentation in the similar manner as Chen and Ramil proposed in [5], this method is similar to RMSHE but it uses median values instead of mean values to divide histogram into sub-histograms. Kim [7] observed that in all previously developed algorithms, increment in level transformation function is given as:

\[
\Delta f(X_k) = f(X_k) - f(X_{k-1}) = (X_{L-1} - X_0)P[X_k].
\] (1)

From the above equation it is clear that the increment is proportional to the probability value \( P(X_k) \). Hence, gray levels of high probabilities will be assigned large dynamic range. In other words gray levels of higher probabilities dominate other gray level of lower probabilities, due to this, image regions with high probabilities are over enhanced and other less probable regions are less enhanced, this leads to losing important visual details present in the image [7]. To solve this problem Kim proposed Recursively Separated and Weighted Histogram Equalization (RSWHE), in this method a power-law function is applied.
to sub-histograms before applying HE process. Using power-law function au-
thors assigned a slightly higher weight to the less frequent gray levels and slightly
less weight to more frequent gray levels. The main advantage of RSWHE is that,
this method not only enhances image contrast but preserves the image bright-
ness as well. In the method a recursive process is used to divide the given
histogram into sub-histogram thus resulting in time complexity.

We propose a new HE based contrast enhancement method named High Speed
Quantile Based Histogram Equalization (HSQHE) which enhances the image
contrast as well as preserves the image brightness in comparatively less time.
The organization of this work is as follows. In section 2 we completely describe
histogram equalization method, Section 3 covers a brief introduction of various
histogram equalization based image enhancement methods. In section 4 we com-
pletely describe proposed method HSQHE. Section 5 provides the simulations
on running time analysis, contrast enhancement and brightness preservation
with visual examples and numerical results. Finally, conclusions are drawn in
Section 6.

2. Histogram Equalization Method

Histogram Equalization is a preprocessing technique to enhance contrast in
all type of images. Equalization implies mapping input gray level distribution
(the given histogram) to another distribution (a wider and having a more flat
gray level distribution) so that the intensity values are spread over the whole
range. Through this adjustment, the gray level distribution becomes closer to
uniform gray level distribution.
In histogram equalization we consider an image as a 2 dimensional array of gray levels. Let the \((i,j)\) element of this array is \(X(i; j)\) be the intensity of \((i, j)\) pixel of the image, where \(X(i; j)\) is from the \(L\) discrete gray levels denoted by \(\{X_0, X_1, \ldots, X_{L-1}\}\). The probability mass function (PMF) of gray level \(X_k\) is denoted by \(P[X_k]\) and defined as:

\[
P[X_k] = \frac{n_k}{n}, \quad k = 0, 1, \ldots, L - 1, \quad (2)
\]

where \(n_k\) be the number of pixel having intensity \(X_k\) and \(n\) be the total number of pixels in image.

The cumulative distribution function (CDF) of \(X_k\), is denoted by \(C[X_k]\) and defined as:

\[
C[X_k] = P[X \leq x] = \sum_{j=0}^{k} P[X_j] = \sum_{j=0}^{k} \frac{n_k}{n}, \quad k = 0, 1, \ldots, L - 1. \quad (3)
\]

It is clear that \(C[X_{L-1}] = 1\). Now we define a transformation function \(f(.)\) for histogram equalization, which maps an input gray level \(X_k\) into an output gray level \(f(k)\), given as

\[
f(X_k) = X_0 + (X_{L-1} - X_0)C[X_k], \quad k = 0, 1, \ldots, L - 1. \quad (4)
\]

3. Enhancement Methods Based on Histogram Equalization

In this section we are providing a brief description for various important HE based image contrast enhancement methods.

1. Brightness Preserving Bi-Histogram Equalization (BBHE) This method basically divides the input image into two sub parts namely \(X_L\) and \(X_U\) based on mean \(X_M\) of brightness of input image (where \(X = X_L \cup X_U\) and \(X_L \cap X_U = \emptyset\)) then the HE process works independently on both the sub-histograms. This method preserves the brightness of input
image to some extent. In this method it is clearly shown that if histogram
of given image \( H(X) \) has a symmetric distribution around \( X_M \), then the
mean brightness of processed image can be calculated using formula \((X_M + X_G)/2\), where \( X_G \) is middle gray level of image and is expressed as \( X_G = (X_0 + X_L - 1)/2 \).

2. Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE)

This method is very similar to BBHE the only difference is that rather
than using mean value it uses median value of the brightness of input im-
age to decompose it into two sub-histogram. If a gray level \( X_D \) satisfies
\( C(X_D) = 0.5 \), then it is called the median of the image \( X \), based on this
value the image is divided into two sub parts namely \( X_L \) and \( X_U \) (where
\( X = X_L \cup X_U \) and \( X_L \cap X_U = \emptyset \)). Each of \( X_L \) and \( X_U \) is then equalized
independently as in BBHE.

3. Recursive Mean Separate Histogram Equalization (RMSHE)

Mean separation is a basic process for preserving certain level of brightness and
this is what RMSHE does. In fact, for \( r = 0 \) and \( r = 1 \) RMSHE is
equivalent to HE and BBHE respectively and for \( r > 1 \) it generates \( 2^r \)
sub-histograms. In order to achieve higher brightness preservation, mul-
tiple recursive mean separations are needed. It is declared that as the
recursion level \( r \) increases, the mean brightness of the processed image
comes near to the mean brightness \( X_M \) of input image.

4. Recursive Sub-Image Histogram Equalization (RSIHE)

As RMSHE
is a generalization of BBHE similarly RSHIE is a generalization of DSIHE,
it performs median based segmentation more than once. Its working pro-
cess is similar to RMSHE but the only difference is that, for histogram
segmentation it uses median values instead of mean values.

5. Recursively Separated and Weighted Histogram Equalization(RSWHE)

Mary Kim and Min Gyo Chung [7] observed that all previously mentioned
algorithms only perform HE process and these algorithms do not modify
the histogram of input image, hence they developed RSWHE which first
modifies the input image histogram using a power law function and then perform the HE process recursively in each sub-histogram. We can say that RSWHE is nothing but a modification in RMSHE or RSIHE where the input image histogram is first modified based on a power law function and then the HE process is applied in each sub-histogram separately.

4. High Speed Quantile Based Histogram Equalization Method

This section contains the detailed description of proposed method High Speed Quantile Based Histogram Equalization for brightness preservation and contrast enhancement. This method consists of three modules namely histogram segmentation module, histogram weighting module and histogram equalization module.

We propose the histogram segmentation based on quantiles values of gray level distribution of image. The quantiles is define as follows:

Quantiles are points taken at regular intervals from the cumulative distribution function (CDF). Dividing ordered data (in our case gray level) into \( q \) essentially equal subsection for \( q \)-quantiles;

For example \( r \)-th, \( q \)-quantile for a random variable \( X \) is the value \( x \) such that

\[
C[x] = P[X \leq x] = \frac{r}{q}, \quad \text{where } r = 0, 1, \ldots, q.
\]  

(5)

4.1. Segmentation by Quantiles.

The image histogram graphically represents number of pixels against the intensity of the pixel of image and denoted by \( H(X) \), defined over \( [X_0, X_{L-1}] \).

Now we divide image histogram \( H(X) \) in \( q \) equal proportions (sub-histograms) using \( q \)-quantiles; \( H_1 = [a_0, a_1], H_2 = [a_1, a_2], \ldots, H_q = [a_{q-1}, a_q] \), such that

\[
P[X \in H_k] = P[a_{k-1} = X \leq a_k] = \frac{1}{q}, \quad k = 1, 2, \ldots, q.
\]  

(6)

where \( a_0 = X_0, a_q = X_{L-1} \) and \( a_k \in \{X_0, X_1, \ldots, X_{L-1}\}, \forall k = 0, 1, \ldots, q \).

The division of histogram using our method can easily be done with the help of...
cumulative distribution function (CDF) and this division requires less time as compared to other methods that use recursive division.

Let $p_k$ be the accumulated probability of sub-histogram $H_k$, and is defined as:

$$p_k = \sum_{h \in [a_{k-1}, a_k]} P[X_j = h], \quad j = 0, 1, 2, \ldots, L - 1. \quad (7)$$

Then the normalize gray level PMF for sub-histogram $H_k$ will be

$$P_k[X_i] = \frac{P[X_i]}{p_k}, \quad (8)$$

and the corresponding CDF of sub-histogram $H_k$ will be

$$C_k[X_i] = \sum_{h=a_{k-1}}^{X_i} \frac{P[X_j = h]}{p_k}. \quad (9)$$

Based on CDF, the transformation function for sub-histogram $H_k$ will be as follows:

$$f_k(X_i) = a_{k-1} + (a_k - a_{k-1})C_k[X_i], \quad k = 1, \ldots, q. \quad (10)$$

Suppose $Y$ be the processed image, then

$$Y = \cup_{k=1}^{q} f_k(X_i). \quad (11)$$

It is clear that the final processed image is basically union of all these sub-histograms.

4.2. Histogram Weighting Module.

Let $H_k$, $k = 0, 1, \ldots, L - 1$, be a sub-histogram of image $X(i, j)$. Let $P_{\text{max}} = \max_k P_k[X_i]$, and $P_{\text{min}} = \min_k P_k[X_i]$, for $i = 0, 1, 2, \ldots, L - 1$; are maximum and minimum probabilities of input image histogram.

Now for each sub-histogram $H_k$, we replace the original PMF $P_k$ by the modified
weighted PMF $P^w_k$, such that:

$$P^w_k[X_i] = P_{\text{max}} \left( \frac{P_k[X_i] - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \right)^{p_k}. \quad (12)$$

The above weighting model gives higher weights to the lower probabilities, i.e., it gives higher weights to those intensities which are less frequent in the image and gives lesser weights to more frequent intensities. These weights help to minimize the distance between gray level distribution and the uniform distribution.

The sum of all the values of $P^w_k$ from $k = 0$ to $L - 1$ is no longer one, as we have changed the weight. To make $P^w_k$ a PMF, it needs to be normalized as follows:

$$P^{nw}_k[X_i] = \frac{P^w_k[X_i]}{\sum_{k=0}^{L-1} P^w_k[X_i]}. \quad (13)$$

Where $P^{nw}_k[X_i]$ is weighted and normalized histogram in the range $[0, L - 1]$. This $P^{nw}_k[X_i]$ is now forwarded to next HE module.

4.3. Histogram Equalization Module.

This module takes weighted and normalized histogram from previous module and then apply HE process in each sub-histograms independently.
4.4. HSQHE - Algorithm

Algorithm 1 HSQHE (IMG[], qValue)

SET GREY_RANGE ← 256

Require: qValue ≥ 1 AND qValue ≤ 255
CALCULATE HISTOGRAM OF IMAGE IMG[]
HIST[]= : HISTOGRAM_IMAGE(IMG[])
CREATE QUANTILE ARRAY OF SIZE qValue
qArray[].SIZE() ← qValue
CALCULATE QUANTILE ARRAY VALUES.
SET COUNTER := 1
while COUNTER ≤ qValue do
SET SUM := 0
SET CNT := 0
while CNT ≤ GREY_RANGE do
SET SUM := SUM + HIST[CNT]
if SUM ≥ COUNTER/qValue then
qArray[COUNTER−1] := CNT
BREAK
end if
SET CNT = CNT + 1
end while
SET COUNTER = COUNTER + 1
end while
SET qArray[qValue−1] = GREY_RANGE − 1
SEGMENT ORIGINAL HISTOGRAM INTO SUB-HISTOGRAM BASED ON QUANTILE ARRAY VALUES.
APPLY HISTOGRAM WEIGHTING MODULE IN EACH SUB HISTOGRAM.
NORMALIZE WHOLE HISTOGRAM.
APPLY HISTOGRAM EQUALIZATION IN EACH SUB-HISTOGRAM INDEPENDENTLY.
CALCULATE PROCESSED IMAGE FROM PROCESSED HISTOGRAM.

4.5. Brightness changed by HSQHE

Let us consider an image as a 2 dimensional array of gray levels and the
(i, j) element of this array is X(i; j) be the intensity of (i, j) pixel of the image,
where X(i; j) is from the L discrete gray levels denoted by \{X_0, X_1, \ldots, X_{L−1}\}.
The probability mass function (PMF) of X_k, is denoted by P[X_k] and defined
as in equation (2). Then gray level probability mass after simple histogram
equalization is as follows:

\[ P[X] = \frac{1}{X_{L−1} − X_0}, \quad \forall X \in \{X_0, X_1, \ldots, X_{L−1}\}. \]  \hfill (14)

The mean brightness E[Y] of the processed image Y, will be:

\[ E[Y] = \frac{X_{L−1} − X_0}{2}. \]  \hfill (15)

This shows that mean brightness of processed image after histogram equalization
does not depend on the gray level distribution of original image.
In proposed HSQHE method, the gray level distribution is as \( \cup_{k=1}^{q} \{ p_k \mid k \in \left[ a_{k-1}, a_k \right) \} \), \( k = 1, 2, \ldots, q \), where \( a_0 = X_0 \) and \( a_q = X_L - 1 \), such that equation (6) satisfied. Than, we have

\[
p_k = \frac{1}{q(a_k - a_{k-1})}, \quad k = 1, 2, \ldots, q. \tag{16}
\]

and the average brightness of \( H_k \) sub-histogram is

\[
E[Y \mid H_k] = \frac{a_k - a_{k-1}}{2q}. \tag{17}
\]

Hence, the average brightness of processed image is

\[
E[Y] = \sum_{k=1}^{q} E[Y \mid a_{k-1} \leq X \leq a_k] = \frac{1}{2q} \sum_{k=1}^{q} (a_k - a_{k-1}). \tag{18}
\]

Figure 1 shows some intermediate results produced by our method. From this figure it is clear that after applying histogram weighting module in a given histogram, less frequent gray levels get higher weights than more frequent gray levels. Due to this, no over enhancement takes place for more frequent gray levels by HSQHE and hence no information is lost during the whole process.
The following Figure 2 shows enhancement results by various methods including our proposed method on elaine and lady images.
5. Experimental Results

In this section, we demonstrate the performance of our proposed method HSQHE in comparison with some other important HE-based methods includ-
implement these methods on the 10 test images bamboosample, bridge, desert, Einstein, elaine, house, icfiller, lady, lena and pirate, which have been previously used in many of these methods to show their performances.

To evaluate the effectiveness of our method, we choose four metrics, PSNR (Peak Signal to Noise Ratio), AMBE (Absolute Mean Brightness Error), Image Entropy (H) and Time taken (in milliseconds for enhancement).

5.1. Assessment of Time

Figure 3: shows graphical representation of time taken by various methods in milliseconds on given set of images.

In figure 3 we compare the time required (in milliseconds) for contrast enhancement by various methods. It is clear from the figure 3 that the proposed method requires less time as compare to all the other methods having recursive segmentation of image histogram in to sub-histogram. The proposed method performs faster, as segmentation process is not recursive as it in the other methods. For measuring processing speed, time taken to enhance given image is taken as base criteria in some of the previously developed methods Wang [8] and Ming [9], these methods prove that they are faster than other methods.

We have implemented all methods (including HSQHE) in Java Programming
Language without using any third party API, to calculate the processing time we have executed these algorithms 1000 times and then we calculated average processing time. This time is displayed in figure 3.

Another advantage of proposed HSQHE method is that it provides the complete control on the level of enhancement. As, in all previously developed methods number of sub-histograms after \( r \)th recursion are \( 2^r \), i.e. after 3rd recursion 8 sub histogram and then in the next levels 16, 32 and so on. In many cases we required- to preserve image brightness up to certain level, for example let in a situation we requirement is to achieve brightness preservation up to a level \( L \), and this level can achieve above 16 and below 32 histogram segmentation of given image \( X \). Then the required level (\( L \)) of brightness preservation cannot be achieved by previously developed methods, whereas in case of our method any value in between 1 to 255 (for images where single pixel intensity value is stored in 8-bits) can be taken to carry out same task that will meet our current requirement of brightness preservation up to level \( L \).

5.2. Assessment of Contrast Enhancement

In this subsection we demonstrate the performance of the proposed HSQHE method in comparison with the other methods in terms of PSNR and Entropy. The PSNR and entropy are widely user measure for comparing the contrast enhancement methods.

5.2.1. Peak Signal to Noise Ratio

PSNR is an approximation to human perception of reconstruction quality of an image. A higher PSNR generally indicates that the reconstruction is of higher quality. To calculate PSNR between two images, mathematical expression is given as:

\[
PSNR = 10 \log_{10} \left( \frac{(L - 1)^2}{MSE} \right),
\]  

(19)
where MSE is Mean Squared Error, is defined as:

\[
MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (X(i,j) - Y(i,j))^2.
\]  

(20)

Practically the mean squared error (MSE) allows us to compare the “true” pixel values of our original image to our degraded image. The MSE represents the average of the squares of the “errors” between our actual image and our noisy image. The error is the amount by which the values of the original image differs from the degraded image. The proposal is that the higher the PSNR, the better degraded image has been reconstructed to match the original image and the better the reconstructive algorithm. This would occur because we wish to minimize the MSE between images with respect the maximum signal value of the image [10].

![Peak Signal To Noise Ratio](image)

Figure 4: shows graphical representation of PSNR values by each method on given set of images

Figure 4 gives the comparison of Peak Signal to Noise Ratio (PSNR) between the proposed HSQHE method and other methods on 10 test images. It is clear from the figure 4 that proposed HSQHE method with \( q = 6 \) have highest PSNR
for all the test images. Also for \( q = 5 \) the proposed HSQHE method have higher PSNR for 7 out of 10 images. The average PSNR values are given in the right most plot of figure 4. The average PSNR values of other methods are smaller than those of HSQHE, which shows that HSQHE outperforms other existing methods in terms of the PSNR.

5.2.2. Entropy

In general, the higher the entropy is, the richer details and information the image holds. Mathematical expression to calculate entropy of an image is given as:

\[
\text{Ent}[P] = - \sum_{k=0}^{L-1} P(k) \log_2 P(k).
\]  

(21)

Where \( P(k) \) is probability mass function PMF of histogram of given image.

Figure 5: shows graphical representation of Entropy values by each method on given set of images

Figure 5 gives the comparison of entropy between the proposed HSQHE method and other methods on same set of 10 test images. It is clear from figure 5 that the proposed HSQHE method have higher entropy for 9 out of 10 images. The average entropy values are given in the right most plot of
figure 5. The average entropy values of other methods are smaller than those of HSQHE, which shows that on an average, HSQHE is better than the other existing methods in terms of the entropy also.

5.3. Assessment of Brightness Preservation

This section shows the efficiency of proposed HSQHE method to deals with “mean-shift” problem. The AMBE is used to measure difference in mean brightness between two images. Mathematical expression to calculate AMBE between two images is given as:

\[
AMBE = |X_M - Y_M|,
\]

where \(X_M\) and \(Y_M\) are mean brightness of input and processed image respectively.

![Figure 6: shows graphical representation of AMBE values by each method on given set of images](image)

Based on results of figure 6 it can be observed that HSQHE \((q=5)\) has least values in 8 images as compared to that of HE, BBHE, DSIHE, RMSHE and RSIHE and it has least values in 6 out of 10 from that of RSWHE-M and RSWHE-D. For HSQHE \((q=6)\), observations in figure 6 shows that the proposed
The HSQHE method produces best results from all other methods in comparatively less time. The average of AMBE of all the 10 test images is given in the right most plot of figure 6. The average of AMBE of proposed HSQHE method is significantly smaller than the other methods.

### 5.4. Inspection of Visual Quality

From quantitative evaluation it is clear that our method produces comparatively better results as compared to other important existing methods. There are situations when we required qualitatively assessment of the image after contrast enhancement, which can be done by judging the visual acceptance and natural appearance of processed image.

The following figure 7 shows results of various methods on Einstein image for visual quality inspection.

![Enhancement results for Einstein image](image)

Figure 7: Enhancement results for Einstein image

From figure 7, it is clear that result produced by HSQHE looks more natural
as compared to other methods. This shows that our method produces results without losing natural appearance of image.

6. Conclusion

In this paper we propose HSQHE; this method is developed to enhance image contrast without much affecting the mean brightness of input image. Also HSQHE is designed such that it takes less time as compared to other methods, as in proposed method we have quantile based histogram segmentation that does not need recursive segmentation of the histogram. Another advantage of quantile based histogram segmentation is that it provides chance to every part of gray level spectrum to play their role in the enhancement process. Experimental results show that HSQHE preserves image brightness more accurately then other existing HE based methods and produces image with better contrast in comparatively less time, without producing unwanted artifacts.

References


Separate Histogram Equalization for Scalable Brightness Preservation,’

Applied to Gray Scale Images,’ Pattern Recognition Letters, 2007,
28, pp. 1209-1221.

Equalization for Brightness Preservation and Contrast Enhancement,’

Image Enhancement,’ 30th Annual International IEEE EMBS Conference

Variation Regularization,’ Science China Press and Springer-Verlag Berlin