

# DELTA-GAMMA-THETA HEDGING OF CRUDE OIL ASIAN OPTIONS

Juraj Hruška<sup>1</sup>

<sup>1</sup> Department of Finance, Faculty of Economics and Administration, Masaryk University, Žerotínovo nám. 617/9, 601 77 Brno, Czech Republic

## Abstract

HRUŠKA JURAJ. 2015. Delta-gamma-theta Hedging of Crude Oil Asian Options. *Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis*, 63(6): 1897–1903.

Since Black-Scholes formula was derived, many methods have been suggested for vanilla as well as exotic options pricing. More of investing and hedging strategies have been developed based on these pricing models. Goal of this paper is to derive delta-gamma-theta hedging strategy for Asian options and compare its efficiency with gamma-delta-theta hedging combined with predictive model. Fixed strike Asian options are type of exotic options, whose special feature is that payoff is calculated from the difference of average market price and strike price for call options and vice versa for the put options. Methods of stochastic analysis are used to determine deltas, gammas and thetas of Asian options. Asian options are cheaper than vanilla options and therefore they are more suitable for precise portfolio creation. On the other hand their deltas are also smaller as well as profits. That means that they are also less risky and more suitable for hedging. Results, conducted on chosen commodity, confirm better feasibility of Asian options comparing with vanilla options in sense of gamma hedging.

Keywords: Asian option, delta, gamma, theta, hedging, investment decision making

## INTRODUCTION

Exotic options are mostly considered as plainly mathematical issue, since they are not widely traded on world derivative markets. In case they are traded it is usually on OTC markets. Asian options are one of those which are at least occasionally listed on the greatest derivative markets such as CBOT or EUREX. Their underlying assets are for example crude oil, ethanol, iron ore and places on the cargo ships between Europe and east coast of USA and between Persian Gulf and Japan.

Papers published about the topic of Asian options are focused either on their pricing, with usage of various mathematical methods or their application in investment process. Black-Scholes methodology was used for pricing Asian options with discrete averaging (Zhang, 1998). This method is sometimes considered as out of date and is being replaced with more sophisticated methods. Dubois is using partial differential equations for their pricing (Dubois, 2004). These equations are solved with various approaches; perturbation method (Zhang, 2003) for example. In modern research jump processes are preferred to simple Wiener process in derivative

pricing, because they can better reflect the nature of stock price movements (Bayraktar, 2010).

Investment strategies in particular assets could be hedged with Asian options or they can be used for direct investment. Lévy's models with jump processes were considered as effective possibility for static hedging (Albrecher, 2005). Gamma hedging itself was used in several researches evaluating investment opportunities of this method (Dengler, 1996 and Jarrow, 1994). It is also used as a tool for hedging mortality and interest rates (Luciano, 2012).

In this paper Asian options are being applied in investment strategy based on creating delta and gamma neutral portfolio. This method was developed for hedging plain-vanilla options (Šturc, 2010). Investment strategy is enhanced with simple decision model based on past movement of underlying asset. Gamma hedging had been proven to be successful with plain vanilla options. The main disadvantage of gamma hedging, as well as in all portfolio optimizing strategies, are the optimal weights in portfolio. They are difficult to achieve in practice, because of trading in lots and limited available capital. This can be solved by using Asian options instead of European. The fact that the spot

price is averaged in pay-off function makes them much cheaper than plain-vanilla options (Haug, 2007). Although this strategy is called hedging it carries certain amount of speculation, because we are always choosing price direction, which is supposed to be hedged. The opposite price direction should bring us profit all the time, if all the requirements hold.

## METHODOLOGY AND DATA

We can distinguish between Asian options with floating strike and with the fixed strike. Position of averaged price in pay-off function is the main difference. Average price replaces strike price in floating strike options and spot price in fixed strike options. The average can be calculated in arithmetic or geometric form. In the next analyses only those with fixed strike are considered, because, they are more common in practice. Hence more data are available for adequate model selection.

### Methodology

All the calculations in this paper are based on Zhang's model for pricing Asian options with fixed strike and continuous geometric averaging (Zhang, 1998). This model was chosen after comparison with other known models for pricing options with continuous as well as discrete averaging. Tests were conducted on real options daily prices with large variety of strike prices and maximal available observations. Pay-off function of these options is using arithmetic discrete averaging. Models using geometric averaging are commonly used for pricing options with arithmetic averaging as an approximation (Zhang, 1995), because pricing models using arithmetic averaging have no closed form (Hull, 2012). But it is still surprising, that models for geometric averaging are more suitable. Mentioned Zhang's model is defined as:

$$V = ke^{\frac{1}{2}(r-g-\frac{1}{6}\sigma^2)T} S_t N(kd) - ke^{-rT} KN \left( k \left( d + \sigma \sqrt{\frac{T}{3}} \right) \right), \quad (1)$$

where

$$d = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - g - \frac{1}{2}\sigma^2\right)\frac{T}{2}}{\sigma\sqrt{\frac{T}{3}}}.$$

Spot price of underlying asset is noted as  $S_t$ ,  $K$  represents strike price,  $r$  is used for risk-free interest rate. Volatility is marked as  $\sigma$ , cost of carry  $g$  and time to maturity as  $T$ . In case call option is calculated  $k = 1$ . On the other hand, if put Asian option price is calculated  $k = -1$ .  $N$  is the distribution function of normal standardized distribution.

Using first and second derivation of option price value with respect to spot price and first derivation with respect to time to maturity the formulas

for delta, gamma and theta of this option were identified. Theta has been added to the model, because one day observations are used for testing. This time step is long enough to cause decline of the time value of Asian options.

$$\Delta = ke^{\frac{(r+g+\frac{\sigma^2}{6})T}{2}} N'(kd), \quad (2)$$

$$\Gamma = e^{\frac{(r+g+\frac{\sigma^2}{6})T}{2}} N'(d) \frac{1}{S_t \sigma \sqrt{\frac{T}{3}}}, \quad (3)$$

$$\begin{aligned} \theta = & S_t e^{\frac{(r+g+\frac{\sigma^2}{6})T}{2}} \left( kN(kd) \left( -\frac{r+g}{2} \right) + N'(kd) \left( d_t + \frac{\sigma}{2} \sqrt{\frac{1}{3T}} \right) \right) + \\ & + kKe^{-rT} \left( rN \left( kd + k\sigma\sqrt{\frac{T}{3}} \right) - N' \left( kd + k\sigma\sqrt{\frac{T}{3}} \right) d_t \right), \quad (4) \end{aligned}$$

where

$$d_t = \frac{-\ln\left(\frac{S_t}{K}\right) \sqrt{\frac{1}{12T}} + \left(r - g - \frac{1}{2}\sigma^2\right) \sqrt{\frac{T}{48}}}{\sigma\frac{T}{3}}.$$

The main idea was to create portfolio of call and put options that would be hedged against the price movement in chosen direction. Therefore the weights of options should reflect the change in price of the opposite option. To estimate anticipated change in the option price we can use Taylor polynomial with deltas, gammas and thetas as function derivations. Expected changes in call and put option prices in up and down direction can be expressed as:

$$call_{up} = \frac{\Delta_{call}}{1!} dS_{up} + \frac{\Gamma_{call}}{2!} (dS_{up})^2 + \theta_{call} dT, \quad (5)$$

$$call_{down} = \frac{\Delta_{call}}{1!} dS_{down} + \frac{\Gamma_{call}}{2!} (dS_{down})^2 + \theta_{call} dT, \quad (6)$$

$$put_{up} = \frac{\Delta_{put}}{1!} dS_{up} + \frac{\Gamma_{put}}{2!} (dS_{up})^2 + \theta_{put} dT, \quad (7)$$

$$put_{down} = \frac{\Delta_{put}}{1!} dS_{down} + \frac{\Gamma_{put}}{2!} (dS_{down})^2 + \theta_{put} dT. \quad (8)$$

These changes are largely depended on anticipated movement of underlying asset ( $dS_{up}$  and  $dS_{down}$ ). In this case they were calculated as average decrease

in prices and average increase in prices of past 20 adequate observations (days) before the portfolio creation. If the change of the asset price is constant in both directions gamma hedged portfolio can keep its value in one selected direction and increase its value in opposite direction. All parameters of call and put options must be the same.

Delta-gamma hedging is not taking time into account. Hence portfolio needs to be rebalanced with high frequency or the strategy must be closed in short time, otherwise hedging would not be effective. High frequency of rebalancing is ineffective, because the gained profit is only small fraction of commissions paid for opening and closing positions. I used daily price changes for my analysis and included thetas to reflect time value decay. Fridays have been excluded, because weekend is already too long time difference, between opening and closing positions. All portfolios created before non-trading days were omitted.

If we want to hedge our portfolio against decline in price it should be constructed as:

$$put_{down} V_{call} - call_{down} V_{put}, \quad (9)$$

or as

$$-put_{up} V_{call} + call_{up} V_{put}, \quad (10)$$

if we are anticipate prices to decrease and want to hedge against the increasing prices. This method was originally used for plain-vanilla options (Šturc, 2010), but there are no boundaries for its application on Asian options.

Certainly, this is only theoretical concept and real price changes could strongly affect the strategy performance. Therefore I had to optimize the choice of the options so the profit from movement opposite to the hedged one, would be maximal. This can be accomplished by maximizing function:

$$put_{down} call_{up} - call_{down} put_{up} \rightarrow max. \quad (11)$$

After applying first order condition with respect to the spot prices of underlying asset we obtain the formula for the optimal spot price at which should be the profit from the strategy maximal.

$$\tilde{S}_t = Ke^{-\left(r-g+\frac{5}{6}\sigma^2\right)T}. \quad (12)$$

It is impossible to choose the spot price, because it is given by market. It is also possible to wait for the spot price to fulfill this rule. On the other hand, we can find option pair with appropriate strike price, which holds the relationship (12) in accordance to observed spot price. Options were chosen if the difference between the real and optimal strike price was less than 20 cents. Expected changes of the underlying asset price have no effect on the value of optimal spot price. That means that they affect only the weights in portfolio.

Even when the positions are theoretically hedged, it is still possible to gain loss after all, because the price changes act as was expected rather occasionally. Simple predictive model was created to decide which price movement is anticipated in next observation (day in this case).

$$up_i = \alpha + \beta S_{i-1} + \sum_{j=1}^7 \gamma_j up_{i-j} + \sum_{k=1}^3 \phi_k t^k + \sum_{l=1}^7 \phi_l \kappa_l + \epsilon_i, \quad (13)$$

where  $up$  is a dummy variable representing upward movement of underlying asset,  $S$  is the spot price of underlying asset,  $t$  depicts time (in number of observations) elapsed since the last occurrence of upward movement and  $\kappa$  is dummy variable identifying levels of variable  $t$ . Maximal time between two upward moments was 7 days, hence it was set as maximal level for variables  $up$  and  $\kappa$ . Variables  $t$  and  $\kappa$  are applied with accordance to the methodology of dealing with time dependency in dummy time series (Carter, 2010). Logit transformation was applied and the coefficients were estimated with MLE (Heij, 2004). This model was rebalanced every 100 trading days.

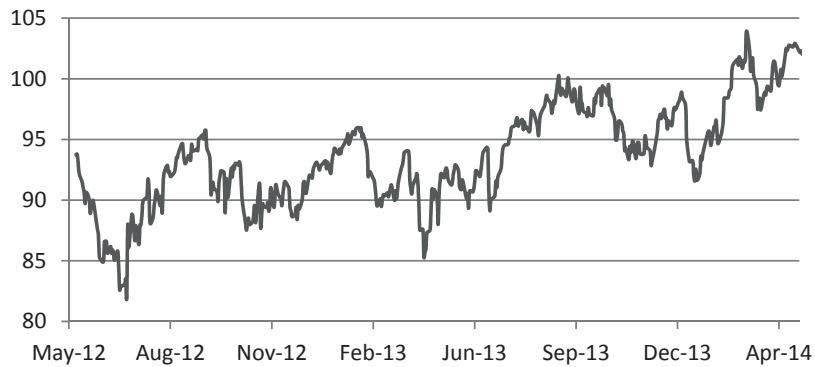
These models were set to suggest growth portfolio in cases when predicted probability of price increase was greater than 0.55, decline portfolio if the predicted probability was lower than 0.45 and in the other cases investor have to wait till next day. Models were estimated in 5 steps. First estimation was based on 560 observations prior the first trade realization. These estimations were applied next 100 trading days. After this time, new observations were added to data set and model was revaluated. This procedure was repeated after every 100 days, until the end of simulation.

## Data

All the calculation are based on the prices of call and put WTI Asian options with fixed strike price and with expiry date on 30. 4. 2014 (G9J4C Comdty). The underlying asset of all analyzed derivatives was Light Sweet Crude Oil futures with maturity on 30. 4. 2014 (CSSJ14). Fig. 1 suggests that there were no huge changes in trend movement. Even volatility seems to be stable for entire time, without any peaks, that would cause radical changes in strategy performance. Same variables for prices of Asian options with maturity on 30. 5. 2014 have been used for comparison.

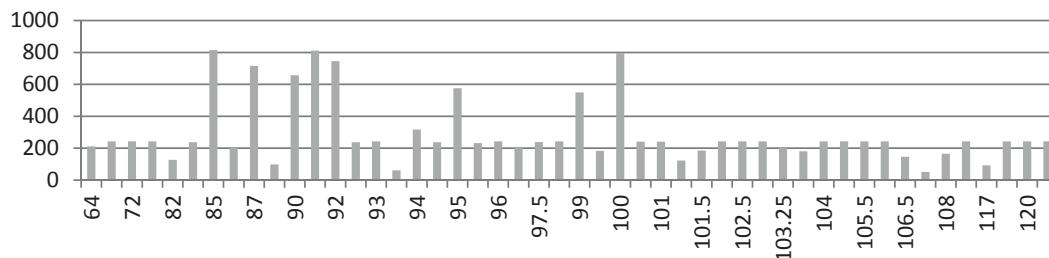
Averaged price is calculated from the prices from the end of last calendar trading day. Including new price into average is called fixation. Options with strike prices from \$64 to \$125 were chosen into selection sample. But only those where both call and put options were available. Fig. 2 shows how many days before maturity were the options with certain strike price issued on the market. As we can see most of options were issued for approximately one year (from 200 to 250 trading days). However option pair with strike price \$100 was available more than 800

## Crude Oil Future Prices



1: Development of Crude Oil futures prices with maturity date on 30. 4. 2014 (in American dollars) from 8. 5. 2012 to 30. 3. 2014

## Number of days to maturity



2: Availability of Crude oil Asian options (in days)

days before maturity. Also options with strike prices \$95 and \$99 were available nearly 600 days before maturity. These three option pairs were used in majority of cases, during the first year of simulation.

Risk-free interest rate is represented by 10-year US government bonds. Volatility of underlying asset was estimated from implied volatility calculated from plain vanilla option with same underlying asset. Costs of carry are considered to be 0. All data were obtained from the Bloomberg database.

Simulated trading was evaluated from the first observation, when there were actually existing options with desired parameters. That occurred on 8. 5. 2012 (476 trading days before maturity). Some of the options were available even before that date, but the optimal strikes defined by relationship (11) were always much higher. Simulation was stopped one month before the expiry date, because even pricing models performed strong inaccuracies during the last days before the options maturity. That could have cause imbalance in portfolio weights, which are derived from these pricing models.

## RESULTS

The hedged portfolios are certainly unable to perform the same results in practice. During the whole observed period the strategy based on gamma hedging without predictive model was

rather inconsistent and outcomes were volatile, which was caused by the errors in price changes predictions. Hedging was not working perfectly, but it was able to reduce significant part of the losses. Hence, the growth portfolios managed to create profit 14.25% and the decline portfolios were slightly worse with profit 6.48%. This positive outcome was mostly created by leverage effect, partial hedging and minimal time between opening and closing of positions.

To enhance the profit and lower the volatility of returns I have combined this strategy with predictive model. Chosen model (12) did not show astonishing predictive power, but its results were significant. The success rate and coefficient estimations of prediction models for selected periods when they were applied are shown in Tab. I.

First and second lag of dependent variable were omitted due to strong correlation with *time* variable and others due to insignificance. The value of coefficients is changing over time, but their character seems to stay the same. Third lag of dependent variable has always positive effect on the probability of the next upward movement of the crude oil futures prices. On the other hand longer lags have completely opposite effect. This suggests that the dependent variable is moving in approximately 3 day trends, during the observed period. Next three *time* variables confirms exponentially rising

## I: Predictive models

Period	8. 5. 2012 24. 9. 2012	24. 9. 2012 14. 2. 2013	15. 2. 2013 11. 7. 2013	12. 7. 2013 2. 12. 2014	3. 12. 2014 30. 3. 2014
Const.	0.0515 (0.7858)	-0.1610 (0.4421)	-9.9599 (0.0091)	-0.1157 (0.4970)	0.0811 (0.5098)
up(-3)	0.4903 (0.0038)	0.3474 (0.0247)	0	0.2586 (0.0573)	0.2225 (0.0882)
up(-5)	-0.4087 (0.0149)	-0.2653 (0.0859)	0	-0.2175 (0.1085)	-0.2599 (0.0427)
up(-7)	-0.3098 (0.0648)	-0.2983 (0.0534)	0	0	0
time	0	0.1986 (0.0057)	9.5949 (0.0074)	0.1403 (0.0255)	0
time^2	0.0324 (0.0203)	0	-2.7598 (0.0076)	0	0
time^3	0	0	0.2457 (0.0080)	0	0.0048 (0.0211)
day after growth	0	0	2.8067 (0.0190)	0	0
Accuracy	58.3%	56.1%	54.5%	56.3%	55.8%
Sommer's D	0.166	0.122	0.09	0.126	0.116

## II: Summary statistics of gamma hedging strategies

Portfolios	Num. of Trades	Avg. Return	Std. deviation	Maximal Drawdown
growth	165	0.0018	0.0444	0.2571
decline	165	0.0026	0.0530	0.4127
predictive	62	0.0144	0.0480	0.0602

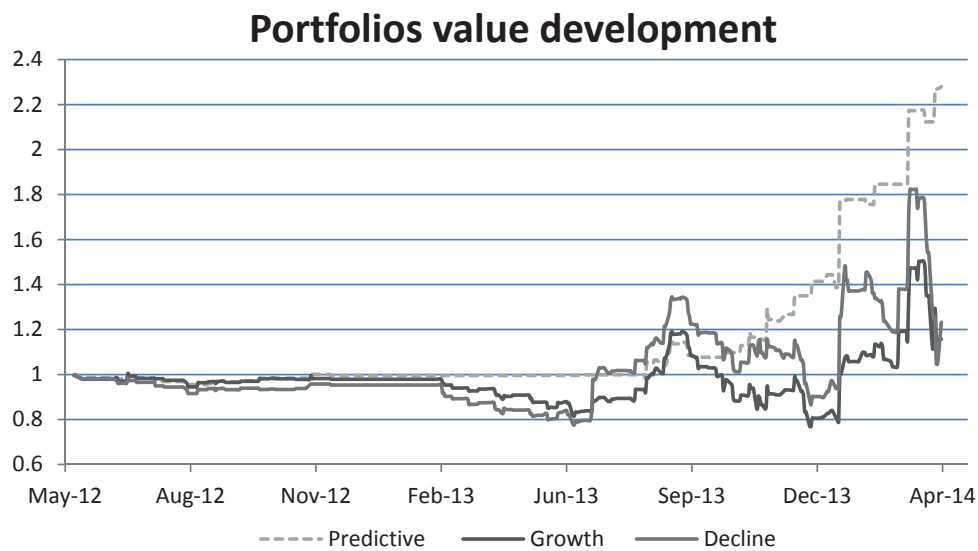
probability of next up movement with the rising time since the last upward movement. The last variable identifying day after upward movement has also positive effect on the explained variable. This can be explained by the herd investing, when most of the investors are expecting the rising trend in prices to continue. Reconfiguration of the model is definitely necessary; probably with even higher frequency than every 100 days.

After the implementation of the predictive model into gamma hedging strategy, significant improvement in its performance is observed. During the observed period of 23 months 62 portfolios were created. Positions were closed next day to minimize time impact on the estimated changes in option prices. Initial capital was increased by 133.24%, with average profit 1.44% per realized portfolio. It means that the average profit was eight times higher than the average profit of growth portfolio and 5 times higher than decline portfolios. All three strategies are relatively similarly risky (measured by standard deviation of returns). If we compare strategies by the maximal drawdown, strategy with predictive model is definitely less risky than other two strategies. Detail comparison can be seen in Tab. II.

Prices of the underlying asset and the prices of Asian call option with strike price \$100 (this option was used in portfolios in most of the cases) were used as benchmark for gamma hedging strategy.

Investment to crude oil futures would bring profit 6.19% during observed period (average return was 0.0217% with riskiness 1.0377%). In case of Asian call option the outcome will be loss 78.67% of initial value (average return was -0.3110% with riskiness 11.2515%). In both cases investments were also combined with the same predictive model used with gamma hedging. Under these circumstances average return of underlying asset is 0.0145% with riskiness 0.7343% and average return of Asian call option is 0.1904% with riskiness 7.6248%. Hence, the risk-reward ratio of these two alternatives was 50.61 and 40.04. If we compare it to the growth portfolio with risk-reward ratio 24.21, decline portfolio with 20.58 and predictive portfolio 3.33, we can see that, even though gamma hedging carries a certain amount of risk, it is still much better option than investing to the crude oil futures and crude oil Asian options directly without hedging.

Gamma hedging was working nearly ideally during the first year of testing. The original value of the portfolio was stable. On the other hand, outcomes of strategies became much more volatile in time closer to the options maturity as can be seen in Fig. 3. This could not be created by the increasing volatility of the underlying asset. There are no evidences of such influences in Fig. 1. The cause of the strategy errors must be intrinsic. One of the explanations could be that the specification of the pricing model is inappropriate. I was using



3: Development of the performance of the strategies from 8. 5. 2012 to 30. 3. 2014

model for pricing Asian options with continuous averaging, because this model best fit the real data. But these options are using discrete monthly averaging, which may not play role when only a few prices were included into the average and many of the prices remain uncertain. Most of all, time value is much higher and covers the differences in averaging type.

However, when the time to maturity was shrinking, this model could not bring adequate results, because the time value of the options were largely reduced and difference between continuous and discrete average became significant; especially after one before the last fixation. During this period daily profits of the strategies were in range from  $-30\%$  to  $40\%$  (with standard deviation  $21.12\%$ , which means four times higher volatility). If I would continue with trading even after the last price fixation, the predictive portfolio would create  $138.58\%$  of total profit, the growth portfolios  $42.41\%$  and the decline portfolios  $70.40\%$ . The profits are much higher but the risk connected with them is not worth of it.

Strategy has been also tested on the other set of options with the expiry date on 30. 5. 2015. Results were significantly worse than in first case. Growth portfolios created loss  $4.85\%$  and decline portfolio loss  $6.99\%$ . The portfolios with predictive model have again performed better, but the overall profit was only  $12.01\%$ .

Effects of the commissions on the strategy performance have also been tested. The commissions and trading costs varies depending on the broker providing the market access. The average commission for trading plain vanilla options is around 50 cents per contract (according to Bloomberg statistics). Those for Asian options are lower, because even the prices are lower. Gamma-delta-theta hedging strategy is strongly sensitive to the effects of spreads and commissions. Even commission such as 1 cent eliminated possibility of any profit. Theirs effect is even stronger during the last month before maturity, when time value of both call and put options is declining faster.

## DISCUSSION AND CONCLUSION

Results presented in this paper confirm that Asian option with fixed strike price can be hedged successfully using the first and second derivation of the option price. The profit of  $133.24\%$  in 62 realized trades is really satisfying outcome. This method could be used in long position in call or put options to assure position for short time or to profit from the short term speculations. Hence I have explained that Asian options are as good for gamma based strategies as plain-vanilla options. Also it is more suitable for these purposes because they are cheaper and therefore much suitable for fitting the portfolios with specific weights.

Gamma hedging could be upgraded to Speed hedging, but adding third derivation with respect to price of the underlying asset would have only slide effect on the portfolios. Amending the model with theta would theoretically allow us to incorporate the time effect into the weights. That would mean that portfolios could be hold for the longer time and hence reduce the commissions originally needed for portfolio rebalancing. Spreads and commissions are one major phenomenon that has been considered in analyses and has the strong potential to reduce or completely eliminate the profit and efficiency of the strategy.

As the time changes are considered to be zero, gamma hedging is most suitable as high-frequency strategy for investors with the possibility to invest with minimal commissions on the market with low spreads. More sophisticated methods for estimating price changes (than simple average) should be used on data with daily frequency. For example Box-Jenkins methodology or other predictive econometrical models or models working with stochastic volatility.

#### Acknowledgement

Support of Masaryk University within the project MUNI/A/1127/2014 "Analýza, tvorba a testování modelů oceňování finančních, zajišťovacích a investičních aktiv a jejich využití k predikci vzniku finančních krizí" (Student Project Grant at MU, Faculty of Economics and Administration, Department of Finance) is gratefully acknowledged.

#### REFERENCES

- ALBRECHER, H., DHAENE, J., GOOVAERTS, M., SCHOUTENS, W. and PANG, L. 2005. Static Hedging of Asian Options under Lévy Models. *The Journal of Derivatives*, 12(3): 63–72.
- BAYRAKTAR, E. and XING, H. 2010. Pricing Asian options for jump diffusion. *Mathematical Finance*, 21(1): 117–143.
- CARTER, D. B. and SIGNORINO, C. S. 2010. Back to the future: Modeling time dependence in binary data. *Political Analysis*, 18(3): 271–292.
- DUBOIS, F. and LELIÈVRE, T. 2004. Efficient pricing of Asian options by the PDE approach. *Journal of Computational Finance*, 8(2): 55–64.
- DENGLER, H. R., JARROW, A., CAPINSKI, M. and KOPP, E. 1996. Option pricing using a binomial model with random time steps (A formal model of gamma hedging). *Review of Derivatives Research*, 1(2): 37–78.
- HULL, J. C. 2012. *Options, futures, and other derivatives*. 8<sup>th</sup> edition. Harlow: Pearson Education.
- HAUG, E. G. 2007. *The complete guide to option pricing formulas*. 2<sup>nd</sup> edition. New York: McGraw-Hill.
- HEIJ, C. et al. 2004. *Econometric methods with applications in business and economics*. 1<sup>st</sup> edition. New York: Oxford University Press Inc.
- JARROW, R. A. and TURNBULL, S. M. 1994. Delta, gamma and bucket hedging of interest rate derivatives. *Applied Mathematical Finance*, 1(1): 21–48.
- LUCIANO, E., REGIS, L. and VIGNA, E. 2012. Delta and Gamma Hedging of Mortality and Interest Rate Risk. *SSRN Electronic Journal*, 50(3): 402–412. [Online]. Available at: <http://www.sciencedirect.com>. [Accessed: 2015, February 5].
- ROGERS, L. C. G., SHI, Z., DAHL, L. O., and BENTH, F. E. 1995. The Value of an Asian Option. *Journal of Applied Probability*, 32(4): 201–216.
- ŠTURC, B. and ČURLEJOVÁ, L. 2010. Teoretické aspekty využitia gama hedgingu opcií. In: *Finance a management v teorii a praxi: proceedings of the International Conference*. Univerzita Jana Evangelisty Purkyně, 30 April. Ústí nad Labem: Univerzita Jana Evangelisty Purkyně, 263–267.
- ZHANG, J. E. 2003. Pricing continuously sampled Asian options with perturbation method. *Journal of Futures Markets*, 23(6): 535–560.
- ZHANG, P. G. 1995. Flexible Arithmetic Asian Options. *The Journal of Derivatives*, 2(3): 133–159.
- ZHANG, P. G. 1998. *Exotic options*. 2<sup>nd</sup> edition. Singapore: World Scientific Publishing Co.Pte.Ltd.

#### Contact information

Juraj Hruška: 206887@mail.muni.cz