Analysis of BER on Sequence Estimation for Digital FM and GMSK with Limiter-Discriminator Detection

Yasunori IWANAMI

Department of Electrical and Computer Engineering
Nagoya Institute of Technology
Nagoya, 466-8555, Japan. E-mail: iwanami@elcom.nitech.ac.jp

Abstract - The sequence estimation scheme using very narrowband IF filter for digital FM signals with Limiter Discriminator (L&D) and Integrate & Dump (I&D) filter has been reported through [1] to [3]. In those papers simulation results were mainly described and 3.1 dB gain for Continuous Phase FSK (CPFSK) and 2.3 dB gain for Gaussian filtered MSK (GMSK) at BER=10⁻⁵ compared with the conventional detection schemes were reported. Here we have analyzed the BER of this scheme using asymptotic upper and lower bounds. Union Chernoff bound and error event path probability having minimum effective distance were used with Gaussian noise assumption for high SNR region. The exact simulation results fall between the upper and lower bounds. The theoretical verification has been examined through this analysis.

Keywords: GMSK, CPFSK, Limiter-Discriminator Detection, Sequence Estimation, Upper and Lower Bounds

I. INTRODUCTION

A communication scheme using digital FM with noncoherent Limiter-Discriminator (L&D) detection has been well known [4], [5]. Historically, the improvement of bit error rate at the receiver side has been carried out through the bandwidth optimization of the IF filter [4], the decision feedback equalization (DFE) [6] or simple two state maximum likelihood sequence estimation (MLSE) [7]. This channel is inherently the intersymbol interference (ISI) channel due to the narrowband IF filter. So the sequence estimation scheme using Viterbi algorithm can be applied successfully, although the channel is not additive white Gaussian and maximum likelihood in a strict sense. In [1] to [3] we had reported the large BER improvement through the sequence estimation scheme for digital FM signals including CPFSK and GMSK signals for very narrowband IF bandwidth such as BT=0.6~1.0, while BT ≥ 1 conventionally. By narrowing the IF bandwidth to BT=0.6, 3.1 dB gain for CPFSK and 2.3 dB gain for GMSK at BER=10⁻⁵ compared with the conventional detection schemes were reported [1]. ITT Industries (http://www.ittind.com/) has modeled a waveform that embodies this approach in the course of their internal development work in support of their wireless communication products on September in 1996, however in those previous works, simulation results have mainly been reported and the theoretical analysis that verify the BER performance remains undeveloped. In this paper, we have obtained the theoretical asymptotic upper bound and lower bound of BER based on the Gaussian noise estimation scheme for narrowband digital FM signals

BER based on the Gaussian noise approximation at the output from L&D in high SNR region. Union Chernoff bound and error event path probability having minimum effective distance have been used for obtaining the upper and lower bounds respectively. The simulated BER points fall between these bounds, thus we have theoretically made clear the effectiveness of the proposed scheme. The sequence estimation scheme analyzed here will be suited to GSM, DECT, HYPERLAN and BLUETOOTH receivers.

II. SYSTEM MODEL AND DERIVATION OF UPPER BOUND

Fig.1 shows the block diagram of the scheme. The data NRZ bits are baseband-filtered, FM modulated and transmitted to the channel. The baseband pulse shape is lowpass-filtered NRZ for GMSK and rectangular for CPFSK, thus the transmitted signal is binary. The channel is the static AWGN channel. At the receiver side, a very narrowband IF filter such as BT=0.6~1.0 is introduced. A L&D is used as a noncoherent detector and an I&D filter is used as a post detection lowpass filter. The output from limiter is expressed as

\[ y(t) = \cos(\omega_0 t + \phi(t) + \eta(t)) \]  (1)

where \( \omega_0 \) is the center angular frequency, \( \phi(t) \) is the IF filtered signal phase and \( \eta(t) \) is the phase noise expressed as

\[ \eta(t) = \tan^{-1} \left( \frac{\xi(t)}{\sqrt{2} \rho(t) + \zeta(t)} \right) \]  (2)

where \( \xi(t) \) and \( \zeta(t) \) are independent Gaussian noise processes with zero means and \( \rho(t) \) is the time varying signal to noise ratio and the probability density of the phase noise \( \eta(t) \) is given as [8]

\[ \rho(\eta) = \frac{e^{-\eta^2}}{2\pi} + \frac{1}{2\pi} \int_0^\infty e^{-\eta \cos \xi} \text{erfc}(\sqrt{\eta \cos \xi}) d\xi \]  (3)

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The output from I&D filter is then given by
\[
\Delta \Phi = \phi(T) - \phi(0) + \eta(T) - \eta(0).
\] (4)

The soft decision Viterbi decoder based on the equivalent Euclidean distance on \( \Delta \Phi \) is used to remove the severe ISI’s caused by the pre-modulation baseband filter as well as the very narrowband IF filter. This is a standpoint where we regard these two filters as the concatenated convolutional encoders. We have considered here the Gaussian IF filter, where the 3dB bandwidth is denoted as \( BT \) and \( T \) is the symbol (bit) duration. The equivalent lowpass transfer function for Gaussian IF filter is given as
\[
H(f) = \exp \left\{ -a (f / f_c)^2 \right\},
\] (5)
where \( f_c = B / 2 \) and \( a = \ln 2 / 2 \) respectively.

The receiving signal point spread due to the ISI introduced by the pre-modulation baseband filter and the narrow-band IF filter is illustrated for GMSK in Fig.2 (a) and for CPFSK in Fig.2 (b). From Fig.2 (a) we observe the receiving signal points are concentrated on the symmetrical 6 points. This is because for the pre-modulation baseband filter and the narrowband IF filter, the neighboring one symbol on each side of the detected symbol affects the detected symbol. Thus we must consider the total three bit pattern. This ISI rule is depicted as the four state trellis diagram in Fig.3. For example, the signal points of \( s^0, s^3 \) and \( s^6 \) are produced from the bit patterns of 000, 101 and 110 respectively. By using the Viterbi algorithm for decoding this trellis, we can enlarge the equivalent minimum free Euclidean distance and it leads to the decoding gain as well as the improvement of BER. The same is also true for CPFSK signaling, but the ISI is only introduced by the IF filter. For high SNR region, the output phase noise can be approximated as
\[
\eta(t) \approx \frac{\xi(t)}{\sqrt{2} \theta(t)} = \frac{\gamma(t)}{\bar{A}} \approx \gamma'(t)
\] (6)
where \( \gamma(t) \) is the IF filtered quadrature component of Gaussian noise and \( \bar{A} \) is the average amplitude of IF-filtered FM signal. The sampled output from the I&D filter is then expressed as
\[
r_n = \phi(T) - \phi(0) + \eta(T) = s_n + y_n' - y_n''.
\] (7)
where \( s_n = \phi(T) - \phi(0) \) and \( n \) indicates the time index.

The output sample \( s_n \) is regarded as a trellis encoded signal sequence denoted by
\[
s = \{ s_{0}, s_{1}, \ldots, s_{N} \}.
\] (8)

The received signal sequence corrupted by the noise is expressed as
\[
r = \{ r_{0}, r_{1}, \ldots, r_{N} \}.
\] (9)
The sequence estimator selects from among all the possible encoded signal sequences and outputs the symbol sequence \( \hat{s} = \{ \hat{s}_{0}, \hat{s}_{1}, \ldots, \hat{s}_{N} \} \) that minimizes the metric. A decision error in a signal sequence occurs when \( \hat{s} \neq s \). This probability is given by
\[
P(s \rightarrow \hat{s}) = \Pr \left\{ \sum_{n=1}^{N} m(r_n, \hat{s}_n) \leq \sum_{n=1}^{N} m(r_n, s_n) \right\}.
\] (11)

The pairwise error probability \( P(s \rightarrow \hat{s}) \) is upper bounded by applying the Chernoff bound
\[
P(s \rightarrow \hat{s}) \leq E \left\{ \exp \left\{ -\lambda \sum_{n=1}^{N} (m(r_n, \hat{s}_n) - m(r_n, s_n)) \right\} \right\}
\] (12)
where \( m(r, s) \) is the branch metric for the specific signal sequence when the encoded signal is \( s \) and the received signal is \( r \) and \( \lambda \) is the Chernoff parameter. Using the squared distance metric criterion
\[
m(r_n, s_n) = (r_n - s_n)^2
\] (13)
and (7), we get
\[
\sum_{n=1}^{N} m(r_n, s_n) - \sum_{n=1}^{N} m(r_n, \hat{s}_n) = \sum_{n=1}^{N} (s_n - \hat{s}_n)^2 + 2 \sum_{n=1}^{N} \Delta s_n (y_n' - y_n'')
\] (14)

Inserting (14) into the right hand side of (12) yields
\[
E \left\{ \exp \left\{ -\lambda \sum_{n=1}^{N} (s_n - \hat{s}_n)^2 - 2 \sum_{n=1}^{N} \Delta s_n (y_n' - y_n'') \right\} \right\}
\] (15)
where \( E \) is the expectation with respect to the Gaussian noise. We use the four-state trellis diagrams in Fig.3.
where $\Delta x_i = s_i - \hat{s}_i$, and we have assumed that $y_i'$ and $y_{i-1}'$ are independent of each other. This assumption is reasonable, because

$$E[y_i' y_{i-1}'] / E[y_i'^2] = \exp\left[-\frac{(\pi BT)^2}{4} \right]$$

and $E[y_i' y_{i-1}'] / E[y_i'^2] = 8.09e-4$ and 0.01 for $BT=1.0$ and 0.8 respectively. Also it follows that

$$E[e^x] = e^{e^x/2}$$

when $Z$ is a Gaussian random variable with the mean 0 and the variance of $\sigma^2$. By assuming

$$\sum_{n=1}^{L-1} \Delta s_n \Delta s_{n+1} > 0$$

in (15), the pairwise error probability is further upper-bounded

$$P_s = \exp\left[-\lambda \sum_{n=1}^{N} \Delta s_n^2 \right] \exp\left[4 \lambda \sigma^2 \left( \sum_{n=1}^{N} \Delta s_n^2 - \sum_{n=1}^{N} \Delta s_n \Delta s_{n+1} \right) \right]$$

$$\leq \exp\left[-\lambda + 4 \lambda \sigma^2 \right] \Delta s_n^2$$

$$\sum_{n=1}^{N} D_i (\lambda)$$

(19)

where $D_i (\lambda) = \exp[(-\lambda + 4 \lambda \sigma^2) \Delta s_n^2]$

and

$$\sigma^2 = E[y_i'^2] = B_i \cdot T \cdot k_p (s_i - E_i / K_0) = 0.532 BT \cdot k_p \cdot E_i / K_0$$

and $B_i$ is the one-sided noise bandwidth of Gaussian IF filter in (5). $0 < p_w < 1$ is the power attenuation constant due to the IF filter and $i$ and $j$ correspond to $s_i$ and $\hat{s}_i$ respectively. Minimization of the right hand side in (19) with respect to $\lambda$ leads to

$$P_s = \exp\left[-\frac{1}{4} \Delta s_n^2 \cdot \Delta s_{n+1} \right] \exp\left[-\frac{1}{4} \Delta s_n^2 \cdot \Delta s_{n+1} \right]$$

where $D = \frac{1}{4} \Delta s_n^2 \cdot \Delta s_{n+1}$ and $D_0 = D_{bc}$. Finally the BER is upper bounded by

$$P_{EB} \leq \frac{1}{k} \frac{D(T(D, L, I))}{I_{\lambda L, \lambda I, \lambda L}}$$

(20)

where $T(D, L, I)$ is the transfer function of the trellis in Fig.3.

$$T = \frac{1}{4} V^T (I - A)^{-1} W$$

where

$$V = \left( 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 \right)$$

and $A$ is the $20 \times 20$ matrix in (24) directly obtained from the trellis in Fig.3 and $I$ is the identity matrix. The matrix calculation and the differentiation with respect to $I$, expressed by (22) and (21) respectively, can be carried out using numerical method.

### III. DERIVATION OF LOWER BOUND

The lower bound is obtained through the probability of the minimum distance error event path having the length $L$ in the trellis of Fig.3. This probability is given by

$$\Pr \left[ \sum_{n=1}^{L-1} m(r_n, s_n) - \sum_{n=1}^{L-1} m(r_n, s_n) < 0 \right].$$

(25)

By defining the random variable $u$

$$u = \sum_{n=1}^{L} m(r_n, s_n) - \sum_{n=1}^{L} m(r_n, s_n)$$

(26)

and $\Delta s_n = \Delta x_n - 2 \Delta x_n \Delta s_n + 2 \Delta s_n^2$

the error event probability having the path length of $L$ is given by $\Pr \{ u < 0 \}$. By assuming that $y_i'$ and $y_j'$ ($i \neq j$) are Gaussian random variables independent of each other having the variance of $\sigma^2$, the characteristic function $\Phi_u (jv)$ with respect to $u$ is evaluated by

$$\Phi_u (jv) = E[e^{jv}]$$

(27)

and

$$\Phi_u (jv) = \exp\left[-\frac{1}{4} \sum_{n=1}^{L-1} \Delta s_n^2 \cdot \Delta s_{n+1} \right] \exp\left[-\frac{1}{4} \sum_{n=1}^{L-1} \Delta s_n^2 \cdot \Delta s_{n+1} \right]$$

The form of (27) clearly shows that $\Phi_u (jv)$ is the characteristic function of Gaussian random variable $u$ having the mean
and the variance of
\[ \sum_{i=1}^{n} \Delta s_i^2 \]
and the variance of
\[ \sigma^2 \Delta_s^2 \]
Thus the probability \( \Pr \{ \mu < 0 \} \) is given by
\[ \Pr \{ \mu < 0 \} = Q \left( - \frac{1}{2\sqrt{2}\sigma} \sum_{i=1}^{n} \Delta s_i \Delta s_{i+1} \right) \]
(30)
where \( Q \) is the Gaussian Q integral. By viewing (30) carefully, we notice that the distance in this case is measured by
\[ d_{\text{effective}} = \sum_{i=1}^{n} \Delta s_i - \sum_{i=1}^{n} \Delta s_i \Delta s_{i+1} \]
(31)
Thus the minimum distance error event path should be measured through the effective distance defined by (31). We have searched the minimum value of \( d_{\text{effective}} \) for the trellis in Fig.3 using a computer. As a result, the minimum value of \( d_{\text{effective}} \) is achieved for \( L=4 \) both for GMSK and CPFSK. Each path having the length 4 produces two bit errors, i.e.,
\[ N_b = 2 \]
for GMSK and CPFSK, thus the lower bound is finally evaluated as
\[ p_{LB} = \Pr \{ \text{Path with } d_{\text{effective}} \leq \text{min} \} \cdot \Pr \{ \mu < 0 \} \]
\[ = \frac{16}{64} \cdot 2 \cdot Q \left( - \frac{1}{2\sqrt{2}\sigma} d_{\text{effective}} \right) \]
\[ = \frac{2}{\pi} \frac{\rho h}{2} \sqrt{\frac{2e_0 E_b}{N_0}} \left( \frac{0.532 BT}{d_{\text{effective}} \text{min}} \right) \]
(32)
where \( \Pr \{ \text{Path with } d_{\text{effective}} \text{min} \} \) is the probability of the path having the minimum effective distance, which is calculated as the ratio of the number of \( d_{\text{effective}} \text{min} \) path to the total number of the path having the length \( L=4 \) in the four state trellis of Fig.3. The number of \( d_{\text{effective}} \text{min} \) path is also searched by a computer and \( d_{\text{effective}} \text{min} = f(\text{th}) \).

IV. NUMERICAL RESULTS

Fig.4–Fig.8 show the numerical asymptotic upper bound, lower bound and simulation results. Fig.4 shows the case of GMSK for \( BT=1.0 \) in the sequence estimation scheme.
Various other noncoherent LD detection schemes [1] are also plotted in Fig.4. Compared with the decision feedback receiver [6], 0.9 dB gain is further achieved by using the sequence estimation receiver at the BER=10⁻³. The upper bound obtained by (21) is not so tight, but the lower bound by (32) is relatively tight within 0.7 dB at the BER=10⁻³. The looseness of the upper bound comes from the approximation of (18) as well as the very small Euclidean distance between the signal point s₁ and s₂ in Fig.2(a), causing one bit error between the two branches in the trellis diagram of Fig.3. As the signal point s₁ and s₂ in Fig.2(a) get closer and closer, the trellis becomes more catastrophic, producing infinite bit errors without increasing the squared distance accumulation between the two different paths. In evaluating the union Chernoff bound, this near catastrophic condition degrades the preciseness of the upper bound. In Fig.4, the specific values are \( d_{\text{effective}} \mid_{\text{min}} = 1.176 \) and \( p_{\text{at}} = 0.925 \).

Fig.5 shows the case of GMSK for BT=0.8. Compared with the decision feedback receiver of [6], 1.6 dB gain is obtained at the BER=10⁻³ for this narrower BT value. However the upper bound in this case is quite loose. Maybe this is brought from more catastrophic trellis in Fig.3 where there exists the closer distance between s₃ and s₄ in Fig.2. The lower bound, on the other hand, is close to the simulation points. In Fig.5, \( d_{\text{effective}} \mid_{\text{min}} = 1.146 \) and \( p_{\text{at}} = 0.889 \).

Fig.6 shows the same BER characteristics as the previous two but for CPFSK with h=0.5 (MSK). In Fig.6, the hard decision BER with L&D and I&D filter is also exhibited. The theoretical curve developed by Pawula [4] agrees well with the simulation results. The simulation results for the sequence estimation are plotted as the black dots. Comparison between the hard decision and the sequence estimation shows that there exists 1.8 dB sequence estimation gain at the BER=10⁻⁵ for this BT=1.0 value. The simulation results just fall between the upper bound and lower bound. We can observe that the upper bound becomes lower than the BER of the hard decision detector at high Eb/No region. The upper bound becomes a little bit tighter than the previous GMSK cases and this is because the distance between s₃ and s₄ shown in Fig.2(b) is larger than GMSK. This is because that there is no premodulation baseband filter for CPFSK. In Fig.6, \( d_{\text{effective}} \mid_{\text{min}} = 1.691 \) and \( p_{\text{at}} = 0.869 \).
Fig. 7 shows the BER curves for CPFSK with $BT=0.8$ in the sequence estimation. The sequence estimation gain now becomes larger compared with the above $BT=1.0$ case. There exists 2.4 dB sequence estimation gain at the BER=$10^{-5}$ to the DFE receiver. The lower bound gets closer to the simulation results compared with the $BT=1.0$ case in Fig. 6. This can be understood that the minimum effective distance paths affect the BER more seriously than the case of $BT=1.0$. In Fig. 7, $d'_{\text{eff}}=1.553$ and $p_{\text{se}}=0.814$.

V. CONCLUSIONS

We have analyzed the BER of the sequence estimation scheme for GMSK and CPFSK signals having very narrowband IF filter, $BT \leq 1$, with noncoherent Limiter-Discriminator detection. Theoretical asymptotic upper bound and lower bound based on Gaussian noise approximation at high input SNR region have been derived. Exact computer simulation results fall between the derived upper and lower bounds. The lower bounds are tighter than the upper bounds. The effective Euclidean distance between two paths is defined in evaluating the lower bounds. These bounds are tighter for higher input SNR region. Up to now, there exists no theoretical upper and lower bounds for this scheme. Through this analysis, the theoretical justification has been made more clear. The sequence estimation scheme analyzed here can be directly applicable to existing GSM, DECT, HYPERLAN and BLUETOOTH receivers.

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VII. REFERENCES