An Improved Flux Observer for Sensorless Permanent Magnet Synchronous Motor Drives with Parameter Identification

Hai Lin*, Kyu-Yun Hwang** and Byung-Il Kwon†

Abstract – This paper investigates an improved stator flux linkage observer for sensorless permanent magnet synchronous motor (PMSM) drives using a voltage-based flux linkage model and an adaptive sliding mode variable structure. We propose a new observer design that employs an improved sliding mode reaching law to achieve better estimation accuracy. The design includes two models and two adaptive estimating laws, and we illustrate that the design is stable using the Popov hyper-stability theory. Simulation and experimental results demonstrate that the proposed estimator accurately calculates the speed, the stator flux linkage, and the resistance while overcoming the shortcomings of traditional estimators.

Keywords: Permanent magnet synchronous motor, Observer, Stator flux linkage

1. Introduction

AC motor drives had been widely used in most industry applications of aircrafts, robots, automobiles and household electric appliance because of its high performance and low cost [1]. Recently, the PMSM is very popular in AC motor applications for its small size, light weight and high efficiency, especially in the application without position or speed sensor [2]. Generally, the rotor position of the PMSM is detected by a Hall sensor, resolver or an absolute encoder. However, these sensors bring not only more cost but also some defects to structure design of the motor. Moreover, the performance of the sensor gets degradation for exterior environment constraints such as air humidity, vibration and temperature. Recently, some sensorless methods of PMSM drives had been developed for the motor drive by the estimated rotor position and velocity. A full order Luenberger observer [3] for stator flux estimation and a simplified Kalman filter for speed estimation have been developed for a direct-torque-controlled IPM motor drive. To simplify the structure of traditional full order observer, a reduced-order position observer [4] with stator resistance adaptation for salient PMSM drives is developed. A second-order observer [5] with the phase-locked loop (PLL) structure and a predictive direct current controller are designed to realize the sensorless control of the PMSM.

To improve the robustness of the traditional observer, the technique of sliding mode variable structure had been introduced to the sensorless control of the PMSM drives. The sliding mode observers [6-8] are proposed to estimate the position and speed of a permanent magnet synchronous motor (PMSM) by the estimated back-emfs. The proposed observers are derived from traditional current model and most employed the discontinuous switching function in sliding mode control. More importantly, much harmonic components among the estimation of back-emf degrade the estimation accuracy of the rotor position. To estimate accuracy rotor position, a simplified Kalman filter [6-7] or a low pass filter [8] was used to eliminate the high frequency components to estimate the rotor position in the traditional sliding mode observer. To reduce the undesirable chattering in the traditional sliding mode technique, a sigmoid switching function [8] with a variable boundary layer was used to replace the traditional sign function. In [9], an adaptive sliding observer for stator flux estimation and a simplified Kalman filter for speed estimation have been developed for a direct-torque-controlled IPM motor drive. The proposed observer based on the motor current model need the estimation of the back-emf of the motor and a Kalman filter to eliminate its high frequency components. The stator flux linkages are calculated by the estimated back-emfs and the measured stator currents. Therefore, current observers have the complex structure and the problem of the chattering from the switching function in the traditional sliding mode technique.

In the paper, an improved adaptive stator flux linkage observer is investigated by a flux model and the sliding mode variable structure with a variable exponent reaching law. The traditional switching function in the reaching law is replaced by a hyperbolic switching function, which can reduce the chattering problem of the traditional sliding mode control. Two estimation laws of the speed and resistance deduced by the given observer model and Popov-stability theory have an accuracy speed and resistance estimation. The simulation and experiment results show the effectiveness of the proposed observer for sensorless PMSM drives.
2. Motor Mathematical Model

For a surface mount permanent magnet synchronous motor, the phase voltages in the rotating frame are expressed as

\[
\begin{align*}
    u_d &= R_s i_d + L_d \frac{di_d}{dt} - \omega L_q i_q \\
    u_q &= R_s i_q + L_q \frac{di_q}{dt} + \omega L_d i_d + \omega \phi
\end{align*}
\]  

(1)

where \(u_d\), \(u_q\), \(i_d\), and \(i_q\) are the stator phase voltages and currents in the rotating frame, respectively, \(L_d\) and \(L_q\) are the inductances in the rotating frame, \(R_s\) is the resistance of the stator winding, \(\omega\) is the motor speed, and \(\phi\) is the rotor magnet flux linkage.

The stator flux linkage in the rotating frame can be expressed by

\[
\begin{align*}
    \lambda_d &= L_d i_d + \phi \\
    \lambda_q &= L_q i_q
\end{align*}
\]  

(2)

where \(\lambda_d\) and \(\lambda_q\) are the stator flux linkages in the rotating frame. The amplitude of the stator flux linkage is

\[
\lambda_s = \sqrt{\lambda_d^2 + \lambda_q^2}
\]  

(3)

Substituting (2) into (1), the phase voltages in terms of the stator flux linkage and the speed in the rotating frame are given by

\[
\begin{align*}
    u_d &= \frac{R_s}{L_d} \lambda_d + \frac{1}{L_d} \frac{d\lambda_d}{dt} - \omega \lambda_q - \frac{R_s}{L_d} \phi \\
    u_q &= \frac{R_s}{L_q} \lambda_q + \frac{1}{L_q} \frac{d\lambda_q}{dt} + \omega \lambda_d
\end{align*}
\]  

(4)

The compact matrix form of (4) can be written as follows:

\[
\frac{dX}{dt} = AX + Bu
\]  

(5)

where

\[
X = \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix}, u = \begin{bmatrix} u_d \\ u_q \end{bmatrix}, A = \begin{bmatrix} -\frac{R_s}{L_d} & \omega \\ -\omega & -\frac{R_s}{L_q} \end{bmatrix}, B = I,
\]

and \(I\) is an identity matrix.

3. Improved Flux Linkage Estimation Scheme

In most methods for estimating the stator flux linkage, the current-based flux model in (2) can be easily used to calculate the flux. It is known from (2) that the flux linkages can be calculated using the currents in the rotating frame and the two motor parameters of rotor flux and stator inductance. However, the current-based flux model suffers from a DC current shift caused by the measured error and signal noise. Subsequently, a voltage-based stator flux linkage model [10] in the stationary frame is more simple and practical, given as follows:

\[
\begin{align*}
    \lambda_{d\alpha} &= \frac{1}{2} (u_{d\alpha} - R_s i_{d\alpha}) dt \\
    \lambda_{d\beta} &= \frac{1}{2} (u_{d\beta} - R_s i_{d\beta}) dt
\end{align*}
\]  

(6)

where \(\lambda_{d\alpha}\), \(\lambda_{d\beta}\), \(u_{d\alpha}\), \(u_{d\beta}\), \(i_{d\alpha}\), and \(i_{d\beta}\) are the stator flux linkages, voltages, and currents in the stationary frame, respectively. In (6), the stator flux linkages are estimated by stator phase voltage, phase currents, the resistance, and an integrator. The voltage-based flux model is used in the stationary frame, and it depends on one motor parameter. However, the problem of the DC shift in two measured currents will cause error in the integrator, which can make it ineffective. To overcome the shortcomings of traditional flux linkage estimation methods (6), in this paper, we propose a new method to accurately estimate the stator flux linkage of the motor using the voltage-based flux linkage mode of (5) and the technology of a sliding mode variable structure.

To estimate the stator flux linkage as well as the resistance and rotor speed, we choose the stator flux linkages of \(\lambda_d\) and \(\lambda_q\) in the rotating frame as the estimated variable, and resistance \(R_s\) and speed \(\omega\) as the regulated objects. Therefore, a traditional flux observer can be given as

\[
\frac{d\hat{X}}{dt} = \hat{A}\hat{X} + B\hat{u} + Ge
\]  

(7)

where the state error \(e = X - \hat{X}\), \(G\) is a constant coefficient matrix, and

\[
\hat{X} = \begin{bmatrix} \hat{\lambda}_d \\ \hat{\lambda}_q \end{bmatrix}, \hat{u} = \begin{bmatrix} u_d + \frac{\hat{R}_s}{L_s} \phi \\ u_q \end{bmatrix}, \hat{A} = \begin{bmatrix} -\frac{\hat{R}_s}{L_s} & \omega \\ -\omega & -\frac{\hat{R}_s}{L_q} \end{bmatrix}
\]

To improve the estimation performance relative to those of traditional flux estimation schemes, a new flux observer with variable exponent reaching law is designed such that:
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\[
\frac{d\hat{X}}{dt} = \hat{A}\hat{X} + \hat{B}\hat{u} - G_1 |e|^p \text{hyp}(S) - G_2 |e|^p S
\]  
(8)

where \( S = e(t) + K\int_0^t e(x)dx \) (\( K \) is a constant coefficient matrix), \( G_1 \) and \( G_2 \) are constant coefficient matrixes, and \( a \) and \( b \) are constants. In (8), \( G_1 |e|^p \text{hyp}(S) + G_2 |e|^p S \) is the designed variable exponent reaching law. In this paper, \( a = b = 1 \). To reduce the chatting problem of the traditional sliding mode control, a hyperbolic function \( \text{hyp}(\cdot) \) is introduced in (8) and defined as follows:

\[
\text{hyp}(x) = 1 - \frac{2}{1 + e^{mx}}
\]  
(9)

where \( m \) is a positive constant that regulates the slope of the function output. The variable speed and variable exponent in the designed observer produce the state variable approach to the sliding mode surface under two different speeds. The state variable quickly approaches the surface under the variable exponent part, and the variable speed part is the main regulator to approach the surface in a small distance. After that, the sliding mode control law allows the state variable to reach the surface and be stable at the origin. The variable exponent reaching law employed in (8) can effectively repress the chatting problem of the traditional sliding mode technology.

Subsequently, based on (5) and (8), the time derivative of the state error \( e \) is

\[
\frac{de}{dt} = Ae + \Delta A\hat{X} + B\Delta u + Z(e)
\]  
(10)

where \( Z(e) = G_1 |e| \text{hyp}(S) + G_2 |e| S \), the error coefficient matrix \( \Delta A \) and the error input matrix \( \Delta u \) are defined as follows:

\[
\Delta A = A - \hat{A} = \begin{bmatrix}
-\frac{\Delta R_s}{L_s} & \Delta\omega \\
-\Delta\omega & -\frac{\Delta R_s}{L_s}
\end{bmatrix}
\]

\[
\Delta u = u - \hat{u} = \begin{bmatrix}
\frac{\Delta R_s}{L_s} \\
0
\end{bmatrix} = \frac{\Delta R_s}{L_s} \phi I_0, \quad I_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]  
(11)

the error resistance is \( \Delta R_s = R_s - \hat{R}_s \), and the error speed is \( \Delta\omega = \omega - \hat{\omega} \).

Defining an output vector as

\[
u = \Delta A\hat{X} + B\Delta u + Z(e)
\]  
(12)

and substituting (12) into (10) yields

\[
\frac{de}{dt} = Ae - I\nu
\]

\[
v = D\nu
\]  
(13)

where \( \nu \) is an input vector, and \( D \) is a constant coefficient matrix.

Defining \( D = I \), then \( \nu = Ie = e \). According to the Popov hyper-stability theory, the system is stable under the following conditions:

(I). \( H(s) = (sI - A)^{-1} \) is a strictly positive matrix.

(II). \( \forall t_0 \geq 0 \), \( \eta(t_0) = \int_{t_0}^t (v^T \nu)dt \geq -\gamma_0^2 \), where \( \gamma_0 \) is a finite positive constant, which is independent of \( t_0 \). Therefore, \( \lim_{t \to \infty} e(t) = 0 \), and the system (10) is stable.

In a designed error system (10), the stable condition (I) described above can be easily satisfied. According to the stable condition (II), substituting (12) into a variable \( v^T \nu \), yields

\[
v^T \nu = e^T \Delta A\hat{X} + e^T B\Delta u + e^T Z(e)
\]  
(14)

Substituting (11) into (14) gives

\[
v^T \nu = -e^T \left( \frac{\Delta R_s}{L_s} I + \Delta\omega J \right) \hat{X} + e^T B \frac{\Delta R_s}{L_s} \phi I_0
\]

\[+ e^T (G_1 |e| \text{hyp}(S) + G_2 |e| S)
\]  
(15)

This leads to the following equations:

\[
\begin{cases}
\mu_1 \frac{d(\Delta R_s)^2}{dt} = \frac{\Delta R_s}{L_s} (-e^T \hat{X} + \phi e^T B I_0) \\
\mu_2 \frac{d(\Delta\omega)^2}{dt} = -e^T J\hat{X} \Delta\omega
\end{cases}
\]  
(16)

where \( \mu_1 \) and \( \mu_2 \) are two positive constants.

Simplifying (16) to

\[
\begin{cases}
2\mu_1 \frac{d\Delta R_s}{dt} = \frac{1}{L_s} (-e^T \hat{X} + \phi e^T B I_0) \\
2\mu_2 \frac{d\Delta\omega}{dt} = -e^T J\hat{X}
\end{cases}
\]  
(17)

From the above definitions, it is known that \( \Delta R_s = R_s - \hat{R}_s \) and \( \Delta\omega = \omega - \hat{\omega} \). Assuming that the derivation of actual speed and resistance in the steady operation is zero, the simplified form of (17) can be simplified as following:
Substituting (18) into (10), the stable condition (II) described above can also be satisfied when parameters $\gamma_0$, $\mu_1$, $\mu_2$, $G_1$, and $G_2$ are chosen appropriately according to the following inequality condition:

$$e^T (G_1 | e | \text{hyp}(S) + G_2 | e | S) \leq 0$$ (19)

Thus, (18) can now be solved for the designed adaptive laws of $\hat{R}_s$ and $\hat{\omega}$. To speed their response, a proportional term is added. The final speed and resistance adaptive laws are

$$\begin{align*}
\hat{R}_s &= k_{p1} \frac{1}{L_s} (e^T I \hat{X} - \phi e^T I_0) + \\
\hat{\omega} &= k_{p2} e^T J \hat{X} + k_{i2} \int (e^T J \hat{X}) dt
\end{align*}$$ (20)

where $k_{p1}$, $k_{p2}$, $k_{i1}$, and $k_{i2}$ are positive constants.

The configuration of the designed sensorless scheme for a PMSM drive with the proposed observer design is shown in Fig. 1. In this scheme, the reference and estimated flux linkage are calculated using a reference flux model and an adaptive flux observer in the rotating frame. The speed and resistance are estimated using two PI regulators with the inputs of two designed adaptive laws. According to (8) to (19), Popov hyper-stability theory can ensure the stability of the designed estimating system if we choose the appropriate constant coefficient in (8), (20), and (21). In the observer, while the regulating objects of speed and resistance are changed synchronously, the regulating model continuously tracks the reference model. That is, when the designed system is stable, the regulating model is close to the actual motor system based on a PLL structure observer.

In Fig. 1, the coordinate conversion from stationary frame to rotating frame is

$$\begin{bmatrix} X_d \\
X_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\
-\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} X_\alpha \\
X_\beta \end{bmatrix}$$ (22)

To verify the estimated flux linkage of (8) in the rotating frame with a voltage-based stator flux linkage model of (6) in the stationary frame, the reverse coordinate conversion of (22) is given by

$$\begin{bmatrix} X_\alpha \\
X_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} X_d \\
X_q \end{bmatrix}$$ (23)

where X is the phase current, voltage, or flux linkage.

5. Verification by Simulation and Experimental Results

The control scheme for the proposed observer is shown in Fig. 2. In this scheme, we use the technique of space vector pulse-width modulation (SVPWM) to obtain the constant inverter switching frequency. The state frame transformations of Clark, Park, and inverse-Park are used to transform the currents and reference voltages. Two proportion integral (PI) controllers are used to regulate the d-axis and q-axis currents, and a PI controller is used to regulate speed. Using the reference voltages from the reverse-Park transformation and two measured stator currents from the Clark transformation, the stator flux linkages, rotor position, and speed are estimated accurately, and the motor’s speed and resistance are simultaneously obtained by the proposed adaptive flux observer.
To verify the proposed scheme shown in Fig. 2, a simulation model is built using Matlab software, and an experiment is performed as shown in Fig. 3. The experiment is implemented in a DSP of TMS320F28035, and a three-phase voltage source inverter is implemented with a chip of DRV8402 from Texas Instruments. In both the simulation and the experiment, we use a PMSM motor with physical parameters of armature resistance 0.79 ohm, inductance 1.2 mH, and number of poles 8. In the digital realization, the PWM switching frequency of the inverter is set as 10 KHz. The DC link voltage for the inverter is set to 24 V. The reference speed of the motor is set to 300 rpm. To reduce the startup current of the motor, a step signal of input speed command is transformed to a ramp signal with a slope value of 0.1. According to the inequality (19), the coefficient matrixes of $G_1$ and $G_2$ in the proposed observer are given by

$$G_1 = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.65 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.08 \end{bmatrix}$$

In the parameter and state estimation laws of (20) and (21), its coefficients of time after time tests are

Fig. 3 The experiment of a sensorless PMSM drive with a proposed adaptive observer

Fig. 4. The simulation and experiment results for the proposed observer in a PMSM drive (Estimated speed and resistance and the errors of speed and rotor position)
$k_{p1} = 0.6, \quad k_{i1} = 2.5, \quad k_{p2} = 0.02, \quad k_{i1} = 1.1, \quad m = 0.45$.

Fig. 4 shows the responses of estimated speed and resistance $\hat{R}_S$ and the speed error $e_\omega$ and rotor position error $e_\theta$ in the simulation and experiment, respectively. In the experimental results, the estimated speed nears its set value of 300 rpm in 2.5 seconds, and the resistance reaches its rated value of 0.79 ohm in 4.5 seconds, as shown in Fig. 4. To further reduce the chatting from the

**Fig. 4.** The estimated flux linkage for the proposed observer in a PMSM drive
traditional switching function, we designed a variable exponent reaching law with a hyperbolic switching function in the proposed observer. The speed error between the reference and estimated speeds is reduced to zero in 2.5 seconds, and the position error between the actual and estimated positions is reduced to zero in 4.0 seconds. During the process of motor starting, the motor has a fast dynamic performance. The error between the reference state and actual state is changed from the maximum close to zero quickly. A hyperbolic switching function will regulate the positive or negative feedback of the error dynamic automatically according to the error. In the simulation results, similar dynamic performance is achieved by the proposed observer.

Figs. 5 and 6 show the response of estimated flux linkage ($\hat{\lambda}_\alpha$ and $\hat{\lambda}_\beta$) and its space distribution in the simulation and experiment for the traditional and proposed observers, respectively. Figs. 5(a) and (c) show the simulation results for the traditional observer (6), and Figs. 5(b) and (d) show the experimental results. These results show that the integration of the traditional estimator (6) will increase the input errors from measurement and calculation, and, unless reset, they will become large and lead to instability. When there are DC shifts of the measured currents and voltages or an unknown initial value of the stator flux linkage, the estimated stator flux linkage from a traditional estimating scheme is divergent, and the origin (O) of its planar circle path leaves that of the coordinates. Subsequently, the traditional method used to estimate the flux linkage with integration is invalid. Figs. 6 (a) and (c) show the simulation results for the proposed observer, and Figs. 6(b) and (d) show the experimental results. The proposed estimator provides the advantage of overcoming the limitation of the traditional integration. The origin of the planar circle path of the flux linkage is fixed to the frame origin (O), as shown in Figs. 6(c) and 6(d). A better steady performance of flux estimation is thus achieved by the proposed observer.

5. Conclusion

An adaptive stator flux linkage observer for sensorless vector control of PMSM drives is presented in this paper. The designed observer has a phase-locked loop structure, which contains two models of an adaptive sliding mode flux observer and a reference flux model. Two adaptive laws are derived by the Popover hyper-stability theory to simultaneously estimate the speed and the resistance. The proposed scheme achieves more accurate speed and stator flux linkage estimation compared to the traditional method, as demonstrated by the simulation and experimental results.

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