Unitary Matrix Pencil Algorithm for Range-Based 3D Localization of Wireless Sensor Network Nodes

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Abstract—Most node localization algorithms for wireless sensor network (WSN) are only applicable to two-dimensional networks. However, in most cases, nodes are placed in three-dimensional (3D) terrains, such as forests, oceans, etc. In this paper, a range-based 3D localization method is put forward based on time-of-arrival (TOA) estimation of ultra wideband signal using unitary matrix pencil (UMP) algorithm. The proposed method combines UMP algorithm, multilateral localization with 3D Taylor algorithm. UMP algorithm is a matrix pencil (MP) algorithm with utilization of a unitary transform, which is traditionally used to estimate angle-of-arrival (AOA). Here it is extended to estimate TOA to measure the propagation distance between an unknown node and an anchor node, which reduces the computational complexity significantly. By simulation, the accuracy of UMP algorithm is compared with MP algorithm to validate the effectiveness in positioning WSN nodes in a 3D space. This method has superiorities over conventional methods in many aspects, such as higher 3D positioning accuracy, smaller computational amount, suppression over non-Gaussian noise, energy saving, faster executing, etc.

Index Terms—Wireless sensor network, three-dimensional positioning, unitary matrix pencil algorithm, multilateral localization

I. INTRODUCTION

The problem of sensor network localization is to determine the positions of the sensor nodes in a network given incomplete and inaccurate pair wise distance measurements. Node localization plays a fundamental role in wireless sensor networks (WSN) [1] operation and management.

In recent years, lots of node localization methods have been proposed for WSN. According to the diverse characteristics, all of the location schemes can be categorized into range-based algorithms and range-free algorithms. Range-based localization methods need the estimation of distance between nodes, which have been discussed in the literatures [11-14], ranging from received signal strength indicator (RSSI), time of arrival (TOA), time difference of arrival (TDOA), to angle of arrival (AOA). Range-free localization methods require no special knowledge for the distance, including DV-Hop algorithm, DV-Distance algorithm, Euclidean algorithm, amorphous localization method, MDS-MAP algorithm, etc. Obviously, the accuracy of the range-based algorithms is higher than that of range-free algorithms.

So far, most localization algorithms for wireless sensor network are only applicable to two-dimensional (2D) network, which cannot be used in actual environment. In most cases, nodes are placed in three-dimensional (3D) terrains, such as forests, oceans, etc. Currently, some 3D node localization algorithms [2, 3] have been proposed, however, some aspects should be improved in accurate localization, small computational amount, suppression over complex noise, energy saving, faster executing, etc.

In this paper, we propose a novel range-based 3D node localization algorithm for ultra wideband (UWB) based wireless sensor network. This method combines unitary matrix pencil (UMP) algorithm, multilateral localization and three-dimensional Taylor algorithm. Matrix pencil (MP) algorithm [4] is a high-resolution parameter estimation method, and UMP algorithm [5, 6] was traditionally a MP algorithm with utilization of a unitary transform to estimate the angle of arrival (AOA), of which the computational amount can be significantly reduced. In this paper, UMP algorithm is extended to estimate the time of arrival (TOA) between nodes so as to measure the propagation distance between them. Unlike other parameter estimation techniques which generally need estimation of the signal covariance matrix, UMP algorithm exhibits a lot of superiorities of MP algorithm. It deals with signal samples directly and requires no independent data samples, which can much reduce the computational complexity [6]. The suppression over non-
Gaussian noise is also a great advantage. Furthermore, UMP utilizes the centro-hermitian property of a matrix and applies a unitary transformation, which can convert a complex matrix to a real matrix with eigenvectors. This significantly reduces the processing time for real time implementation [7]. As for node position computation algorithm, multilateral localization other than trilateral localization is used here. Taylor algorithm [10] is extended to 3D to solve nonlinear equations.

II. TOA ESTIMATION USING UNITARY MATRIX PENCIL ALGORITHM

In this section, UMP algorithm is extended to the application of UWB wireless sensor network to reduce computational load and improve time resolution. A UMP-based TOA estimation algorithm is proposed to measure the distance between two nodes. The estimation results will be used in 3D position computation.

A. UWB Channel Model

Assume that an unknown node transmits signals to an anchor node by \(M\) paths, the received signal in noise-free environment is

\[
r(t) = \sum_{m=1}^{M} \alpha_m s(t - \tau_m) \quad (1)
\]

where \(\tau_m\) and \(\alpha_m\) are the propagation delay and amplitude of the \(m\)-th path, respectively. \(\tau_t\) denotes the direct-path delay, which is the desired TOA.

\[
s(t) = \sum_{p=0}^{N-1} \sqrt{E_p} P(t - pT_{pr}) \quad (2)
\]

is the sending UWB signal [8, 9], where \(\sqrt{E_p}\) is the pulse amplitude, \(P(t)\) is the pulse waveform, and \(T_{pr}\) is the pulse repetition period.

Channel impulse response (CIR) is modeled as a sum of \(M\)-path components shifted according to the corresponding time delays. The estimated channel and its frequency domain representation is written as

\[
\hat{h}(t) = \sum_{m=1}^{M} \alpha_m \delta(t - \tau_m). \quad (3)
\]

\[
\hat{H}(j\omega) = \sum_{m=1}^{M} \alpha_m e^{-j\omega\tau_m}. \quad (4)
\]

In noisy environment with complex additive white noise, the discrete frequency-domain CIR model is

\[
\hat{H}(k) = \sum_{m=1}^{M} \alpha_m z_m^k + W(k). \quad (5)
\]

for \(k = 0, 1, \ldots, K - 1\), where \(K\) is the number of sample data segments, \(T_s\) is the symbol period. \(W(k)\) denotes the noise in frequency-domain, and the parameter is

\[
z_m = e^{-j\frac{2\pi}{K}\tau_m} \quad (6)
\]

B. Unitary Matrix Pencil Algorithm for TOA Estimation

Consider a Hankel matrix \(Y\) obtained from \(\hat{H}(k)\), \(k = 0, 1, \ldots, K - 1\), each column of which is a windowed part of the discrete frequency-domain CIR data vector \([\hat{H}(0), \hat{H}(1), \ldots, \hat{H}(K - 1)]^T\), i.e.,

\[
Y = \begin{bmatrix}
\hat{H}(0) & \hat{H}(1) & \cdots & \hat{H}(L) \\
\hat{H}(L) & \hat{H}(2) & \cdots & \hat{H}(L+1) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{H}(K-L-1) & \hat{H}(K-L) & \cdots & \hat{H}(K-1)
\end{bmatrix}_{(K-L)(L+1)} \quad (7)
\]

where \(L\) is the pencil parameter, \(K/3 \leq L \leq K/2\). According to the MP algorithm [4], the parameters \(z_m\), \(m = 1, 2, \ldots, M\), are the generalized eigenvalues of the matrix pair \(\{J_1Y, J_2Y\}\), where the matrices

\[
J_1 = \begin{bmatrix}
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}_{J(K-L)(K-L)} \quad (8)
\]

and

\[
J_2 = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix}_{J(K-L)(K-L)} \quad (9)
\]

therefore, \(\tau_m\) can be obtained from \(z_m\).

In this paper, in order to significantly reduce the complexity of the computation, we estimate the UWB time delays based on UMP algorithm by using a unitary matrix transformation.

Based on the data matrix of (7), a combined matrix \(\tilde{Y} \in \mathbb{C}^{(K-L)(2K+1)}\) is formed, which is written as

\[
\tilde{Y} = [Y, \Pi_{(K-L)}, Y\Pi_{(L+1)}]. \quad (10)
\]

where \(\Pi_{(K-L)}\) and \(\Pi_{(L+1)}\) are \((K-L)\times(K-L)\) and \((L+1)\times(L+1)\) dimensional real exchange matrix, respectively, in which only the anti-diagonal elements are 1, while the other elements are 0.

From Theorem 2 and Theorem 3 in [7], we know that \(\tilde{Y}\) is a central complex conjugate symmetric (centro hermitian) matrix, and the matrix

\[
X_k = U_{(K-L)}^{H} \tilde{Y} U_{(2L+1)} \quad (11)
\]

is a real matrix, where \(U\) is a unitary matrix, and its even-order and odd-order matrices are \(U_{2n}\) and \(U_{2n+1}\), respectively,
According to (10), we know that the parameters $z_m$ are the generalized eigenvalues of the matrix pair $\{J_j \tilde{Y}, J_j \tilde{Y}\}$.

Perform the singular value decomposition (SVD) of $X_R$, and assume that $A_s$ is the singular vectors of $X_R$ that corresponds to the largest $M$ singular values. According to (11) [8], $z_m$ can also be regarded as the generalized eigenvalues of the matrix pair $\{(U^H J_j U) A_s, (U^H J_j U) A_s\}$, and further, as the generalized eigenvalues of the matrix pair $\{(U^H J_j U) A_s, (U^H J_j U) A_s\}$. Take the real and imaginary parts of the matrix pair, respectively, then

$$\tan\left(-\frac{\pi}{K T_s} \tau_m\right)$$

can be as the generalized eigenvalue of the matrix pair $\{\text{Im}(U^H J_j U) A_s, \text{Re}(U^H J_j U) A_s\}$. Therefore, the estimation of the $m$-th path delay can be calculated by

$$\tau_m = -\frac{K T_s}{\pi} \tan^{-1}(\lambda_m).$$

for $m = 1, 2, \cdots, M$, where $\lambda_m$ is the generalized eigenvalue of the matrix pair $\{\text{Im}(U^H J_j U) A_s, \text{Re}(U^H J_j U) A_s\}$.

In summary, UMP-based TOA estimation algorithm can be summarized as follows:

1) Obtain discrete frequency-domain CIR $\hat{H}(k)$, $k = 0, 1, \cdots, K - 1$ from the received $M$-path UWB signals.

2) Form a Hankel matrix $Y$ using the data $\hat{H}(k)$.

3) Compute the real matrix $X_R = U_{K-L} \tilde{Y} U_{2(L+1)}$, where $\tilde{Y} = \begin{bmatrix} Y, \Pi_{(K-L)}, Y', \Pi_{(L+1)} \end{bmatrix}$.

4) Evaluate $\text{Im}(U^H J_j U)$ and $\text{Re}(U^H J_j U)$, then perform a SVD of $X_R$ and calculate $A_s$.

5) Calculate the generalized eigenvalues of the matrix pair $\{\text{Im}(U^H J_j U) A_s, \text{Re}(U^H J_j U) A_s\}$.

6) Calculate the delays $\tau_m$, $m = 1, 2, \cdots, M$, according to (14), and the smallest one is the desired TOA.

### III. MULTILATERAL LOCALIZATION AND 3D TAYLOR ALGORITHM FOR WSN

Node position computation makes use of geometry relationship and available information concerning both TOA and positions of anchor nodes. The cross points of sphere or hyperbolic are calculated using the geometric relations, and then the 3D location coordinates are determined.

![Figure 1. The geometry relationship of 3D localization](image1)

The geometry relationship of 3D node localization using 4 anchor nodes is shown in Fig.1, and the process of positioning an unknown node using 5 anchor nodes is described in Fig.2.

![Figure 2. The process of an unknown node localization using 5 anchor nodes](image2)
\[ \Delta t_i = t_i - t, \text{ between the unknown node to the } i\text{-th and the 1st anchor nodes, we form an equation set} \]
\[
\begin{align*}
D_i &= \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}, \\
D_{i,1} &= c\Delta t_i = D_i - D_1, \quad i = 2, 3, ..., N
\end{align*}
\]

where \((x, y, z)\) and \((X_i, Y_i, Z_i)\) are the coordinates of unknown nodes and anchor nodes. \(D_i\) is the distance between unknown nodes and the \textit{i}-th anchor node, \(c\) is the light speed. We use multilateral position computation method for WSN, i.e. \(N \geq 5\).

Here 3D Taylor algorithm is exploited to solve the nonlinear equations. 3D Taylor algorithm starts with an initial estimate \((x^{(0)}, y^{(0)}, z^{(0)})\), and uses (16) to iteratively calculate the estimations of \(\Delta\).

\[
\Delta = \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} = \left(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G}^T\right)^{-1} \mathbf{G}^T \left(\begin{bmatrix}
D_{1,2} - (D_2 - D_1) \\
D_{2,3} - (D_3 - D_2) \\
\vdots \\
D_{n,1} - (D_n - D_1)
\end{bmatrix}
\right)
\]

(16)

where

\[
\mathbf{G} = \begin{bmatrix}
(X_1 - x) & (X_2 - x) & (X_3 - x) & \cdots & (X_n - x) \\
D_1 & D_2 & D_3 & \cdots & D_n \\
(X_1 - x) & (Y_2 - y) & (Y_3 - y) & \cdots & (Y_n - y) \\
D_1 & D_2 & D_3 & \cdots & D_n \\
(X_1 - x) & (Z_2 - z) & (Z_3 - z) & \cdots & (Z_n - z) \\
D_1 & D_2 & D_3 & \cdots & D_n \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
(X_1 - x) & (Y_2 - y) & (Y_3 - y) & \cdots & (Z_n - z) \\
D_1 & D_2 & D_3 & \cdots & D_n
\end{bmatrix}
\]

(17)

where \(\mathbf{Q}\) is the covariance matrix of TDOA.

IV. SIMULATIONS

In the section, some simulations are performed to validate the effectiveness of our algorithm in high-resolution TOA estimation using UMP algorithm and accurate 3D node localization in a non-Gaussian noise environment, including to the performance comparison of UMP algorithm and MP algorithm.

A. TOA Parameter Estimation Results

Assume that the distance between two nodes is 6m, i.e., the propagation delay of direct path between them is 20 ns. The second order derivative function of Gaussian function is used as UWB pulse waveform, its period is 1ns. \(L\) is set as 3 and \(K\) is set as 6.

Fig. 3 and Fig. 4 show the simulation result of TOA estimation using UMP algorithm and MP algorithm on the condition of 10 trials in noise-free environment and noisy environment with exponentially distributed noise (SNR=10dB), respectively.

From fig. 4 we can see that the errors of TOA estimation by both algorithms are small (ns-level), even in non-Gaussian noise environment. The accuracy of UMP algorithm is higher than that of MP algorithm. It can estimate TOA of UWB signals with high-resolution.

B. Performance Comparison of MP and UMP Algorithms in TOA Estimation

Fig. 5 shows the simulation comparison of root mean square error (RMSE) vs. SNR using both UMP algorithm and MP algorithm on the condition of 1000 trials. Here SNR varies from 10dB to 30dB in fig. 5.
It shows that the performance of UMP algorithm is obviously better than MP algorithm. And as is mentioned above, the computational amount of UMP algorithm is much less than that of MP algorithm. So in the next part with respect to 3D position computation, UMP algorithm other than MP algorithm is utilized to improve the localization performance.

C. 3D Position Computation with Taylor Algorithm

In fig. 6 ~ fig. 9, there are 8 anchor nodes distributed in a $40m \times 40m \times 40m$ area, and $R$ (the coverage radius of anchor nodes) is 10m. We randomly generate a number of unknown nodes within the area. First, we estimate the time delays from an unknown node to anchor nodes. Then, the node is localized using three-dimensional Taylor algorithm.

Fig. 6 shows the simulation results of 3D localization of 15 unknown nodes using three-dimensional Taylor algorithm in a noise-free environment.

Fig. 7 shows the case in a non-Gaussian noisy environment with an exponentially distributed noise, SNR=10dB.

It shows in fig. 6 and fig. 7 that combining UMP-based TOA estimation with three-dimensional Taylor algorithm, the proposed method can effectively and efficiently realize 3D node localization of WSN.

Further, we increase the number of unknown nodes to see the performance when the density of unknown nodes is decreased.

Fig. 8 shows the simulation results of 3D localization of 100 unknown nodes using three-dimensional Taylor algorithm in a noise-free environment, and fig. 9 shows that in a noisy environment with an exponentially distributed non-Gaussian noise (SNR=10dB).
In fig. 8 and fig. 9, it is shown that the proposed method can still well realize three-dimensional localization of WSN nodes even when the density of anchor nodes is decreased.

D. Accuracy of the proposed algorithm

In fig. 10 ~ fig. 11, we randomly generate 100 unknown nodes in a 40m × 40m × 40m area, and perform the simulations to show the accuracy of the proposed algorithm with the changes of SNR and the number of anchor nodes, respectively.

Let us define

\[
\text{Relative Error} = \frac{\text{Error}}{R} = \frac{\sum_{n=1}^{N_{\text{node}}} [(x_n - \hat{x}_n)^2 + (y_n - \hat{y}_n)^2 + (z_n - \hat{z}_n)^2]}{R} \quad (18)
\]

where \( \text{Error} \) denotes the ranging error, \( \text{Relative Error} \) denotes the relative error of the algorithm, \((\hat{x}_i, \hat{y}_i, \hat{z}_i)\) denotes the coordinate of the estimated position of the \(i\)-th unknown node. And \( N_{\text{node}} \) denotes the number of unknown nodes.

In fig. 10, the number of anchor nodes is set to be 8, and SNR varies from 10dB to 30dB. We replaced UMP algorithm with MP algorithm when estimate TOA and compare the accuracy of both methods.

In fig. 11, SNR is 10dB, and the number of anchor nodes varies from 5 to 8. We generate randomly 100 unknown nodes and localize them. Then \( \text{Relative Error} \) is calculated.

Also, in fig. 10 and fig. 11, the accuracy of both MP-based and UMP-based localization methods is compared.

It is shown that the accuracy has a positive correlation with SNR and the number of anchor nodes even in environment with non-Gaussian noise. When the density of anchor nodes is 7.4% and SNR is 10dB, the relative error of the proposed algorithm can be about 0.12. The performance based on UMP algorithm is obviously better than using MP algorithm.

V. SUMMARY

We propose a novel three-dimensional range-based method for node localization in WSN. We extend the UMP algorithm to estimate the TOA, which significantly reduces the computational amount. Furthermore, we extend Taylor algorithm to three-dimension, and utilize it to solve nonlinear equations. Multilateral localization promotes the accuracy of the proposed algorithm. It is proved by simulation that the error of TOA estimation using UMP algorithm is very low (ns-level) even in the environment of non-Gaussian noise, which can also be inhibited by increasing SNR. The accuracy of the proposed method is higher than many traditional methods.
and by increasing SNR or the number of anchor nodes, it can be even improved.

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