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## Analytical and Numerical Investigation on Depth and Pipe Configuration for Coaxial Borehole Heat Exchanger, A Preliminary Study

Hongshan Guo[1], Forrest Meggers[1,*]

[1] Princeton University, Princeton, NJ, United States
[2] Andlinger Center for Energy and the Environment, Princeton, NJ, United States

*Corresponding email: fmeggers@princeton.edu

### ABSTRACT

Existing research on the performance of shallow geothermal systems are prone to investigate the ground as a large thermal mass at a constant temperature despite possible temperature increase at depths - otherwise commonly known as the geothermal gradient. Most of the existing analytical models that predict the heat exchange between a borehole heat exchanger with the soil does not allow for the geothermal gradients to be accounted for. The few models that actually does account for the geothermal gradients, on the other hand, does so by enforcing a pre-existing temperature gradient only. We are presenting a bottom up approach in this paper to solve the temperature distribution by accounting for both the convective heat transfer from the working fluid and the conductive heat transfer through both the pipe and the soil. Assuming the heat transfer is entirely axisymmetric, we approach the problem by solving the Navier-stokes equation and energy equation with a finite difference solver that calculates the temporal change of temperature with given diameter, depth of borehole and geothermal gradient. The heat transfer through the pipe and into the ground can therefore be further calculated. We were able to determine a CBHE configuration that allows maximized thermal output by assuming a synthetic heating/cooling load for year-round production of heat.

### KEYWORDS

geothermal gradient; CBHE; geothermal energy; ground-source heat pump

### INTRODUCTION

Existing research on the harvesting geothermal energy categorizes the harvesting process with respect to the depth of the ground roughly into shallow geothermal systems and deep use systems. The prior are often associated with harvesting the energy solely for thermal purposes, in particularly exploiting the benefits from geothermal energy as a source of large thermal mass where energy can be deposited and/or extracted while the source maintains relatively consistent temperatures (Lund, 1999). The deep use systems, on the other hand, are usually associated with geothermal basins and highly pressurized steams that can be used for power generation. For the shallow geothermal system, the temperature of the working fluid extracted from the boreholes commonly rises up to 25 degree Celsius, while for the deeper geothermal systems, this temperature could go up to 225 degree Celsius (Lu, 2018). Obtaining the working fluid at a temperature in-between is a much less practiced approach.

This would have been made possible by exploiting the geothermal gradient that was known to be at 25 to 50 K/km for different types of geological conditions. According to data released from NGDS where the temperature at the bottom of 17,462 boreholes were made public, the temperatures at the bottom of deeper boreholes could go up to 75 degree Celsius since some

of the boreholes went as deep as 2.4 km. Admittedly, these boreholes were neither designed to be delivery heat to households, nor were they to ever be connected to municipal district heating networks. Yet the temperatures alone could cast questions as to whether spending much more money to drill deeper boreholes could in fact repay such investments with higher quality of energy coming out from the boreholes. To fully appreciate the scope of this questions, we are presenting a numerical study that we are currently investigating in this paper as we try to determine the most optimized design for a coaxial borehole heat exchanger (CBHE) since it provides the maximized amount of surface area for heat exchange comparing to other alternatives.

**METHODS**
We have created a three-step solver to investigate the flow regime within a CBHE with a set of known design parameters so that the temperature inside the borehole, at the casing and outside of the borehole can all be calculated. At every single time step, the temperature distribution and velocity field within the borehole heat exchanger will first be solved. The temperature at the wall and pipe interface are then used to calculate the temperature within the pipe and casing. The resulting temperature at the outside of the casing will then be used to compute the temperature distribution at the end of this time step.

Assuming water to be the main working fluid, the flow that is being modelled should be considered incompressible. Solving the hydraulic performance of a CBHE while considering the convective heat exchange between the water and the borehole casing, we are essentially solving the flow conditions with respect to the following three equations:

$$\nabla \cdot u = 0 \tag{1}$$
$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + \nu \nabla^2 u + g\alpha\Delta T \tag{2}$$
$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa \nabla^2 T \tag{3}$$

Equation 1 denotes the continuity of an inviscid fluid, Equation 2 is the Navier-Stokes equation which can be solved to understand the fluid condition for an inviscid fluid, while Equation 3 is the Boussinesq approximation that can be used to solve for temperature distribution. For the in-pipe fluid flow, we solve the N-S equation for an incompressible flow condition. The incompressibility acts as a constraint for the pressure. Within the borehole, for every single time step, the velocity profile will be imported, following which the pressure field can be updated with the new velocities. The further influence of the boundary conditions can then be used to further constraint the new pressure condition within the heat exchanger, creating new pressure conditions, leading to an updated velocity profile, thus updating the temperature profiles in Equation 3.

Since we are working with an axisymmetric coordinate system, it is possible to assume Equation 2 can be further simplified into a 2D form, with an exception of its final term .

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \tag{4}$$
$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial z} + \rho g\alpha\Delta T + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \tag{5}$$

To solve for the pressure term, we took divergence of Equation 4 and 5, adding them up, and simplified with the continuity constraint Equation 1, it is possible to arrive at a Pressure Poisson Equation of

$$\frac{1}{\rho}\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) + \frac{g\alpha\Delta T}{\partial y} = \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 \tag{4}$$

After updating the temperature for the fluid is determined, the temperature of the fluid that is most adjacent to the casing and inner pipe can then be introduced Equation 5, where the conduction within the casing. The ρ stands for the pipe density, C stands for their specific heat capacity ($J/kg \cdot K$), while k stands for the pipe's thermal conductivity.

$$\rho C \frac{\partial T(x,t)}{\partial t} = \nabla \cdot [k \nabla T(x,t)] \tag{5}$$

As we are solving energy equation for a incompressible fluid in an internal space before moving on to the heat conduction happening in the soil, it is also necessary to non-dimensionalize both the momentum equation and the energy equations. The non-dimensionalize terms that will be used are the following:

$$u = \frac{u^*}{U_\infty}, \; v = \frac{v^*}{U_\infty}, \; \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$x = \frac{x^*}{L}, \; y = \frac{y^*}{L}, \; t = \frac{t^*}{L/U_\infty} \tag{6}$$

such that Equation 5 can be re-written in the form of non-dimensionalized form as

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{Pe}\left[\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right] + \frac{Ec}{Re}\phi \tag{7}$$

Since the flow velocity is very small ($M \to 0$), $Ec$ also disappear so that the last term on the right hand side disappears, the temperature can therefore be fully written as a function that can be obtained from the u and v velocity components with the help of the Peclet number which we can obtain with Equation 8.

$$Pe = Pr \cdot Re = \frac{Lu}{\alpha} = \frac{Lu\rho c_p}{k} \tag{8}$$

where $L$ represents the characteristic length, $u$ is the local velocity, $\rho$ is the density, $c_p$ is the specific heat capacity, and $k$ is the thermal conductivity of water. We will be using the depth of the borehole as the characteristic length hence L = H. The resulting temperature distribution, specifically the temperature at the outside of the casing resulting from the calculations were then introduced into Equation 4 again, with different set of parameters, where ρ stands for the soil density, C stands for soil specific heat capacity ($J/kg \cdot K$), while k stands for the soil thermal conductivity. As all the computation were resulted from discretized analytical modeling, we were able to construct this solver in a light-weight, easy-to-interpret Python module that takes in only diameter of borehole, depth of borehole as inputs to produce the geothermal energy. The parameters this study was subjected to are as the following in Table 1:

Table 1. Parameters used in the numerical simulation

| Material | Density, (kg/m³) | Specific heat capacity ($kJ/kg \cdot K$) | Thermal conductivity ($W/m \cdot K$) |
|---|---|---|---|
| Soil | $1.8 \times 10^3$ | 0.8 | 1.59 |
| Pipe | $0.95 \times 10^3$ | 1.9 | 0.5 |
| Casing | $7.9 \times 10^3$ | 0.47 | 14.9 |
| Water | $1.0 \times 10^3$ | 4.1844 | 0.609 |

To further decrease the computational costs, both the control equations and inputs were further non-dimensionalized in both the vertical and axial directions, more specifically with

respect to 2*H*, where *H* stands for the depth of the proposed borehole depth designs. This nondimensionalization method is consistent with a few existing studies. The parameters from Table 1 were then used to run a series of iterations of simulations to simulate the temperature out at different flow velocities. The pseudocode for the algorithm used can be found in Table 2 as is shown below.

Table 2. Algorithm used to calculate the temperature field inside and around CBHE

*Input*: CBHE Diameter ( $D$ ), Depth ( $H$ ), Mass Velocity( $\dot{m}$ ), Length of Time for simulation ( $T$ )

**Initialize** grid network (j = [1,J], k = [1,K]) within, inside and outside of the CBHE shell
mass velocity converted to linearized velocity for inlet and outlet $u_{in}$ and $u_{out}$ from $\dot{m}$ and $T_{i,j}$
**Start** of Simulation for a total of *N* time steps where $N = T/d$t
  *For* i to *N*:
    *While* Ti-Ti+1 > 0.001 do
      Velicity ($u_{j,k}, v_{j,k}$) and Temperatures($t_{j,k}$) inside the CBHE solved by N-S & Energy Equations;
      Temperature inside the inner and outer tube solved by heat conduction equations;
      Temperature outside of the outer tube and inside the soil solved from heat conduction equations;
    $t = t + dt$
    Output Velocity fields *U,V* and Temperature fields *T*
**End**

Assigning the time of simulation (*T*) to be 1 h and the size of time step at 0.005s for the simulation, the temperature distribution inside and outside of the borehole can be simulated. We will be assuming a constant temperature of 10˚C within the scope of this paper, but could switch to a varying *Q(t)* since the temperature development was strictly temporal hence can handle varying heating/cooling demand. This is a placeholder for variable heat input that can better simulate the load coming from actual buildings in the future. It is also important to point out that we will be assuming a geothermal gradient of 30 K/km, and no underground water flow that interacts with the CBHE within the context of this study. We will be modeling the analytical problem with the central difference approximation scheme as it provides a much better smoothness and could serve our purpose better. Due to the page limit imposed on this paper, the actual discretization expressions will be omitted.

**RESULTS**
The proposed solver was very time consuming to develop, and was very time-consuming to develop. We were able to determine the flow pattern using the 2D solver we developed. For every time step, the velocity and temperature can be computed by solving the Navier-Stokes equation with the Boussinesq Approximation. At every step in time, the pressure term is first solved from the velocity and temperature profile of the last time step. Velocity and temperature profile are then calculated and developed over time, as is being shown in Figure 1.
The in-borehole temperature distribution development within the borehole can be qualitatively captured over time. Since we used dt = 5.4e-6s for the simulation, a fully developed flow pattern at the 1st hour requires a total of 61 minutes to compute. This was computationally very expensive comparing to similar solvers in TRNSYS and COMSOL.
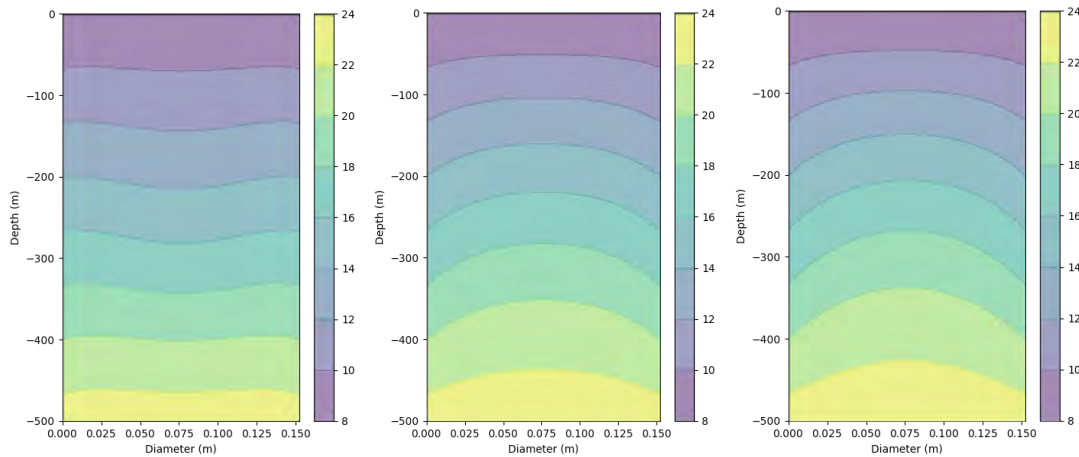
Figure 1. Temporal change of the temperature distribution within the in-bore model from the solver as time steps forward t = 0.1s (left), 0.5s (middle), 0.7s (right).

**DISCUSSIONS**

To save computation time and avoid excessive read/write to the hard drive, we did not store the temporal change of the temperature field. What was stored, was rather only data at the end of calculation, i.e. by the end of the 1st hour of hypothetical borehole operation. Understanding the temporal change of temperature was not of significant interest for this paper, but we would like to acknowledge its importance with follow-up studies, since the temporal response of the borehole is of major research interests to many preliminary borehole assessment simulations where thermodynamic properties can be compared against simulation results. We believe the first step to further this investigation is to compare their performances both in terms of computation time and the resulting temperature and velocity profiles. A clear benefit of our method would be the simplification of creating meshes as it'd be fully automated for any given diameter and depth of borehole, while any change in the geometry of a CBHE requires a new mesh generated to be calculated. Also highly simplified is the geotechnical conditions around the borehole. As we assumed homogeneous soil condition for this study, the actual soil conditions are often observed to vary significantly and needs to be described with an entire set of thermodynamic properties instead of the one set that was used in this study. As this study is, again, purely numerical, it might be possible to further customize the soil conditions through assigning different soil properties that exist by layers but is off the focus of this research and are hence not pursued within the scope of this paper.

Additionally, there is a missing link that this solver does not provide information for, but could prove helpful to further analyze: stress/strain on the borehole wall, in particular the stress and strain on the outer tube/casing interface. Better characterization of the temperature increases' influence on the harvestation of geothermal energy may very likely increase the awareness of how to better quantify the heat exchange with the ground in other modeling practices, i.e. helix geothermal heat exchanger, single and double U-tube geothermal heat exchanger as well as multiple inlet/outlet energy pile systems. There are currently no existing research that quantitatively compare the differences in between their performances, which we are also very interested in quantifying in the near future.

Last but not the least, it is important to stress that existing commercial softwares does not achieve the similar level of resolution during design-stage analysis. In short, refined solution

of the temperature profile within a CBHE is relevant to harvest the most amount of thermal energy to ground level, but cannot be achieved without fine grids in numerical simulations - which is impossible when the depth of a CBHE is yet to be determined through parametric analysis. The proposed method, on the other hand, will allow such analysis at the expense of computational time but has its own merits in the ease of use as steady-state results after prolonged periods of operation can be calculated and compared against another - as well as using variable insulation levels for the inner pipe of the annulus. We believe this is research that is crucial to further the current understanding of borehole heat exchangers and could be brought further by putting additional attention to rewrite the algorithm in compiled language to reduce the computational time in the near future.

**CONCLUSIONS**

We have devised the very basics of fluid mechanics in conjunction with the heat exchange by solving the hydraulic condition within the CBHE. Working with the basic assumption that the flow inside a CBHE can be assumed to be axisymmetric, we based our model on the fundamentally solving the Navier-Stokes equation with the boundary constraints of a CBHE with known inlet and outlet information, we demonstrated that it is possible to use a simple 2D model to predict the temperature distribution within and outside of the borehole. Admittedly, the result we reached can also be completed with commercially available softwares, we believe our method is much more simplified to run parametric studies and/or for further optimization.

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