ABSTRACT

The operator-based signal separation approach, which formulates the signal separation as an optimization problem, uses an adaptive operator to separate a signal into additive subcomponents. Furthermore, it is possible to design different operators to fit different signal models. In this paper, we propose a new kind of differential operator to separate multicomponent AM-FM signals. We then use the estimated operators to calculate each sub-component’s envelope and instantaneous frequency. To demonstrate the efficacy of the proposed method, we compare the decomposition and AM-FM demodulation results of several signals, including real-life signals.

Index Terms— Operator-based; Null Space Pursuit; Multicomponent AM-FM signals; separation and demodulation.

1. INTRODUCTION

Single-channel signal separation and estimation has attracted a great deal of attention in recent years. The most widely used approach models a signal as a superposition of additive coherent basic signals. Each basic subcomponent is extracted from the signal and the parameters of the subcomponent are then estimated. In this work, we consider two aspects: 1) the separation of a signal into a summation of amplitude modulated (AM) and frequency modulated (FM) components; and 2) the estimation of the AM and FM parameters of each component.

For a multicomponent AM-FM signal, usually, a bandpass filter-bank is used; then the energy tracking algorithm, for example the Teager-Kaiser energy operator (TKEO) [1], is used to estimate the AM and FM parts of a component [2]. The approach assumes that a multicomponent AM-FM signal has approximately disjoint spectra in a time-frequency representation of the signal. However, when a signal is the sum of two signals with close instantaneous frequencies (IFs), the resulting interference may cause a strong perturbation of the phase term in the time-frequency representation of the signal. As a result, the ridge extraction will inevitably leads to erroneous estimation of the envelope and the instantaneous frequency (IF) of the signal [3]. Instead of in a time-frequency representation of the signal, several approaches have been proposed to separate and demodulate a multicomponent AM-FM signal in the time domain, for example the PASED algorithm [4], the EMD algorithm [5], the IHT approach [6] and so on.

In this paper, we propose a novel multicomponent AM-FM separation and demodulation method. Our method is based on the Null Space Pursuit (NSP) [7] algorithm, which is a variant of the operator-based approach [8] in which an operator in the time domain is used to separate the signal and to demodulate the envelopes and IFs of the signal from the parameters of the operator. The attraction of using the operator-based approach to solve the signal separation problem is that the design of the operator can be customized to the target signal. To separate multiple AM-FM signals with the operator-based approach, we design and build a parameterized differential operator customized to a mono-component AM-FM signal. We also show that, with moderate assumptions on $a(t)$ and $\phi(t)$ of an AM-FM modulated signal $a(t) \cos(\phi(t))$, the envelope and IF of the signal can be derived from the parameters of the proposed operator. The proposed differential operator is used to separate multicomponent AM-FM signals and to demodulate the envelope and IF of a mono-component AM-FM signal. We demonstrate the performance of the proposed method on several synthetic and real-life signals in the experimental part. And our numerical results show that the proposed separation and demodulation method is robust when multicomponent AM-FM signals are immersed in an additive noise environment.

The remainder of this paper is organized as follows. In Section 2, we review the NSP algorithm. In Section 3, we propose a differential operator and a demodulation algorithm for a mono-component AM-FM modulated signal based on the proposed operator; and in Section 4, we introduce an AM-FM separation algorithm based on the NSP algorithm. In Section 5, we provide some implementation details and discuss the results of experiments on simulated and real-life signals. Sec-
2. NULL SPACE PURSUIT (NSP) ALGORITHM

The operator-based signal separation approach separates a signal $S$ into $U$ and $R$ such that $U = S - R$ is in the null space of an operator $T_S$ [8]. The sub-index, $S$, of the operator indicates that the parameters of the operator can be estimated from the signal $S$. The objective of signal separation in the operator-based approach is to solve the following optimization problem:

$$\hat{R} = \arg \min_R \left\{ \|T_S(S - R)\|^2 + \lambda \|D(R)\|^2 \right\},$$

where $R$ is the residual signal, and $D$ is a differential operator that regulates $R$. Minimizing the term $\|T_S(S - R)\|^2$ indicates that $S - R$ is in the null space of the operator $T_S$.

The main difficulty in applying the operator-based approach to a real-life signal is determining the correct value of $\lambda$, since different $\lambda$ values can result in different signal separations. In addition, during the optimization of Equation (1), most of the signal in the null space of $T_S$ is removed from $S$; however, a small portion of $S$ is required to regulate $\hat{R}$, so it is retained in the residual signal $\hat{R}$. In fact, for many signals, better solutions can usually be obtained if less information is removed from the null space of $T_S$ than that required by Equation (1). This suggests that a less greedy approach is needed to preserve more information in the null space of $T_S$ in the residual signal.

To solve the above problem, the NSP algorithm was proposed [7]. The algorithm uses the following optimization method to estimate the signal $\hat{R}$ that minimizes the problem:

$$\hat{R} = \arg \min_R \left\{ \|T_S(S - R)\|^2 + \lambda \|D(R)\|^2 \right\} + \gamma \|S - R\|^2 + F(T_S),$$

(2)

The first and the second terms of Equation (2) correspond to the corresponding terms in Equation (1). The leakage parameter $\gamma$ in the third term of Equation (2) determines the amount of $S - U$ to be retained in the null space of $T_S$; and the last term is the Lagrange term for the parameters of the operator $T_S$. When the leakage parameter $\gamma$ is set to zero, Equation (2) is reduced to Equation (1). The parameters $\lambda$ and $\gamma$ can be adaptively estimated in the NSP algorithm. The more precisely, details and performance of the NSP algorithm can be referred to [7].

3. DIFFERENTIAL OPERATORS FOR AM-FM DEMODULATION

In this section, we propose a new differential operator and introduce an AM-FM demodulation algorithm based on the operator that can estimate the envelope and IF of an AM-FM signal.

3.1. Monocomponent AM-FM annihilating operator

In [9], the differential operator $(d^2/dt^2) + \pi(t)^2$ is used to annihilate the FM modulated signal $\cos(\phi(t))$, where $\phi(t)$ is a local linear function and $\pi(t)$, defined as $\pi(t) = d\phi(t)/dt$, is the IF of the signal at $t$. A signal $s(t)$, in the null space of the operator defined above, is the solution of the following differential equation:

$$\frac{d^2 s(t)}{dt^2} + \pi(t)^2 s(t) = 0,$$

(3)

which is a simple harmonic vibration equation, corresponding to a free vibratory motion on the mass-spring model without damping in a mechanical system [10]. The operator defined in Equation (3) cannot annihilate an AM-FM modulated signal $a(t)\cos(\phi(t))$, i.e., $(d^2/dt^2 + \pi(t)^2) a(t) \cos(\phi(t)) \neq 0$. To annihilate such a signal, we can define the operator as follows:

$$\frac{d^2}{dt^2} + P(t) \frac{d}{dt} + Q(t),$$

(4)

which describes the vibratory motion of a mass-spring-damper model, i.e., adding a viscous damper to the mass-spring model [10]. If we substitute $a(t)\cos(\phi(t))$ into Equation (4) and set the result to zero, we obtain

$$\left(\frac{d^2}{dt^2} + P(t) \frac{d}{dt} + Q(t)\right) a(t) \cos(\phi(t)) = 0.$$

(5)

If we assume that for any $t$, there is a small interval of $t$ such that $\phi''(t) \approx 0$, then by using calculations to solve the ordinary differential equation in (5) and letting the notation $x'(t)$ stand for $dx(t)/dt$, we obtain:

$$P(t) = -2a'(t)\frac{a''(t)}{a(t)} = -2(\ln(a(t)))',$$

(6)

where $[a'(t)/a(t)]$ is the instantaneous bandwidth of the signal [3], and

$$Q(t) = \pi(t)^2 + 22\left(\frac{a'(t)}{a(t)}\right)^2 - \frac{a''(t)}{a(t)}.$$

(7)

Since the amplitude function varies much slower than the phase function, we can assume that $\pi(t)^2 + 22\left(\frac{a'(t)}{a(t)}\right)^2 >> \left(\frac{a''(t)}{a(t)}\right)$ and obtain

$$Q(t) \approx \pi(t)^2 + 2\left(\frac{a'(t)}{a(t)}\right)^2.$$

(8)

After substituting the derived $P(t)$ and $Q(t)$ into Equation (4), we obtain our AM-FM operator as follows:

$$\frac{d^2}{dt^2} - 2\frac{a'(t)}{a(t)} \frac{d}{dt} + \pi(t)^2 + 2\left(\frac{a'(t)}{a(t)}\right)^2.$$

(9)

Note that the operator defined in Equation (3) is a special case of the AM-FM operator when the instantaneous bandwidth $a'(t)/a(t)$ is assumed to be zero.
3.2. Monocomponent AM/FM demodulation

We have shown that an AM-FM signal \( s(t) = a(t) \cos(\phi(t)) \) is in the null space of the AM-FM operator defined in Equation (4). The parameters \( P(t) \) and \( Q(t) \) of the AM-FM operator are specified in Equations (6) and (8) respectively; and the derivation of the parameters based on the NSP algorithm is detailed in the next section. Here, we assume that the parameters are given and use \( \hat{P}(t) \) and \( \hat{Q}(t) \) to indicate the estimated parameters \( P(t) \) and \( Q(t) \) respectively.

From Equations (6) and (8), it is straightforward to obtain the estimated envelope \( \hat{a}(t) \) and phase functions \( \hat{\phi}(t) \) of a mono-component AM-FM modulated signal \( a(t) \cos(\phi(t)) \) as follows:

\[
\hat{a}(t) = e^{i \int_{t_0}^{t} -\hat{p}(\tau) \, d\tau + c} = c_1 e^{i \int_{t_0}^{t} -\hat{p}(\tau) \, d\tau}, \tag{10}
\]

\[
\hat{\phi}(t) = \int_{t_0}^{t} \sqrt{Q(\tau) - \hat{P}^2(\tau)/2} \, d\tau + c_2, \tag{11}
\]

where we assume that the signal begins at \( t = 0 \). To recover \( a(t) \) and \( \phi(t) \) from the signal, we need to estimate the constants \( c_1 \) and \( c_2 \) respectively. Since \( c_1 \) only depends on the amplitude and \( c_2 \) only depends on the phase of the signal, we estimate \( c_1 \) with \( c_1 = \sqrt{\|a(t) \cos(\phi(t))\|^2 / \|s(t)\|^2} \). After recovering \( c_1 \) and \( c_2 \) can be derived from the difference between the phase of the estimated AM-FM signal and that of the original signal. We use the Hilbert transform to estimate the phase angles of the original signal and the recovered signal. Then, we measure the mean difference of the two signals to obtain the value of \( c_2 \).

4. SEPARATING MULTICOMPONENT SIGNALS WITH NULL SPACE PURSUIT

We use the model proposed in [11] as our real multicomponent signal

\[
x(t) = \sum_{i=1}^{M} x_i(t) = \sum_{i=1}^{M} a_i(t) \cos(\phi_i(t)), \tag{12}
\]

where the signal \( x(t) \) is comprised of \( M \) mono-component signals \( x_i(t) \), with \( i = 1, \ldots, M \). To separate a multicomponent AM-FM signal into a sum of mono-component AM-FM signals, we need to tailor Equation (2) to suit our purpose. We use the NSP algorithm to search for \( P(t) \), \( Q(t) \) and \( \hat{R}(t) \) that minimize the equation

\[
\| \mathcal{T}_S(S(t) - R(t)) \|^2 + \lambda_1 \left( \| R(t) \|^2 + \gamma \| S(t) - R(t) \|^2 \right) + \lambda_2 \left( \| D_2 Q(t) \|^2 + \| P(t) \|^2 \right), \tag{13}
\]

where \( \mathcal{T}_S \) denotes our AM-FM operator, which is \( D_2 + P(t)D_1 + Q(t) \); \( D_1 \) and \( D_2 \) are the first and the second order differential operators; \( \lambda_1 \) and \( \lambda_2 \) are Lagrangian parameters; and \( \gamma \) is the leakage parameter, the value of which determines the amount of information about \( S(t) - R(t) \) retained in the null space of \( \mathcal{T}_S \). The terms following \( \lambda_2 \) represent the regularization of the AM-FM operator’s parameters. To minimize Equation (13), we adopt the smooth variations of \( D_2 Q(t) \), which is the weighted sum of the squared \( \mathbf{IF} \) and the squared instantaneous bandwidth, and the small \( P(t) \), which is the squared instantaneous bandwidth caused by amplitude modulation. The NSP algorithm can estimate \( \lambda_1 \) and \( \gamma \), adaptively; however, \( \lambda_2 \) is insensitive to signals, so we set the value as a constant in our implementation.

In a discrete representation, for ease of presentation, we use bold upper case, e.g. \( \mathbf{A} \), to represent matrices and bold lower case, e.g., \( \mathbf{a} \), to represent vectors. The matrix \( \mathbf{A}_x \) denotes a diagonal matrix in which the diagonal elements are equal to the vector \( \mathbf{x} \). In a discrete case, \( S(t), R(t), P(t) \) and \( Q(t) \) can be represented as column vectors \( s, r, p \) and \( q \) respectively; and \( D_1 \) and \( D_2 \) can be represented as the matrices of the first and second order differences, \( D_1 \) and \( D_2 \) respectively. Equation (13) can then be rewritten as

\[
\mathcal{F}(p, q, r) = ||(D_2 + \mathbf{A}_p D_1 + \mathbf{A}_q)(s - r)||^2 + \lambda_1 (||r||^2 + \gamma ||s - r||^2) + \lambda_2 (||D_2 q||^2 + ||p||^2), \tag{14}
\]

Let \( \Phi \) be the vector that contains all the parameters of the operator \( \Phi = [p^T, q^T]^T \). Then, Equation (14) becomes

\[
\mathcal{F}(\Phi, r) = ||(D_2 + B_3 M_1)(s - r)||^2 + \lambda_1 (||r||^2 + \gamma ||s - r||^2) + \lambda_2 ||M_2 \Phi||^2, \tag{15}
\]

where \( B_3 = [ \mathbf{A}_p \mathbf{A}_q ], \ M_1 = [ D_1^T \mathbf{E}^T ]^T \), and \( M_2 = [ D_2 \mathbf{E} ] \) in which \( \mathbf{E} \) is the identity matrix. By taking the partial derivative of \( \mathcal{F} \) with respect to \( \Phi \) and setting the result to zero, we have

\[
\frac{\partial \mathcal{F}}{\partial \Phi} = \mathbf{A}^T (D_2(s - r) + \mathbf{A} \Phi) + \lambda_2 M_2^T M_2 \Phi = 0, \tag{16}
\]

where \( \mathbf{A} = [ \mathbf{A}_{D_1(s-r)} \mathbf{A}_{s-r} ] \). This leads to

\[
\hat{\Phi} = -(\mathbf{A}^T \mathbf{A} + \lambda_2 M_2^T M_2)^{-1} \mathbf{A}^T D_2(s - r). \tag{17}
\]

Similarly, to estimate \( \hat{r} \), we use the equation \( \frac{\partial \mathcal{F}}{\partial r} |_{\Phi=\hat{\Phi}} = 0 \) and obtain

\[
\hat{r} = (\mathbf{T}^T \mathbf{T} + (1 + \gamma) \lambda_1 \mathbf{E})^{-1} (\mathbf{T}^T \mathbf{T} s + \lambda_1 \gamma s), \tag{18}
\]

where \( \mathbf{T} = D_2 + B_3 M_1 \) and \( \mathbf{E} \) is the identity matrix.

Following the setting of the NSP algorithm, described in [7], the parameters \( \lambda_1 \) and \( \gamma \) can be calculated as follows:

\[
\lambda_1 = \frac{1}{1 + \gamma} \frac{s^T M(\lambda_1, \gamma, \hat{T}) s}{s^T M(\lambda_1, \gamma, \hat{T}) s}, \tag{19}
\]
where $M(\lambda_1, \hat{\gamma}, \hat{T}) = (\hat{T}^T \hat{T} + (1 + \hat{\gamma})\lambda_1 E)^{-1}$ with $\hat{T} = D_2 + B_2 M_1$, and
\[
\gamma = \frac{(s - \hat{r})^T s}{|s - \hat{r}|^2} - 1. 
\tag{20}
\]

Based on Equations from (17) to (20), we can use the NSP algorithm to separate a multicomponent AM-FM signal. By executing Algorithm NSP-AMFM, we can separate a multicomponent AM-FM signal $u_1$ from signal $s$ and obtain residual $s - u_1$. The residual signal can be used as input for the algorithm to extract other mono-component AM-FM signals, iteratively. Thus, by repeating the process $M$ times, the signal $s$ can be decomposed into a sum of $M$ mono-component AM-FM signals as $s = \sum_{j=1}^{M} u_j + r_M$.

**Algorithm 1** NSP-AMFM

1. Input signal $s$ and parameter $\lambda_2$, choose a stopping threshold $\epsilon$ and the values of $\lambda_0$ and $\gamma^0$.
2. Set $j \leftarrow 0$, $\hat{r} \leftarrow 0$, $\lambda_j^0 \leftarrow \lambda_0^0$ and $\gamma^j \leftarrow \gamma^0$.
3. repeat
   4. Compute $\Phi_j$ to obtain $\hat{p}_j$ and $\hat{q}_j$ according to Equation (17) using $\hat{r}_j$.
   5. Compute $\lambda_j^0$ according to Equation (19) using $M(\lambda_j^0, \gamma^j, \hat{T}_j)$.
   6. Compute $\hat{r}_{j+1}$ according to Equation (18) using $\gamma^j, \hat{p}_j, \hat{q}_j$ and $\lambda_j^0$.
   7. Compute $\gamma^{j+1}$ according to Equation (20) using $\hat{r}_{j+1}$ and set $j = j + 1$.
8. until $||\hat{r}_{j+1} - \hat{r}_j||^2/||\hat{r}_j||^2 < \epsilon$
9. return Extract AM-FM mono-component $\hat{u} = (1 + \gamma^j)(s - \hat{r})$ and the residual signal $\hat{r} = s - \hat{u}$.

Algorithm NSP-AMFM is close to an automatic procedure because the parameters $\lambda_1$ and $\gamma$ that are more sensitive to the separation results can be estimated adaptively. However, different values of $\lambda_j^0$ can affect the convergence rate. The value of $\gamma^0$ is set at 1. The threshold $\epsilon$ can be set as low as $1e^{-7}$. The value of the parameter $\lambda_2$, which is less sensitive to the separation result, is set at 0.0001.

**5. EXPERIMENT RESULTS**

In this section, we consider some implementation issues of the proposed algorithm, and evaluate our proposed algorithm using several simulated and real-life signals.

**5.1. Implementation details**

In line 4 of Algorithm NSP-AMFM, we use Equation (17) to estimate all the $P$ and $Q$ values of the signal simultaneously in each iteration. However, because there are two parameters $P(t)$ and $Q(t)$ to estimate at each $t$, Equation (16) is an under-determined system; and matrix $A^T A$ in Equation (17) is a singular matrix. Thus, in our implementation, at each point $t_0$, we select a neighborhood $B_{t_0}$ of the point and assume that, in $B_{t_0}$, the values of parameters $P(t)$ and $Q(t)$ equal $P(t_0)$ and $Q(t_0)$ respectively with $t \in [t_0 - \Delta t, t_0 + \Delta t]$. Then, the parameters at $t_0$ are estimated by
\[
[\hat{P}(t_0), \hat{Q}(t_0)]^T = (A_{t_0}^T A_{t_0} + \lambda_2 E_2)^{-1} A_{t_0}^T D_2(s_{t_0} - r_{t_0}),
\tag{21}
\]
where $A_{t_0}$ has the form of matrix $A$, defined in Equation (16), but it is restricted to the data point in the neighborhood $B_{t_0}$ of $t_0$. Similarly, $s_{t_0}$ and $r_{t_0}$ are the corresponding data points restricted in $B_{t_0}$. The matrix $E_2$ is a $2 \times 2$ identity matrix. After estimating $\hat{P}(t)$ and $\hat{Q}(t)$, we use a low pass filter to smooth the values of the two functions. Note that this is an approximation of applying $M_2$ to the estimation of the parameters.

**5.2. Performance evaluations and discussions**

In the first simulated example, we aim to demonstrate the accuracy and robustness of using the proposed method to remove the noise from a noisy AM-FM signal and then demodulate the denoised signal. Figure 1(a) shows the monocomponent AM-FM signal $s(t) = (2 + \cos(2\pi t)) \cos(16\pi t +...
stronger than those of the others. The input simulated signal is a multi-component chirp signal represented as follows:

\[ y(n) = \sum_{i=1}^{4} s_i(t) = \sum_{i=1}^{4} A_i (2 + \cos(v_{a1}t)) \cos(v_{b1}t^2 + v_{c1}t). \]  

The values of the four sets of parameters in Equation (22) are chosen as follows:

- \( A_1 = 2.0, v_{a1} = 1, v_{b1} = 0.1, v_{c1} = 10 \)
- \( A_2 = 5.0, v_{a2} = 2, v_{b2} = 0.2, v_{c2} = 20 \)
- \( A_3 = 0.6, v_{a3} = 3, v_{b3} = 0.3, v_{c3} = 30 \)
- \( A_4 = 0.3, v_{a4} = 4, v_{b4} = 0.4, v_{c4} = 40 \)

We choose the values so that the amplitudes of \( s_1(t) \) and \( s_2(t) \) are much stronger than those of \( s_3(t) \) and \( s_4(t) \). Figure 2 shows the input multicomponent AM-FM signal, its spectrum, and the decomposition results derived by the NSP-AMFM algorithm and the EEMD algorithm [12]. From the second row to the fifth row of Figure 2, the left column and right column show the derived results of the NSP-AMFM algorithm and the EEMD algorithm, respectively. The first two subcomponents extracted by both methods, shown in the second and the third row of Figure 2, are similar. However, the EEMD method cannot successfully separate \( s_3(t) \) and \( s_4(t) \), as shown in the third and the fourth row of Figure 2. The amplitude of \( s_2(t) \) is so large than those of \( s_3(t) \) and \( s_4(t) \) that the extrema of \( s_3(t) \) and \( s_4(t) \) are completely covered in \( s_2(t) \). This phenomenon makes the extraction of the extrema of \( s_3(t) \) and \( s_4(t) \) difficult and it leads to the failure of the EEMD algorithm to separate the signals correctly [13].

A speech signal can be modeled as a multicomponent AM-FM signal. In the third experiment, we demonstrate the efficacy of separating the phonics structure of a speech signal by our algorithm. The input signal, shown at the top row of Figure 3, is the digitalized sound of the word, /ga¨o/, at 16KHz digitization. Under NSP-AMFM, the frequency of the first extracted component is the fundamental frequency of the sound. The frequency of the fourth subcomponent is an overtone, whose frequency is double that of the first subcomponent. The second, third and fifth subcomponents are pronounced as /g/, /¨o/, and /a/ respectively. From the time axis of the subfigures in Figure 3, it is clear that the order of appearance of the energy peaks of the three subcomponents is in accordance with the pronunciation order.

\[ \sin(\pi t) \]

We add white Gaussian noise to \( s(t) \) and acquire nine noisy signals, whose SNRs range from 2\( \text{dB} \) down to \(-9\text{dB}\). Figures 1(b) and 1(c) show the noisy signal of SNR \(-7.4\text{dB}\) and its denoised signal of SNR 9.8\( \text{dB}\) by our method, respectively. The X-axis and Y-axis of Figure 1(d) indicate the SNR of a noisy signal and the denoised signal, respectively. In Figure 1(d), we compare the performance of noise reduction by using Algorithm NSP-AMFM and Algorithm NSP [7]. From which, we can find that the performance of the proposed method is better than that of the NSP algorithm. This is because our operator annihilates an AM-FM signal; while that of NSP annihilates a FM signal. In Figures 1(e) and 1(f), we compare the demodulation results of our operator and T-K operator on the signals extracted by using the NSP-AMFM algorithm. Our operator and T-K operator are comparable in demodulating IF. However, our operator is slightly better than that of T-K operator in demodulating an envelope.

In the second example, we aim to demonstrate that the proposed method can separate multicomponent chirp signals, even when the amplitudes of some components are much

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**Fig. 2.** The subfigures in the left and right columns of the first row are the input signal and its spectrum, respectively. The frequency axis is normalized so that \( \pi \) corresponds to 0.5. The left column: From the second row to the fifth row are the extracted subcomponents by the NSP-AMFM algorithm and the residual signal is at the end row. Similarly, shown in the right row are the subcomponents extracted by the EEMD algorithm and its residual is at the end row.

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Fig. 3. The phonic structure of /gä̂o/ signal. First row: the input signal. From the second row to the sixth row: the fundamental frequency, sound /g/, sound /ä̂/, the overtone of the fundamental frequency, sound /a/, and the residual signal.

results of experiments on several simulated and real-life signals demonstrate that the proposed algorithms can robustly separate and demodulate multicomponent AM-FM signals.

In addition, we find that different types of operators can characterize different properties of a signal. And hence, analyzing the properties and abilities of different kinds of operators are important issues that warrant further study.

7. REFERENCES


