

Single j Shell Observations

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In this report we examine two topics relating to previous work. However we feel that there are points to be made which we have not made before. A common thread in the two problems is that they both involve the isospin variable in an important way.

In a publication by Devi et. al. [1] we calculated the excitation energies of $T=T_{min}+1$ states in even-odd nuclei and of $T=T_{min}+2$ states of even-even nuclei in the f-p shell where $T_{min}=\frac{|N-Z|}{2}$. We performed a linear fit to these excitation energies

$$\begin{aligned} E(SA) &= b(T + X) \\ E(DA) &= 2b(T + X + \frac{1}{2}) \end{aligned}$$

For a simple interaction of the form $a+b t(1)\cdot t(2)$, the value of X is unity.

We point out that with a simple adjustment we can also convert this to a formula for binding energies and hence obtain a term linear in isospin, which was first found by Wigner. [2,3] We simply assume that the binding energy is given by

$$E = \frac{b}{2}T(T + Y) \tag{1}$$

It is then easy to show that $Y=2X-1$. For the $t(1)\cdot t(2)$ interaction we have $X=1$, $Y=1$. For the Wigner SU(4) limit, we have $X=2.5$, $Y=4$. It is worthwhile to note that in mean field theories we cannot obtain a linear term in T, but in shell model calculations it is impossible to not get such a term.

In [1] we performed a fit to the single j shell calculation. We found that a good fit was obtained with $b=2.32$ and $X=1.3$. This leads to a binding formula in the single j shell

$$E = \frac{b}{2}T(T + 1.6) \tag{2}$$

In Talmi's book [4], expressions for the binding energy in both the SU(4) limit and the seniority conserving limit are shown. In the former case the binding energy goes as $T(T+4)$

and in the latter as $T(T+1)$. It has been pointed out by McCullen et. al. [5] that although seniority may be a pretty good quantum number for a system of identical nucleons, e.g. the Calcium isotopes, seniority is badly broken when we have both protons and neutrons in open shells. This formula $T(T+1.6)$ lies in between the two extremes - one of seniority conservation for mixed protons and neutrons and the other of the $SU(4)$ limit in spin and isospin variables.

The next problem we consider takes note of the fact that the ground state wavefunctions of even-even Ti isotopes bear some resemblance to IBA wavefunctions. There are various versions of the Interacting Boson Approximation, IBA1 [6], IBA2 [7], and IBA3 [8]. The format of the Ti wavefunctions in MBZ [5] most closely resembles that of IBA2. We here define a model which we refer to as single j IBA.

In Table 1 we show the MBZ wavefunctions for the $J=0^+$ T_{min} ground states of $^{44,46,48}\text{Ti}$ as well as the unique (in the single j shell model) $T_{min}+2$ states. The wavefunction is written as

$$\psi^I = \sum D^I(J_p J_n) [(j^2)^{J_p} (j^r)^{J_n}]^I \quad (3)$$

where I is the total angular momentum and $D^I(J_p J_n)$ is the probability amplitude that the protons couple to J_p and the neutrons couple to J_n . For $I=0$ $J_p = J_n = J$.

In the single j model space, the states with the higher isospin $T_{min}+2$ are not affected by any isospin conserving two nucleon interaction. In fact for these states the coefficients $D^I(J_p J_n)$ are two particle coefficients of fractional parentage. The reason for this is that these states in Ti are double analogs of corresponding states in Ca, and for Ca we are dealing with a system of identical particles i.e. only $f_{7/2}$ neutrons. A two particle cfp will be an expansion in which neutrons are separated into $(n-2)$ and 2 . We can then easily see the following for $I=0$

$$D^I(JJ) = (j^n J; j^2 J \} j^{n+2} 0) \quad (4)$$

And as the $D^I(J_p J_n)$ satisfy the orthonormality conditions

$$\sum_{J_n J_p} D^{I\alpha}(J_p J_n) D^{I\alpha'}(J_p J_n) = \delta_{\alpha\alpha'} \quad (5)$$

So that in particular any T_{min} state is orthogonal to a state with $T=T_{min}+2$

We now define the single j IBA. Note that the ground state wavefunction amplitudes for all three Ti isotopes. The largest amplitudes have $J_p=J_n=0$ and $J_p=J_n=2$. The other amplitudes ie (4,4), (6,6) etc are very small. This motivates us to consider a simple model where the only non-vanishing D's are $D(00)$ and $D(22)$.

$$\psi \approx D(00)[(j^2)^0(j^n)^0] + D(22)[(j^2)^2(j^n)^2] \quad (6)$$

With the conditions that

$$D(00)^2 + D(22)^2 = 1 \quad (7)$$

and that the orthonormality to the $T_{min}+2$ state is maintained

$$D(00)(j^n 0; j^2 0 | j^{n+2} 0) + D(22)(j^n 2; j^2 2 | j^{n+2} 0) = 0 \quad (8)$$

But these two conditions mean that $D(00)$ and $D(22)$ are completely determined - there is no freedom. We can show that the wavefunctions, written as two component vectors for the various Ti isotopes are

$$\begin{aligned} \psi_{44Ti} &= \frac{1}{\sqrt{14}}(\sqrt{5}, 3) = (0.5976, 0.8018) \\ \psi_{46Ti} &= \frac{1}{\sqrt{8}}(\sqrt{5}, \sqrt{3}) = (0.7906, 0.6124) \\ \psi_{48Ti} &= \frac{1}{\sqrt{6}}(\sqrt{5}, 1) = (0.9129, 0.4082) \end{aligned}$$

Comparing with the results of Table 1 we see that for ^{44}Ti there is too much $J=2$ coupling - more than $J=0$. However the trend as one goes through the Ti isotopes is quite reasonable and the wavefunctions for ^{48}Ti are remarkably similar.

The important point we wish to make here is that in the single j shell model, once we make the assumption that the $T=T_{min}$ state consists of only $J_p=J_n=0$ and $J_p=J_n=2$ couplings, the relative amounts of the couplings is fixed. There is no freedom. The reason for this is that the states with T_{min} must be orthogonal to the states with $T_{min}+2$. A small amount of the higher J couplings restores the freedom to adjust the relative amounts of $J=0$ and $J=2$. However, this effect remains small at midshell (ie. ^{48}Ti).

This work was supported by the U.S. Dept. of Energy under Grant No. DE-FG02-95ER-40940.

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TABLES

TABLE I. Wave functions of $I = 0_1$, T_{min} and $I = 0$, $T_{min} + 2$ states of ^{44}Ti , ^{46}Ti and ^{48}Ti .

The symbol * means $v = 4$.^a

^{44}Ti	J_P	J_N	$I = 0 \ T = 0$	$I = 0 \ T = 2$
	0	0	0.7608	0.5000
	2	2	0.6090	-0.3727
	4	4	0.2093	-0.5000
	6	6	0.0812	-0.6009
^{46}Ti	J_P	J_N	$I = 0 \ T = 1$	$I = 0 \ T = 3$
	0	0	0.8224	0.3162
	2	2	0.5420	-0.4082
	2	2*	0.0563	0.0
	4	4	0.0861	-0.5477
	4	4*	-0.1383	0.0
	6	6	-0.0127	-0.6583
^{48}Ti	J_P	J_N	$I = 0 \ T = 2$	$I = 0 \ T = 4$
	0	0	0.9136	0.1890
	2	2	0.4058	-0.4226
	4	4	0.0196	-0.5669
	6	6	-0.0146	-0.6814

a)The phases have been adjusted to fit with the cfp conventions of [4] and differ in some way with those in [5].