High-Confidence Off-Policy Evaluation

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Overview

This paper develops a method to compute the confidence that the expected return of a policy exceeds some lower bound, using only trajectories generated from other policies.

Terminology:

- *Evaluation Policy* - The policy which we wish to estimate the expected return of.
- *Behavior Policy* - The policies used to estimate the return of the evaluation policy.

Motivation

"..execution of a new policy can be costly or dangerous if it performs worse than the policy that is currently being used..."

Examples:

- News recommendation systems
- Patient diagnosis systems
- Neuroprosthetic control
- Automatic drug administration

Problem Setup

This work follows the standard Markov Decision Process (MDP) formalism:

- $\mathcal{S}$: State space.
- $\mathcal{A}$: Action space.
- $r_t \in [r_{max}, r_{min}]$: Reward at time t.
- $\gamma \in [0, 1]$: Discount factor.
- $\pi(a \mid s, \theta)$: Probability of taking action a in state s given policy parameters θ.
- $\tau = \{s_1, a_1, r_1, \ldots, s_T, a_T, r_T\}$: A trajectory.

Problem Setup

Define the normalized and discounted *return* of a trajectory to be:

$$R(\tau) = \frac{(\sum_t \gamma^{t-1} r_t) - R_-}{R_+ - R_-}$$

Where R_+ and R_- are upper and lower bounds for $\sum_t \gamma^{t-1} r_t$.

Ideally, we want to know the expected return given the *evaluation policy* parameters θ:

$$\rho(\theta) = \mathbb{E}\left[R(\tau) \,|\, \theta\right]$$

Generating Unbiased Estimates of $\rho(\theta)$

Key Idea

Given a dataset $\mathcal{D} = \{(\tau_i, \theta_i) : \tau_i \text{ generated using } \theta_i\}$ estimate $\rho(\theta)$ using *importance sampling*.

Importance Sampling

Given a *target distribution* p and *sampling distribution* q and a function f:

$$\mathbb{E}_{x \sim p}(f(x)) = \frac{1}{n} \sum_{i=1}^{n} f(x_i) \frac{p(x_i)}{q(x_i)}$$

where $x_i \sim q$.

Generating Unbiased Estimates of $\rho(\theta)$

In the context of our problem, the *target distribution* is $Pr(\tau_i \mid \theta)$ (e.g. the probability of a trajectory under the *evaluation policy*), and the *sampling distribution* is $Pr(\tau_i \mid \theta_i)$ (e.g. the probability of a trajectory under the *behavior policy* it was generated by).

The expected return of the *evaluation policy* can then be estimated by:

$$\begin{aligned}
\rho(\theta) &\approx \frac{1}{n}\sum_{i=1}^{n} R(\tau_i)\frac{Pr(\tau_i \mid \theta)}{Pr(\tau_i \mid \theta_i)} \\
&= \frac{1}{n}\sum_{i=1}^{n} \underbrace{R(\tau_i)\prod_t \frac{\pi(a_t \mid s_t, \theta)}{\pi(a_t \mid s_t, \theta_i)}}_{\hat{\rho}(\theta,\tau_i,\theta_i)}
\end{aligned}$$

Interlude

Recall our motivation:

"..execution of a new policy can be costly or dangerous if it performs worse than the policy that is currently being used..."

"Classical" Results

In the following slides, we will review a few "classical" results providing lower bounds for the expectation of a random variable that can be estimated using samples.

We introduce the following variables/assumptions:

- X_i: real-valued, positive, bounded random variables (e.g. importance weighted returns).
- μ: $\mathbb{E}(X_i)$ for all X_i.
- b: A real-number satisfying $Pr[X_i < b] = 1$ for all X_i.

"Classical" Results

Chernoff-Hoeffding (CH) inequality

With probability at least $1 - \delta$:

$$\mu \geq \frac{1}{n} \sum_{i=1}^{n} X_i - b \sqrt{\frac{\ln\left({}^{1}\!/\!_{\delta}\right)}{2n}}$$

Anderson (AM) inequality

With probability at least $1 - \delta$:

$$\mu \geq z_n - \sum_{i=0}^{n-1} (z_{i+1} - z_i) \min\left\{1, \frac{i}{n} + \sqrt{\frac{\ln({}^{2}\!/\!_{\delta})}{2n}}\right\}$$

where z_i are the samples X_i in increasing order, and $z_0 = 0$.

"Classical" Results

Maurer & Pontil's empirical Bernstein (MPeB) inequality

With probability at least $1-\delta$:

$$\mu \geq \frac{1}{n}\sum_{i=1}^{n} X_i - \frac{7b\ln\left({}^{2}\!/\!_{\delta}\right)}{3(n-1)} - \frac{1}{n}\sqrt{\frac{\ln\left({}^{2}\!/\!_{\delta}\right)}{n-1}\sum_{i,j=1}^{n}(X_i - X_j)^2}$$

New Result

The author's use the **MPeB** inequality to prove the following:

Theorem 1

Let $Y_i = \min\{X_i, c_i\}$ where $c_i > 0$, with probability at least $1 - \delta$:

$$\mu \geq \left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sum_{i=1}^{n} \frac{Y_i}{c_i} - \left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \frac{7n \ln\left(2/\delta\right)}{3(n-1)}$$
$$- \left(\sum_{i=1}^{n} \frac{1}{c_i}\right)^{-1} \sqrt{\frac{\ln\left(2/\delta\right)}{n-1} \sum_{i,j=1}^{n} \left(\frac{Y_i}{c_i} - \frac{Y_j}{c_j}\right)^2}$$

Additional Remarks

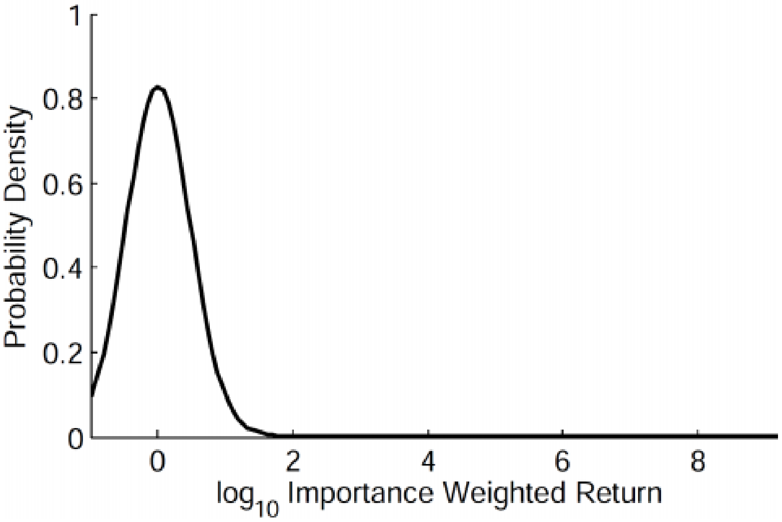
- These inequalities can be inverted so that given a lower bound μ_- we can determine the confidence (e.g. value of δ) that $\mu \geq \mu_-$.
- Theorem 1 requires pre-specified thresholds c_i. In the paper, the authors select a single value c^* and set all $c_i = c^*$. They show how an optimal value of c^* can be determined by splitting the dataset and performing cross-validation.

Experiment 1: The Mountain Car Problem

Demo

https://www.youtube.com/watch?v=x_qDs2kA7H4&feature=youtu.be

Experiment 1: The Mountain Car Problem



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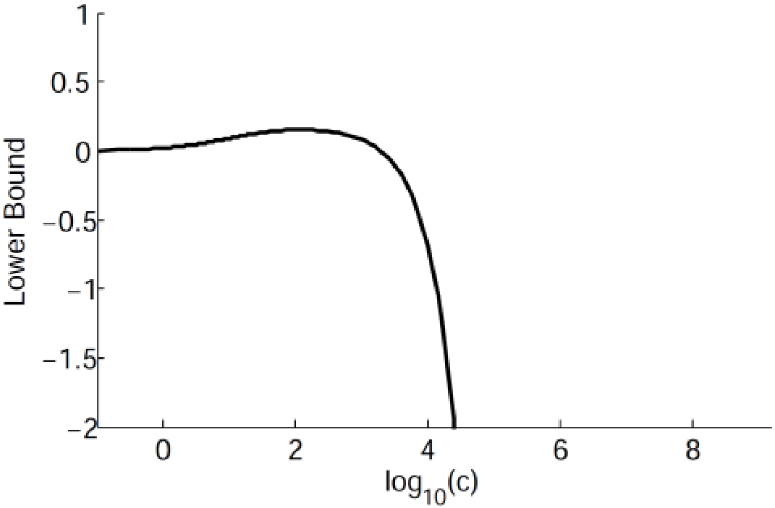


Experiment 1: The Mountain Car Problem

Method	ρ_-
Thm. 1	0.154
CH	-5,831,000
MPeB	-129,703
AM	0.055

Table: 95% confidence lower bounds

Experiment 2: Digital Marketing using Real-World Data

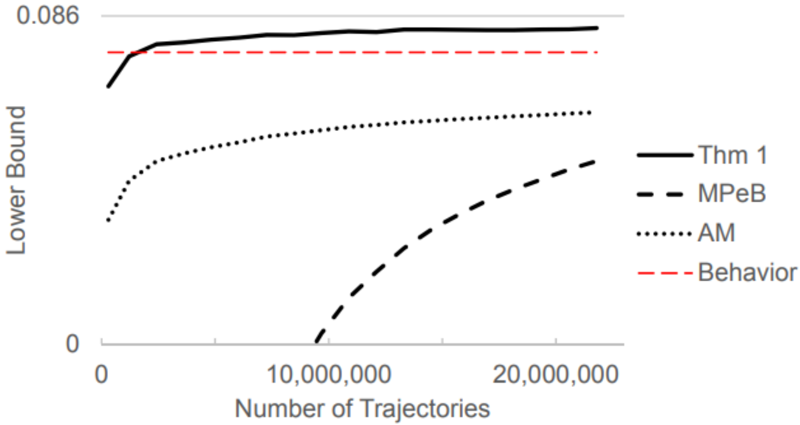
Problem

Decide the optimal policy for user-specific targeting of advertisements. Rewards are measured by advertisement click rates (e.g. agent recieves $+1$ if a user clicks the ad, and 0 otherwise).

Data

- From a website for a Fortune 50 company.
- $> 100,000$ visitors a day.
- Each user has 31 features.
- Agent must select from two clusters of advertisements.

Experiment 2: Digital Marketing using Real-World Data



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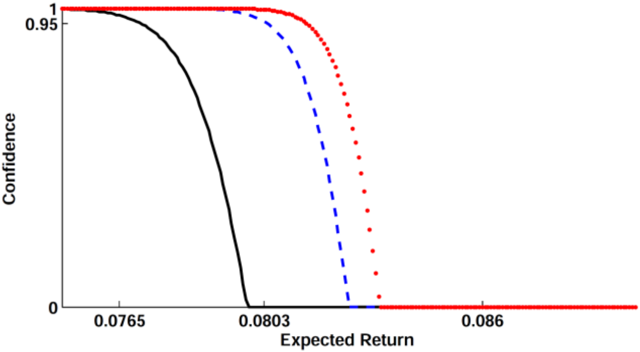

Black - 2 million off-policy trajectories
Blue - 5 million off-policy trajectories
Red - 5 million off-policy trajectories + 1 million on-policy trajectories