Indoor Wireless Localization via Convex Feasibility Problem

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Abstract—Node localization in wireless sensor networks (WSNs) aims to determine unknown target position given anchors’ known positions and available relative pairwise measurements among directly-connected neighboring nodes. From this point of view, WSNs can be divided into two groups based on collaboration between targets: cooperative networks in which the measurements between the targets are also used for positioning procedure, and non-cooperative networks in which only measurements between pairs of anchor nodes and target nodes are utilized. Obtaining a good estimation of where targets are located at is critical in sensor network applications such as monitoring, geographic routing, and industrial manufacturing. Since the positions of the targets are unknown and only local distance information is given, we need to find true positions from these local distance measurements. Maximum likelihood estimation is one of the state-of-art method for this problem, which is non-convex and consumes much effort to solve. Moreover, the quality of obtained solution is highly dependent on the chosen search methods and the used initial points. In this paper, we formulate the non-cooperative sensor network localization as a convex feasibility problem where the unknown node belongs to a sufficient intersection set of a family of closed convex sets, then apply a projection method to find the solutions. This approach leads us to the use of different algorithm which is fast, robust and convergent. With an appropriate step-size sequence, estimated sensor locations will converge to the global optimal solution in a finite number of steps, as shown by the numerical results. The proposed approach has good stability, robustness and lower complexity, which is effective for the localization problems with small sensor size constraints.

Index Terms—wireless sensor network; localization; convex feasibility problem; projection method; convex optimization

I. INTRODUCTION

Wireless sensor networks have been considered for numerous applications in routing, density control, tracking, and a number of other communication network applications. Localization in wireless sensor networks is often required to ensure that the collected information is useful. The sensor network localization (SNL) problem refers to the process of estimating and computing the positions of unknown sensor nodes given positions of the known anchor nodes and their corresponding measurements. These measurements may include the information of time-of-arrival [1], [2], time-difference-of-arrival [3], [4], angle-of-arrival [5], [6], received signal strength (RSS) [7]–[9], or any combinations of them. However, the distances between sensor nodes are also required to estimate from any type of measurements. In this paper, we consider the combination of RSS measurements and time interval, which provides a perspective on improving ranging accuracy for target location.

Recently, many approaches for localization have been proposed. Convex optimization localization algorithms are popular due to their global convergence property. The maximum likelihood (ML) estimator was studied in [10], but it is evidently nonlinear and nonconvex. The linear least squares methods provide closed form solutions. However, since the solution is obtained based on many approximations such as [11]–[13], the performance is not good in highly noisy cases. Solving SNL problems by the semidefinite programming (SDP) guarantees the convergence to the global solution with expense of high computational complexity, and it cannot achieve the best performance in all conditions [14]–[16].

In sum, most of previous approaches are not scalable and require high computational complexity. Thus, we tackle this issue by using an iterative algorithm to find the solution that minimizes the cost function. Inspired by the approach in [17], we formulate the SNL problem as a convex feasibility problem (CFP) and apply a modified projection method to solve it efficiently. Projection methods are iterative algorithms used to split CFP by performing a projection algorithm onto the individual sets from a given family of sets. This is the main advantage to allow them to have low complexity and ability to handle the huge-size problems of dimensions. In this paper, we consider the indoor non-cooperative scenario localization scenario, where the SNL problem is formulated as a CFP. A modified projection algorithm is applied for both consistent and non-consistent cases to find the solutions that make it easier to solve the nonlinear objective function with an appropriate number of iterations.

The organization of this paper is as follows. In Section II, we present the mathematical framework for the wireless sensor network localization problem, and briefly introduce the CFP. In Section III, we present our approach for node localization in wireless sensor networks by using a modified projection method and its efficiency with a simple example. The concluding remarks are presented in Section IV.

Notations: The following notations are used throughout our paper. $(\cdot)^T$ denotes the transpose operation, $\| \cdot \|$ denotes the $\ell_2$-norm, $|S|$ denotes the number of elements in a given set $S$ and $\mathbb{E}(\cdot)$ denotes the expectation operator. $P_{\Omega}(x)$ denotes the orthogonal of a point $x \in \mathbb{R}^2$ onto set $\Omega$, which is $P_{\Omega}(x) = \arg \min_{\omega \in \Omega} \|x - \omega\|$.
II. PROBLEM FORMULATION

A. Indoor Localization Scenario

We consider a two-dimensional localization problem in an indoor environment with $N$ devices called anchors whose positions $\{a_i\}_{i=1}^N$ are known, and a single point called target whose position $x \in \mathbb{R}^2$ is unknown. Assume that $x$ follows the uniform distribution which has the probability density function (pdf) as follows.

$$f(x) = \begin{cases} \frac{1}{S_R} & x \in \mathcal{R} \\ 0 & \text{elsewhere} \end{cases},$$

where $\mathcal{R}$ is the region where the target is bounded (e.g., $\mathcal{R} = \{x \in \mathbb{R}^2 : ||x||_2 \leq r, r \text{ is given}\}$) and $S_R$ is its area. Let $T$ be the set of indices associated with the anchors connected with the target, which is given by

$$T = \{i : \text{anchor } i \text{ connected with the target}\}.$$  \hspace{1cm} (2)

Taking $N_{\text{obs}} = |T|$ will return the number of observations. Then, the localization system can find the target position by searching inside the region

$$R^T = \left( \bigcap_{i \in T} \mathcal{R}^i \right) \bigcap \mathcal{R},$$ \hspace{1cm} (3)

where $\mathcal{R}^i$ denotes the coverage region of $i$-th anchor centered at $a_i$. The expected region where the target belongs to, is

$$P^T = \left( \bigcap_{i \in T} \mathcal{R}^i \right) \bigcap B(\mathcal{R}),$$ \hspace{1cm} (4)

where $B(\mathcal{R})$ is the support of $\mathcal{R}$, i.e., the bounding box of $\mathcal{R}$. Note that the target can be located in line-of-sight (LOS) or non-line-of-sight (NLOS) conditions. If we denote $N_o(a, b)$ as the number of obstructions between two points $a$ and $b$, the set $T$ can be divided into two subsets as follows.

$$T^{\text{LOS}} = \{i \in T : N_o(x, a_i) = 0\};$$

$$T^{\text{NLOS}} = T \setminus T^{\text{LOS}}.$$ \hspace{1cm} (5)

The range estimate of target $x$ from a set of $N_{\text{obs}}$ observations can be modeled as

$$z_i = d_i(x) + b_i(x) + n_i(x),$$ \hspace{1cm} (6)

where $d_i(x) = ||x - a_i||$ is the true distance between the target position $x$ and the $i$-th anchor, $b_i(x)$ represents the NLOS bias appeared from obstructions affecting the $i$-th wireless link, and $n_i(x) \sim N(0, \sigma^2_i)$ is a zero-mean Gaussian random variable with the variance $\sigma^2_i$.

For RSS measurements, $z_i$ can be expressed in dB as

$$z_i = \psi(P_{r,i}),$$ \hspace{1cm} (7)

where

$$P_{r,i}(\hat{d}_i) = P_0 - 10\eta \log_{10}\left(\frac{\hat{d}_i}{d_0}\right) + X,$$

$$\psi(P_{r,i}) = (P_0 - P_{r,i})/\rho.$$ \hspace{1cm} (8)

Here, $P_0$ denotes the RSS measurement at a reference distance $d_0$ from the transmitter, $\eta$ is the path-loss exponent, $X$ denotes the shadowing effect and $\rho$ is a specific value which is optimized in [18]. If $[t_1, t_{N_{\text{obs}}}]$ is the time interval in which we obtain the RSS values between the target and the anchors, we calculated each distance measurement in the following manner.

$$\hat{d}_i \approx 10^{(P_0 - P_{r,i})/10\eta},$$ \hspace{1cm} (9)

where $P_{r,i}$ is the average received power over the time interval. Let $\Phi = [P_0; \eta]$, then the received power at cooperative sensor can be expressed as

$$P_r = H\Phi,$$ \hspace{1cm} (10)

B. Convex Feasibility Problem

Given finite closed convex sets $C_1, \cdots, C_N$ in $\mathbb{R}^n$ with nonempty intersection $C = C_1 \cap \cdots \cap C_N$, a CFP is to find a point $x$ in $C$. If $C \neq \emptyset$, the CFP is called consistent. Otherwise, it is called inconsistent. One of popular methods for solving CFP are projection algorithms due to their simplicity and numerous applications [19], [20]. In this paper, we focus on the case where each set $C_i$ is a convex set in the sense that there exists $\zeta_i \in \mathbb{N}$ and convex polynomial function $g_{ij}, j = 1, \cdots, \zeta_i$ such that

$$C_i = \{x \in \mathbb{R}^n \mid g_{ij}(x) \leq 0, j = 1, \cdots, \zeta_i\}.$$ \hspace{1cm} (11)

Then, we can generalize the CFP by considering the explicit CFP: Find a point $x \in \bigcap_{i=1}^N C_i$. Given a current iterate $x^k$, the next iterate $x^{k+1}$ is obtained by

$$x_{k+1} = x^k + \lambda_k \left( \sum_{i=1}^N w_i^k P_{C_i}(x^k) - x^k \right),$$ \hspace{1cm} (12)

where $w_i^k$ are the weighting factors such that $w_i^k > 0$, and $\sum_{i=1}^N w_i^k = 1$, ($\lambda_k$)$k \in \mathbb{N}$ is a sequence of strictly positive relaxation parameters. If $\lambda_k = 1$, (15) is also known as the projection onto convex sets (POCS) method, which is given by

$$x_{k+1} = \sum_{i=1}^N w_i^k P_{C_i}(x^k).$$ \hspace{1cm} (13)

III. PROPOSED LOCALIZATION APPROACH

A. Localization as a Convex Feasibility Problem

Suppose that the measurement error is non-negative, the CFP for indoor localization is formulated as the following. Find $x$ such as

$$x^* \in \bigcap_{i \in T} \mathcal{R}^i \cap \mathcal{R} = \bigcap_{i \in T} C_i,$$ \hspace{1cm} (14)
where \( C_i = \mathcal{R}^i \cap \mathcal{R} \). The basic idea is to minimize the cost function \( f(X) \) defined by

\[
    f(x) = \sum_{i \in T} \left( \max \left\{ z_i - d_i(x), 0 \right\} \right)^2.
\]

Thus, to formulate our problem as a CFP, we rewrite it into the following form

\[
    f(x) = \sum_{i \in T} w_i \|x - P_i(x)\|^2,
\]

where \( P_i = P_{C_i} \). If \( C_i \) is a disc, \( P_i(x) \) can be defined as

\[
    P_i(x) \triangleq \begin{cases} 
        a_i + \frac{x-a_i}{\|x-a_i\|}, & \|x-a_i\| \geq z_i, i \in T^{LOS} \\
        a_i + (z_i-b) \frac{x-a_i}{\|x-a_i\|}, & \|x-a_i\| \geq (z_i-b), i \in T^{NLOS} 
    \end{cases}
\]

where \( b = E \left( \{b_i(x)\}_{i=1}^{N_{obs}} \right) \).

The situation can fall into two cases: consistent and inconsistent. For the consistent case, if \( x \) is prescribed, then finding the minimum of (19) can be efficiently solved. By choosing a suitable starting point \( x^0 \) for the target, we defined \( x^1, x^2, \ldots, x^k, \ldots \) by

\[
x^{k+1} = x^k + \lambda_k \left( \sum_{i \in T} \frac{w_i P_i(x^k) - x^k}{\sum_{i \in T} w_i P_i(x^k)} \right).
\]

Convergence for the general parameters \( \lambda_k \in [\epsilon, 2 - \epsilon] \) has been established for special classes of \( C_i \) [21], where \( \epsilon \) is an arbitrary small positive number. The sequence in POCs is given by the iterative step

\[
x^{k+1} = x^k + \lambda_k \left( P_i(x^k) - x^k \right).
\]

If \( \|x^{k+1} - x^k\| \) is small enough then we stop the iteration. This sequential method converges in the consistent case. In sum, for each iteration \( k \), we update

\[
x^{k+1} = \frac{1}{|T|} \sum_{i \in T} (z_i - ||x^{k+1} - a_i||)^2.
\]

In the inconsistent case, for given \( x^0 \) and \( \beta_k, \epsilon \in (0, 1) \), we consider a combination scheme given by

\[
x^{k+1} = \sum_{i \in T} w_i x^k + \beta_k \left( \sum_{i \in T} w_i P_i x^k \right) - \sum_{i \in T} w_i x^k,
\]

for some \( \beta_k \geq \epsilon \). Let \( u^k = \sum_{i \in T} w_i^k x^k \), we obtain

\[
x^{k+1} = (1 - \beta_k) u^k + \beta_k P_i(u^k).
\]

Here progress is measured in terms of \( \sum_{i \in T} \|x^k - P_i(x)\|^2 \). Thus, it would seem reasonable to consider the line given in (26). We define a new sequence \( \{\delta_k\}_{k \in \mathbb{N}} \) which is called the projection error such that

\[
P_i(u^k) = P_{C'_i}(u^k) + \delta_k,
\]

where \( C'_i = \bigcap_{i \in T} C_i \neq \emptyset \). Let \( x^* \) be the optimal solution which belongs to \( C'_i \). Thus, we have

\[
x^{k+1} - x^* = (1 - \beta_k)(u^k - x^*) + \beta_k P_{C'_i}(u^k) - x^* + \beta_k \delta_k.
\]

Then,

\[
\sum_{i} ||x^{k+1} - x^*|| \leq (1 - \beta_k) \sum_{i} ||u^k - x^*|| + \beta_k \sum_{i} ||\delta_k|| + \beta_k \sum_{i} ||P_{C'_i}(u^k) - x^*||
\]

\[
\leq \sum_{i} ||u^k - x^*|| + \beta_k \sum_{i} ||\delta_k||,
\]

where \( \delta \in \mathbb{T} \). We want \( x^{k+1} \) to converge to \( x^* \) at step \( k + 1 \), then the series \( \sum_{k \geq 0} \sum_{i \in T} \beta_k ||\delta_k|| \) must hold. It leads to some assumptions made on \( \{\beta_k\}_{k \in \mathbb{N}} \) and \( \{\delta_k\}_{k \in \mathbb{N}} \).

(i) \( \lim_{k \to +\infty} \beta_k = 0 \).

(ii) \( \lim_{k \to +\infty} \beta_k 1/\beta_k < 1 \).

(iii) \( \sum_{i \in T} ||\delta_i|| \) is bounded.

The advantage here is that convergence is ensured even when \( \bigcap_{i \in T} C_i = \emptyset \). We show that if the above conditions are satisfied, the series \( \sum_{i} ||x^{k+1} - x^*|| \) will converge.

Conversely, let \( x^{k+1} = (1 - \beta_k) u^k + \beta_k P_{C'_i}(u^k) \), we observe that

\[
||x^{k+1} - x^*||^2 \leq ||x^{k+1} - x^*||^2 + 2\beta_k ||x^{k+1} - x^*|| \cdot ||\delta_k|| + (\beta_k)^2 ||\delta_k||^2.
\]

Similarly, we also get an upper bound for \( ||x^{k+1} - x^*||^2 \) as

\[
||x^{k+1} - x^*||^2 \leq ||(u^k - x^*) + \beta_k (P_{C'_i}(u^k) - u^k)||^2 \\
\leq ||u^k - x^*||^2 + (\beta_k)^2 ||P_{C'_i}(u^k) - u^k||^2 \\
+ 2\beta_k ||u^k - x^*|| \cdot ||P_{C'_i}(u^k) - u^k||.
\]

Taking the summation according to \( i \) and \( k \) yields to

\[
\sum_{k \geq 0} \sum_{i} ||x^{k+1} - x^*||^2 \\
\leq \sum_{k \geq 0} \sum_{i} ||u^k - x^*||^2 + \sum_{k \geq 0} \beta_k (1 - \beta_k) \sum_{i} ||P_{C'_i}(u^k) - u^k||^2 \\
+ 2 \sum_{k \geq 0} (||x^{k+1} - x^*|| + \beta_k ||\delta_k||) \sum_{i} \beta_k ||\delta_k||.
\]

The convergence of \( \{x^k\}_{k \in \mathbb{N}} \) leads to \( \sup_{k \geq 0} (||x^{k+1} - x^*|| + \beta_k ||\delta_k||) < +\infty \), and the series \( \sum_{k \geq 0} \beta_k (1 - \beta_k) \sum_{i} ||P_{C'_i}(u^k) - u^k||^2 \) must converge, i.e., \( \sum_{k \geq 0} \beta_k (1 - \beta_k) \sum_{i} ||P_{C'_i}(u^k) - u^k||^2 < +\infty \). Then, \( \lim_{k \to +\infty} P_{C'_i} = u^k \).
B. Performance of Algorithm: An Example

In this section, we present a simple scenario to illustrate the aforementioned algorithms. The 9 anchors are located at (0, 0), (5, 0), (10, 0), (0, 5), (5, 5), (10, 5), (0, 10), (5, 10) and (10, 10). The unknown target node location is deployed according to (1). Assume that there are some anchor nodes which are directly connected with the target. Figure 1 illustrates the indoor room layout with the pre-defined anchor node locations.

We assume that the target was connected with four anchor nodes and their ranges are bounded by a disc with radius $R$. A summary description of simulation parameters is given in Table 1.

![Fig. 1. An example of indoor room layout.](image)

**TABLE 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSS model</td>
<td>$d_0 = 1$ m, $\rho = -0.7$ dBm/m, $P_0 = -36$ dBm, $\gamma = 3$</td>
</tr>
<tr>
<td></td>
<td>Set $\sigma_s(x) = \sigma$ in range $[0, 6]$ dB</td>
</tr>
<tr>
<td></td>
<td>$X \sim U[2, 3]$ dBm. $U(\cdot)$ denotes uniform distribution</td>
</tr>
<tr>
<td>Environments</td>
<td>NLOS $b \sim \exp(0.5 \times \left(\frac{\text{RSS}}{120}\right)^2)$</td>
</tr>
<tr>
<td>CFP</td>
<td>$u_1 = 1/7$, $b_1 = 1/7$, $R = 10$ m</td>
</tr>
<tr>
<td>Iterations</td>
<td>Maximum number of iterations $N_{\text{max}} = 300$</td>
</tr>
</tbody>
</table>

According to the distribution of target given by (1), we evaluate the robustness of our proposed algorithm in LOS and NLOS environments by considering the normalized cumulative distribution function (cdf) of the term $||x^* - x||/||x||$, where $x^*$ is the corresponding estimated solution. Figure 2 depicts the normalized cdf errors for our proposed method in LOS and NLOS environments. It is observed that the method achieves a good performances when the measurement error is small. However, when the measurement error becomes large, the method performs poorly in both LOS and NLOS cases, especially in NLOS case. This is because it might fall into the situations where the measured distance from the target node to the anchor node is much larger than the real distance, even when the target is closer to the anchor. Then, it causes a large error to the results.

![Fig. 2. Normalized cdf errors in LOS and NLOS cases.](image)

We define the location estimation error for the unknown target node $x$ as $e \triangleq ||x^* - x||$. Then, the root mean square error (RMSE) is given by $\text{RMSE} \triangleq \sqrt{E(e^2)} = \sqrt{E(||x^* - x||^2)}$.

Figure 3 illustrates the RMSE versus $\sigma$ of the ML, the approach in [17] and our proposed solutions. When $\sigma$ is small (0 and 1), it is observed that the RMSE of our approach gives the performance that close to the ML solution, and performs better than the existing approach in [17]. When $\sigma$ becomes large, our approach still has the same estimation accuracy with the approach [17]. Note that since the quality of the ML solution depends on the initial starting points for a particular search method, then it can obtain a global solution. However, it is still non-convex and takes a long run-time to find a better solution compared to our approach and the approach [17].

IV. CONCLUDING REMARKS

In this paper, we propose an approach to locate unknown target nodes in cooperative wireless sensor network scenario, where the SNL problem is considered as a CFP and solved by a modified projection method. Analysis and simulation show that by iteratively running for all target sensors of the considered network, our algorithm ensures global convergence to the correct positions of all sensors even in the presence of large measurement error.
of small measurement errors. There are some interesting points which should be remarked. First, further analysis is required for both consistent and inconsistent cases without the assumptions we noted before. Second, we have assumed that the measurement noise is positive, which may not be realistic and will be consider as a future work.

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