Near Maximum-Likelihood Detector and Channel Estimator for Uplink Multiuser Massive MIMO Systems with One-Bit ADCs

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Abstract—In massive multiple-input multiple-output (MIMO) systems, it may not be power efficient to have a high-resolution analog-to-digital converter (ADC) for each antenna element. In this paper, a near maximum likelihood (nML) detector for uplink multiuser massive MIMO systems is proposed where each antenna is connected to a pair of one-bit ADCs, i.e., one for each real and imaginary component of the baseband signal. To achieve low complexity in the proposed nML detector, a strict constraint on the possible transmitted symbols in the original maximum likelihood (ML) detection problem is relaxed to formulate an ML estimation problem. Then, the ML estimation problem is converted into a convex optimization problem which can be efficiently solved. After obtaining the estimate by solving the convex problem, the base station can perform simple symbol-by-symbol detection for the transmitted signals from multiple users.

In the proposed detector, it is necessary for different transmit vectors to output different quantized signals to prevent detection error. Therefore, the exact probability for two different transmit vectors to output the same quantized signal is derived in a certain condition. The probability goes to zero as the number of receive antennas goes to infinity, which indicates that with enough antennas it is possible to overcome the fundamental limitations of one-bit ADCs. The results are generalized to other scenarios using classical combinatorial geometry, which shows the benefit of massive MIMO for one-bit ADCs.

Numerical results show that the proposed nML detector is efficient enough to simultaneously support multiple uplink users adopting higher-order constellations, e.g., 16 quadrature amplitude modulation. Since our detector makes use of the channel as part of the decoding, an ML channel estimation technique with one-bit ADCs that shares the same structure with our proposed nML detector is also developed. The proposed detector and channel estimator provide a complete low power solution for the uplink of a massive MIMO system.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is a transmission technique for cellular systems that leverages a large number of base station antennas to support many single antenna users. With enough antennas, massive MIMO can eliminate inter-user interference completely using matched beamforming for downlink and matched combining for uplink if the base station has full channel state information (CSI). Because of its simple beamforming and combining structures, massive MIMO will be beneficial not only for cellular systems but also for other wireless communication systems, e.g., vehicular-to-everything (V2X) communications using many antennas.

There are several practical constraints that are encountered when implementing massive MIMO systems. Because of the large number of antennas, it may not be possible to deploy expensive and powerful hardware with small noise and distortion at the base station. Prior work studied the impact of several hardware impairments including phase-drifts due to non-ideal oscillators and distortion noise caused by analog-to-digital converters (ADCs) for massive MIMO systems. It was shown in [3]–[5] that having a large number of antennas helps to mitigate these hardware impairments, which confirms the benefit of massive MIMO.

In this paper, we focus on uplink multiuser massive MIMO systems using extremely low-resolution of one-bit ADCs at the base station. Because the power consumption by ADCs grows exponentially with their resolution level [6], [7], using one-bit ADCs may be a practical way of implementing cost-efficient and green massive MIMO systems. It is expected that using one-bit ADCs is particularly beneficial for wideband communication systems (when properly handling frequency selectivity) that require high sampling frequency, which will become common in millimeter wave (mmWave) communication systems [8]. Adopting one-bit ADCs for uplink massive MIMO, however, is challenging because of the severe threshold applied to the received signal (after it has passed through the channel and noise was added).

Using low-resolution ADCs for wireless communications has been investigated under various assumptions. It was shown in [9] for one-bit ADCs and in [10] for low-resolution (one to three bits) ADCs that the capacity maximizing transmit signals for single-input single-output (SISO) channel are discrete, which is different from the unquantized output case. It was shown in [11], [12] that oversampling recovers some of the loss in the SISO channel capacity occurred by one-bit quantization. The mutual information of the MIMO channel quantized output was studied without optimizing input distribution in [13] for one-bit ADCs and in [14], [15] for low-resolution ADCs. The input distributions were optimized to achieve the capacity of the quantized MIMO channel using one-bit ADCs in [16], [17]. Interestingly, the impact of having low-resolution ADCs is not that severe. For example, [15] showed that the rate loss due to using one-bit ADCs is 1.5793 bits/s for 4×4 MIMO with a quadrature phase shift keying.
(QPSK) constellation. For SISO channels, it was shown in [13] that even a low-complexity suboptimal QPSK detector with one-bit ADC suffers only 1-3 dB signal-to-noise-ratio (SNR) loss compared to the unquantized case. It was proven in [16] that the mutual information of MIMO with one-bit ADCs is only $2/\pi$ times smaller compared to that of the unquantized MIMO case in the low SNR regime. With optimized threshold, this gap can vanish in the low SNR regime [19].

Aforementioned work on quantized MIMO [13–17] was restricted to point-to-point communications and focused on studying capacity or designing optimal transmit signals for quantized channel outputs using low-resolution ADCs without proposing specific signal processing algorithms for detecting the received signals. For point-to-point communications with low, but more than one-bit resolution (e.g., 2-3 bits) ADCs, a modified minimum mean square error (MMSE) detector was proposed in [20] and later extended to an iterative decision feedback equalizer in [21]. It is possible to extend the detection technique developed in [21] to multiuser scenarios; however, the detector parameters become dependent on the constellation. In [22], digital and analog combiners were compared in terms of the achievable rate for point-to-point mmWave systems where the derived achievable rates were tight only for the low SNR regime.

There are several recent papers on massive MIMO with low-resolution ADCs. It was shown in [23] and [24] that linear detectors with one-bit ADCs work well for multiuser scenarios with QPSK and 16 quadrature amplitude modulation (QAM) constellations, respectively. The mixed use of one-bit and high-resolution ADCs in massive MIMO was analyzed in [25], which showed that using one-bit ADCs with a few high-resolution ADCs can achieve similar performance with the unquantized case. Joint channel and data estimation using low-resolution ADCs was proposed in [26]. While the performance was comparable to the unquantized case, the technique in [26] required hundreds of iterations to converge.

In this paper, we propose a practical near maximum likelihood (nML) detector for uplink multiuser massive MIMO systems with one-bit ADCs. The complexity of the maximum likelihood (ML) detector grows exponentially with the number of users, which makes it difficult to use for massive MIMO with multiple users. The proposed nML detector, however, can be implemented with standard convex optimization techniques with marginal performance degradation when the number of receive antennas at the base station is large. Our nML detector also can support higher order constellation structures in a unified way.

Work that deals with detectors for massive MIMO with low resolution ADCs is found in [27–30]. An uplink multiuser massive MIMO detector was developed in [27] but the detector was based on several bit ADCs and developed for the spatial modulation transmission technique [31]. For general symbol transmission in uplink massive MIMO systems with one-bit ADCs, a message-passing algorithm-based multiuser detector was proposed in [28] for special constellation structures. The detector was later extended to arbitrary constellations in [29]. Our numerical studies show that our nML detector works well with both perfect and normalized CSI while the detector in [29] experiences the performance degradation with the normalized CSI. In [30], a multiuser detector using low-resolution ADCs was developed based on convex optimization, which has a similar structure with our nML detector. The detector in [30], however, should be optimized separately for each constellation, which is similar to [21], while our nML detector is able to detect arbitrary constellations without any modification.

Since our detector makes use of the channel information, we also propose an ML channel estimation technique using one-bit ADCs that is in line with our proposed nML detector. There has been related work on channel estimation with low-resolution ADCs [32] and with one-bit ADCs [33–37]. In [33], [34], the expectation-maximization (EM) algorithm was exploited to estimate channels. The problem solved in [33], [34], however, is not convex, and the EM algorithm may converge to a local optimal in the high SNR regime. The work [35], [36] used generalized approximate message passing (GAMP) algorithms for channel estimation with one-bit ADCs where the techniques heavily relied on the sparsity of the target vector. While the work in [34], [36] was not able to estimate the norm of the channel due to the simple zero-threshold setting for one-bit quantization, [35] adopted asymmetric one-bit quantizers to deliver the the norm information of the target vector. Our ML channel estimator, which is an extension of [37], is shown to estimate not only the channel direction but also the channel norm and does not make an assumption about sparsity in the channel.

Our contributions are summarized as follows.

- We propose an nML detector for uplink multiuser massive MIMO systems. The proposed nML detector is based on the ML detector developed for distributed reception in [38]. We show that the two problems, i.e., distributed reception and multiuser detection, are essentially the same problem, which makes it possible to exploit the detectors from [38] for our multiuser detection problem. The complexity of the ML detector in [38], however, grows exponentially with the number of uplink users, which prevents its use in massive MIMO with many users. We reformulate the ML detector in [38] to derive an ML estimator and convert the ML estimation problem into a convex problem. Therefore, we can rely on efficient convex optimization techniques to obtain an ML estimate and perform symbol-by-symbol detection based on the ML estimate.

- In the high SNR regime, we derive the exact probability that two different transmit vectors output the same quantized received signal with one-bit ADCs, which may cause detection error. The result shows the relationship between the probability and the numbers of antennas and users, where the probability of having the same quantized outputs goes to zero as the number of antennas goes to infinity. By using classical combinatorial geometry results [39–41], we claim that the same result may hold in general, i.e., different transmit vectors output different quantized signals when the number of receive antennas is large. This shows the benefit of massive MIMO for one-bit ADCs. Note that the classical combinatorial geometry results were also exploited in [17] to bound the high SNR
We propose an ML channel estimator that has the same structure with the proposed nML detector. The proposed estimator can estimate the direction and norm of the channel more accurately than other channel estimators using one-bit ADCs. Because of the similar structure, it is possible to implement both the nML detector and the ML channel estimator using the same algorithm.

The paper is organized as fellows. We describe our system model in Section II. In Section III we briefly discuss the detectors which were originally developed for distributed reception in [36]. Then, we propose our nML detector and present the analyses in the high SNR regime in Section IV. We propose an ML channel estimator in Section V. In Section VI we evaluate the proposed techniques by simulations, and the conclusion follows in Section VII.

Notation: Lower and upper boldface letters represent column vectors and matrices, respectively. $||a||$ is used to denote the $\ell_2$-norm of a vector $a$, and $A^T$, $A^*$, $A^\dagger$ denote the transpose, Hermitian transpose, and pseudo inverse of the matrix $A$, respectively. $\text{Re}(b)$ and $\text{Im}(b)$ represent the real and complex part of a complex vector $b$, respectively. $0_m$ is used for the $m \times 1$ all zero vector, and $I_m$ denotes the $m \times m$ identity matrix. $\mathbb{C}^{m \times n}$ and $\mathbb{R}^{m \times n}$ represent the set of all $m \times n$ complex and real matrices, respectively.

II. SYSTEM MODEL

We explain our system model and several assumptions that are relevant to our detector design. We also define expressions for one-bit ADCs in this section.

A. Massive MIMO Received Signal Model

We consider a uplink multiuser cellular system with $N_c$ cells. Each cell consists of a base station with $N_r$ received antennas and $K$ users equipped with a single transmit antenna. All $K N_c$ users transmit independent data symbols simultaneously to their serving base stations. Assuming all users transmit data with power $P$, the received signal at the $i$-th base station $y_i = \begin{bmatrix} y_{1,i} & y_{2,i} & \cdots & y_{N_r,i} \end{bmatrix}^T$ is

$$y_i = \sqrt{P} \sum_{k=1}^{K} h_{i,ik} x_{ik} + \sqrt{P} \sum_{m=1}^{N_c} \sum_{k \neq m}^{K} h_{i,mk} x_{mk} + n_i$$

for all $i$ where $x_i = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{iK} \end{bmatrix}^T$. If $\mathcal{S}$ is a phase shift keying (PSK) constellation satisfying $|s_m|^2 = 1$ for all $m$, then the norm constraint is trivially satisfied. For a QAM constellation, the law of large numbers (LLN) gives

$$\sum_{k=1}^{K} |x_{ik}|^2 \approx K$$

as $K$ becomes large if all users select their data symbols independently and with equal probability within $\mathcal{S}$ (with proper normalization). Although the norm constraint in (3) plays an important role in implementing the proposed nML detector, numerical results in Section VII shows that the nML detector works well even with moderate numbers of users.

Assumption 1: The data symbols $x_{ik}$ are from an $M$-ary constellation $\mathcal{S} = \{s_1, \cdots, s_M\}$ and have been normalized such that

$$||x_i||^2 = K$$

for all $i$.

We assume the base station knows perfectly.

B. Received Signal Representation with One-Bit ADCs

We focus on a massive MIMO system that uses one-bit ADCs for the real and imaginary parts of the each element of $y$. The conceptual figure of our system is depicted in Fig. 1. The output of the $n$-th receive antenna after the one-bit ADCs is given as

$$\hat{y}_n = \text{sgn}(\text{Re}(y_n)) + j \text{sgn}(\text{Im}(y_n))$$

where $\text{sgn}(\cdot)$ is the sign function which is defined as

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Therefore, we have

$$\hat{y}_n \in \{1 + j, -1 + j, -1 - j, 1 - j\}$$

for $1 \leq n \leq N_r$. The collection of $\hat{y}_n$ is given as

$$\hat{y} = [\hat{y}_1 \ \hat{y}_2 \ \cdots \ \hat{y}_{N_r}]^T.$$
III. POSSIBLE DETECTORS USING ONE-BIT ADCS

In [38], distributed reception techniques for spatially multiplexed data symbols were studied. Assuming one-bit quantization for the real and imaginary parts of the received signal at each receive node, ML and zero-forcing (ZF)-type detectors at the fusion center were proposed assuming an error-free link between the fusion center and each receive node.

If we map each receive node in [38] into a receive antenna in our system model, the detectors proposed in [38] can be applied to our problem directly. In this section, we reformulate the detectors for distributed reception proposed in [38] into the uplink multiuser massive MIMO setting. We also discuss their characteristics and limitations. This discussion is useful for developing our nML detector in Section IV.

A. ML Detector Reformulation

Let \( g_{n}^{T} \in \mathbb{C}^{1 \times K} \) be the \( n \)-th row of the channel matrix \( H \), i.e.,

\[
H = [g_1 \ g_2 \ \cdots \ g_N]^{T}.
\]

Note that \( g_n \) is the channel between the \( K \) users and the \( n \)-th receive antenna. Assuming the real and imaginary components of the Gaussian noise are IID, it is useful when approaching one-bit ADC problems to rewrite the signal model in the real vector form instead of complex form as

\[
G_{R,n} = \begin{bmatrix} \text{Re}(g_n) & \text{Im}(g_n) \\ -\text{Im}(g_n) & \text{Re}(g_n) \end{bmatrix}^{T} = \begin{bmatrix} g_{R,n,1}^{T} \\ g_{R,n,2}^{T} \end{bmatrix} \in \mathbb{R}^{2 \times 2K},
\]

\( \mathbf{n}_{R,n} = \begin{bmatrix} \text{Re}(n_n) \\ \text{Im}(n_n) \end{bmatrix} = \begin{bmatrix} n_{R,n,1} \\ n_{R,n,2} \end{bmatrix} \in \mathbb{R}^{2 \times 1}, \)

\[
x_{R} = \begin{bmatrix} \text{Re}(x) \\ \text{Im}(x) \end{bmatrix} \in \mathbb{R}^{2K \times 1},
\]

where

\[
g_{R,n,1} = \begin{bmatrix} \text{Re}(g_n) \\ -\text{Im}(g_n) \end{bmatrix}, \quad g_{R,n,2} = \begin{bmatrix} \text{Im}(g_n) \\ \text{Re}(g_n) \end{bmatrix}
\]

and

\[
n_{R,n,i} \sim \mathcal{N} \left( 0, \frac{\sigma^2}{2} \right)
\]

for all \( n \) and \( i \). The received signal at the \( n \)-th receive antenna can be also rewritten as

\[
y_{R,n} = \begin{bmatrix} \text{Re}(y_n) \\ \text{Im}(y_n) \end{bmatrix} = \begin{bmatrix} y_{R,n,1} \\ y_{R,n,2} \end{bmatrix} = \sqrt{P}G_{R,n}x_{R} + \mathbf{n}_{R,n},
\]

and the vectorized version of the quantized \( \hat{y}_k \) in the real domain is given as

\[
\hat{y}_{R,n} = \begin{bmatrix} \text{sgn}(\text{Re}(y_n)) \\ \text{sgn}(\text{Im}(y_n)) \end{bmatrix} = \begin{bmatrix} y_{R,n,1} \\ y_{R,n,2} \end{bmatrix}.
\]

Based on \( \hat{y}_{R,n} \), the base station generates the sign-refined channel matrix for the \( n \)-th receive antenna as

\[
\hat{G}_{R,n} = \begin{bmatrix} g_{R,n,1}^{T} \\ g_{R,n,2}^{T} \end{bmatrix}
\]

where \( \hat{g}_{R,n,i} \) is defined as

\[
\hat{g}_{R,n,i} = \hat{y}_{R,n,i}g_{R,n,i}.
\]

Note that \( \hat{y}_{R,n,i} = \pm 1 \) depending on the sign of \( y_{R,n,i} \). Define \( S_{R} \) to be

\[
S_{R} = \left\{ \begin{bmatrix} \text{Re}(s_1) \\ \text{Im}(s_1) \end{bmatrix}, \ldots, \begin{bmatrix} \text{Re}(s_M) \\ \text{Im}(s_M) \end{bmatrix} \right\}
\]

where \( M \) is the size of the data symbol constellation \( S \).

With these definitions and using similar logic from [38], the ML detector can be defined as

\[
\hat{x}_{R,\text{ML}} = \arg\max_{\hat{x}_{R} \in S_{R}^{K}} \prod_{n=1}^{N} \prod_{i=1}^{K} \Phi \left( \sqrt{2P}g_{R,n,i}^{T}\hat{x}_{R} \right)
\]

where \( \Phi(t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \) and \( S_{R}^{K} \) is the \( K \)-ary Cartesian product set of \( S_{R} \), which is ordered with the real parts of the constellations first and the imaginary parts later. The \( \sqrt{2} \) term in (21) comes from the distribution of \( n_{R,n,i} \) given in (15).

B. ZF-Type Detector Reformulation

By brute-force search, the complexity of the ML detector in (21) is \( K^{M} \) which grows exponentially with the number of users \( K \). To support a large number of \( K \), the ZF-type detector was proposed in [38]. The base station first obtains the ZF estimate as

\[
\hat{x}_{ZF} = H^{\dagger}\hat{y}.
\]

Because the norm square of \( \hat{x}_{ZF} \) may not equal to \( K \), the base station normalizes \( \hat{x}_{ZF} \) as

\[
\hat{x}_{ZF} = \frac{\hat{x}_{ZF}}{||\hat{x}_{ZF}||} \quad (23)
\]

and performs symbol-by-symbol detection using \( \hat{x}_{ZF} \) as

\[
\hat{x}_{ZF,i} = \arg\min_{\hat{x} \in S} ||\hat{x}_{ZF,i} - \hat{x}||^2 \quad (24)
\]

where \( \hat{x}_{ZF,i} \) is the \( i \)-th element of normalized \( \hat{x}_{ZF} \). The normalization is not an issue for PSK constellations; however, it is crucial for QAM constellations.

It is shown numerically in [38] that the ZF-type detector approaches the inferior error rate floor than the ML detector as SNR increases; the error rate floors in both cases are inevitable due to the one-bit ADCs. Therefore, we propose the nML detector that outperforms the ZF-type detector and requires much less complexity than the ML detector.
IV. NEAR ML DETECTOR IMPLEMENTATION

We derive our nML detector by converting the original ML detection problem as a convex optimization problem. This can be done by relaxing constraints on the transmitted vector. We also analyze the nML detector in the high SNR regime and derive the exact probability that two different transmit vectors output the same quantized signal in a certain condition. The relationship between the probability and the number of receive antennas show that the probability goes to zero as the number of antennas goes to infinity, which shows the benefit of massive MIMO for one-bit ADCs. Then we generalize the result by using combinatorial geometry. Finally, we extend the proposed nML detector to a multicell scenario.

A. Convex Optimization Formulation of ML Detector

Because of the norm constraint \((2)\), we define the ML estimator by relaxing the constraint \(\hat{x}_R \in S^K_R\) in \((21)\) as

\[
\hat{x}^{(1)}_{R, ML} = \arg \max_{\|x_R\|^2 = K} \sum_{i=1}^{N_t} \sum_{n=1}^{N_r} \Phi(\sqrt{2\rho R_{R,R,n,i}} \hat{x}_R) \tag{25}
\]

It was shown in \((23)\) that

\[
\hat{x}^{(1)}_{R, ML} \rightarrow x_R \tag{27}
\]

in probability as \(N_t \rightarrow \infty\) for arbitrary \(\rho > 0\). Therefore, if \((26)\) can be solved, then the detector is guaranteed to achieve good performance when \(N_t\) is large.

The problem is that \((26)\) is not easy to solve in general. The function \(\log \Phi(\cdot)\) is log-concave but the optimization problem in \((26)\) is not convex due to the norm constraint \(\|x_R\|^2 = K\). To sidestep this challenge, we relax the norm constraint as \(\|x_R\|^2 \leq K\) and reformulate the problem as

\[
\hat{x}^{(2)}_{R, ML} = \arg \max_{\|x_R\|^2 \leq K} \sum_{i=1}^{N_t} \sum_{n=1}^{N_r} \log \Phi(\sqrt{2\rho R_{R,R,n,i}} \hat{x}_R) \tag{28}
\]

which is a convex optimization problem that can be efficiently solved \((23)\). Similar to \((20)\), \((45)\), we provide a simple yet effective iterative algorithm to solve \((28)\) in Algorithm 1.

In Step 1, we initialize to

\[
\hat{x}^{(0)}_R = \sqrt{K} \frac{G_R^{T} 1_{2N_r}}{\|G_R^{T} 1_{2N_r}\|} \tag{29}
\]

Algorithm 1 Iterative algorithm to solve \((28)\)

**Initialization**
1. Set the initial point \(\hat{x}_R^{(0)}\)
2. Set \(G_R = [\bar{G}_{R,R,1}, \ldots, \bar{G}_{R,R,N_r}, \bar{G}_{R,R,2}, \ldots, \bar{G}_{R,R,N_r}]^T\)
3. Set the step size \(\kappa\) and the termination threshold \(\epsilon\)

**Iterative update**
4. While \(\|\hat{x}_R^{(k)} - \hat{x}_R^{(k-1)}\|^2 \geq \epsilon\)
5. \(\hat{x}_R^{(k)} = \hat{x}_R^{(k-1)} + \kappa G_R \nabla f(\hat{x}_R^{(k-1)})\)
6. If \(\|\hat{x}_R^{(k)}\|^2 > K\)
7. \(\hat{x}_R^{(k)} = \sqrt{K} \frac{x^{(k)}_R}{\|x^{(k)}_R\|}\)
8. end If
9. end While

set to \([-\sqrt{M-1} A, (\sqrt{M-1} A]\) in \((30)\) where \(A\) is the average power normalization factor.

After obtaining the estimate \(\hat{x}^{(2)}_{R, ML}\), the base station needs to perform normalization followed by symbol-by-symbol detection similar to the ZF-type detector in \((24)\). If we let \(\hat{x}_{R, ML,i}\) be the \(i\)-th element of normalized \(\hat{x}_{R, ML}\), the symbol-by-symbol detection is

\[
\hat{x}_{ML,i} = \arg \min_{\hat{x} \in S} (|\hat{x}_{R, ML,i} + j\hat{x}_{R, ML,K+i} - \hat{x}|^2) \tag{30}
\]

considering the fact that \(\hat{x}_{R, ML}\) is a \(2K \times 1\) real vector.

**Remark 1**: As also discussed in \((30)\), the complexity of Algorithm 1 is dominated by the number of iterations because there are only two matrix-vector multiplications and numerical calculations to get \(\nabla f(\hat{x}_i)\) in the iterative update. The numerical calculations for updating \(\nabla f(\hat{x}_i)\) in each iteration prevent performing the direct comparison with the ZF-type detector but it is reasonable to assume that the complexity of one iteration is less than that of the ZF-type detector that requires matrix inversion. As shown in Section VI the number of iterations for Algorithm 1 to converge can be moderate, e.g., around 20–40.

**Remark 2**: Note that either nML detector based on \((26)\) or \((28)\) is suboptimal compared to the original ML detector \((21)\). The proposed nML detector is based on the ML estimation \((28)\) over the \(2K\)-dimensional real space with the norm constraint. It can be the case that

\[
\sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi(\sqrt{2\rho R_{R,R,n,i}} \hat{x}^{(2)}_{R, ML}) > \sum_{i=1}^{2N_r} \sum_{n=1}^{N_t} \log \Phi(\sqrt{2\rho R_{R,R,n,i}} x_R) \tag{31}
\]

with

\[
\|x_R - \hat{x}^{(2)}_{R, ML}\|^2 > \|x_R - \hat{x}^{(2)}_{R, ML}\|^2 \tag{34}
\]
where \( x_\mathbf{R} \) is the true transmitted vector and \( \hat{x}_\mathbf{R} \in S^2_N \setminus \{ x_\mathbf{R} \}. \) In words, it can happen that the original ML detector makes the correct decision as in (32) and (33) while the estimated vector \( \hat{x}_{\mathbf{R}, \text{ML}} \) from (28) is closer to \( x_\mathbf{R} \), which is not the true transmitted vector. The suboptimality, however, will not come into play when \( N_t \) is large as shown in [38].

**Remark 3:** All detectors discussed in this paper are based on the assumption that \( \tilde{g}_{\mathbf{R},n,i}^T x_\mathbf{R} \geq 0 \) for all \( n \) and \( i \). When \( \rho \) is not large enough, it is possible that \( \tilde{g}_{\mathbf{R},n,i}^T x_\mathbf{R} < 0 \) for some \( n \) and \( i \), which degrades the performance of all the detectors. The numerical results in Section VI show that \( \rho \) can be not-so-large for the proposed nML detector to achieve good performance, e.g., it is enough to have 5dB SNR for the proposed nML detector to achieve the symbol error rate performance less than \( 10^{-3} \) with \( N_t = 128 \) antennas and \( K = 8 \) users for 8PSK constellation.

**B. Analyses in high SNR regime**

The estimates \( \hat{x}_{\mathbf{R}, \text{ML}}^{(1)} \) in (26) and \( \hat{x}_{\mathbf{R}, \text{ML}}^{(2)} \) in (28) may not be the same in general because \( \hat{x}_{\mathbf{R}, \text{ML}}^{(1)} \) is selected from a circle \( \| \hat{x}_1 \| = K \) while \( \hat{x}_{\mathbf{R}, \text{ML}}^{(2)} \) is selected from a ball \( \| \hat{x}_2 \| \leq K \). Note that (26) can have multiple local optimal solutions due to non-convexity of the constraint. If we let \( \lambda^{(1)} \) be a set of all possible solutions of (26), the following lemma shows the relation between \( \lambda^{(1)} \) and \( \hat{x}_{\mathbf{R}, \text{ML}}^{(2)} \) in the high SNR regime.

**Lemma 1.** When \( \rho \to \infty \), we have

\[
\hat{x}_{\mathbf{R}, \text{ML}}^{(2)} \in \lambda^{(1)}.
\]  

**Proof:** Please see Appendix A. ■

Lemma 1 shows that the nML detector based on (28) will perform the same as the one based on (26) in the high SNR regime.

As shown in the proof of Lemma 1 the estimator (28) will output \( \hat{x}_{\mathbf{R}, \text{ML}}^{(2)} \) that satisfies

\[
\tilde{g}_{\mathbf{R},n,i}^T \hat{x}_{\mathbf{R}, \text{ML}}^{(2)} > 0
\]  

in the high SNR regime. Because \( \tilde{g}_{\mathbf{R},n,i} \) is based on the sign refinement from the quantized received signal \( \hat{y}_{\mathbf{R},n,i} \) in (19), it is necessary that different transmit vectors output different quantized received signals to avoid possible detection errors. Although it is difficult in general, we can explicitly derive the probability of which two different transmit vectors output the same quantized received signal for a special case.

**Special case:** Consider two transmit vectors \( x_1 = [x_1 \ x_2 \ \cdots \ x_K]^T \) and \( x_2 = [-x_1 \ x_2 \ \cdots \ x_K]^T \) where \( x_k \), which is selected from a standard \( M \)-ary constellation \( S \) with equal probability, is the transmit symbol of the \( k \)-th user. Assume each entry of \( \mathbf{H} \) follows IID Rayleigh fading, and the high SNR regime which gives two received signals \( y_1 = \mathbf{H} x_1 \) and \( y_2 = \mathbf{H} x_2 \). Define \( y_1 \) and \( y_2 \) as the quantized outputs of \( y_1 \) and \( y_2 \) using one-bit ADCs.

**Proposition 1.** For the special case,

\[
\Pr (\hat{y}_1 = \hat{y}_2) = \left( \frac{2}{\pi} \arctan \sqrt{K-1} \right)^{2N_t}.
\]  

**Proof:** Please see Appendix B. ■

Proposition 1 shows that the probability of possible detection error between \( x_1 \) and \( x_2 \) in the special case decreases exponentially with \( N_t \), which shows the benefit of massive MIMO with one-bit ADCs. The following corollary is a direct consequence of Proposition 1.

**Corollary 1.** For the special case, \( \Pr (\hat{y}_1 = \hat{y}_2) \to 0 \) as \( N_t \to \infty \).

Although it is difficult to generalize Proposition 1 and Corollary 1 for an arbitrary pair of transmit vectors and channel models, the result from the classical combinatorial geometry [39]–[41] suggests that the same conclusion would hold for the high SNR regime in general.

Using the result in the combinatorial geometry, it is possible to derive the number of distinguishable regions constructed by the channel vectors \( \tilde{g}_{\mathbf{R},n,i} \). First, we restate the lemma in [17] that is useful for our claim.

**Lemma 2.** \( N \) hyperplanes in general position passing through the origin of a \( d \)-dimensional space divide the space into

\[
2 \sum_{k=0}^{d-1} \binom{N-1}{k}
\]  

regions.

General position is a strengthened rank condition, and we refer to [17] for the exact definition of general position. Applying Lemma 2 to our problem,

\[
d = 2K, \quad N = 2N_t
\]  

assuming all channel vectors \( \tilde{g}_{\mathbf{R},n,i} \) are in general position. The number of distinguishable regions in \( 2K \)-dimensional space becomes

\[
2 \sum_{k=0}^{2K-1} \binom{2N_t-1}{k} \geq \binom{2N_t}{K} = O \left( N_t^K \right).
\]  

If \( N_t \geq K \), the number of distinguishable regions can always be represented with one-bit ADCs because

\[
2 \sum_{k=0}^{2K-1} \binom{2N_t-1}{k} \leq 2 \sum_{k=0}^{2N_t-1} \binom{2N_t-1}{k} = 2^{2N_t}.
\]  

The transmitted vector \( x \) can be detected without error when \( x \) is the only vector located in a specific region. Note that the total number of possible transmit vectors is \( M^K \). In massive MIMO, it is likely to have \( N_t \gg M \), and the number of distinguishable regions will be much larger than the number of possible transmit vectors. For example, \( N_t = 32 \) with \( K = 5 \) gives around \( 5.6 \times 10^10 \) distinguishable regions. If all five users adopt an 8PSK constellation for their data symbols, the total number of possible transmit vectors becomes 32768, which is much less than the number of regions. Therefore, it seems reasonable that the probability of having two or multiple possible transmit vectors in a single region would become very low in massive MIMO. Considering the fact that the channel vectors are random in general and the transmit vectors are independent of the channel vectors, however, there
is a probability that multiple candidate transmit vectors can be located in the same region, i.e., they output the same quantized received signal, which leads to a possible detection error.

C. Extension to Multicell Setting

For the multicell scenario, the inter-cell interference should be taken into account for the proposed nML detector. It is reasonable to assume that the base station can accurately estimate the long-term statistic of the inter-cell interference. Therefore, we assume the $i$-th base station knows the average inter-cell interference power at the $n$-th receive antenna

$$\eta_{i,n}^2 = P \sum_{m=1}^{N_c} \sum_{k=1}^{K} E[\|h_{i,mk,n}\|^2]$$  \hspace{1cm} (42)

where $h_{i,mk,n}$ is the $n$-th entry of $h_{i,mk}$ in $\{1\}$. Note that large-scale fading, e.g., path loss, is reflected in $h_{i,mk}$. We further assume that

$$\eta_{i,1}^2 = \eta_{i,2}^2 = \cdots = \eta_{i,N_i}^2 = \eta_i^2$$ \hspace{1cm} (43)

considering the fact that all receive antennas at the base station are closely deployed.

Without having any CSI from out-of-cell users, the base station will consider the inter-cell interference as additional AWGN, i.e.,

$$\sqrt{P} \sum_{m=1}^{N_c} \sum_{k=1}^{K} h_{i,mk} \tilde{z}_{mk} \sim \mathcal{C}\mathcal{N}(0_{N_i}, P\eta_i^2 \mathbf{I}_{N_i}).$$ \hspace{1cm} (44)

Then the effective signal-to-interference-noise ratio (SINR) at the $i$-th base station is

$$\rho_{i,MC} = \frac{P}{P\eta_i^2 + \sigma^2}.$$ \hspace{1cm} (45)

and the proposed nML detector can be adapted to the multicell scenario by substituting $\rho$ in (28) with $\rho_{i,MC}$.

V. CHANNEL ESTIMATION WITH ONE-BIT ADCs

Note that the detectors discussed in the previous sections assume perfect CSI at the base station. For a coherent detector, however, the CSI is normally obtained through an estimate of the channel. In this section, we develop efficient channel estimation techniques assuming one-bit ADCs at the base station. We focus on estimating $g_n$, i.e., the channel between the receive antenna $n$ and $K$ users, instead of $h_i$.

We consider a block fading channel to develop channel estimation techniques. We assume the channel is static for $L$ channel uses in a given fading block and changes independently from block-to-block. The received signal at the $n$-th antenna for the $\ell$-th channel use in the $m$-th fading block is given as

$$y_{n,m}[\ell] = \sqrt{\rho} g_{n,m}^* x_{m}[\ell] + w_{n,m}[\ell].$$ \hspace{1cm} (46)

Let the first $T < L$ channel uses be devoted for a training phase and the remaining $L - T$ channel uses be dedicated to a data communication phase. Put the first $T$ received signals during the training phase into a vector form as

$$y_{n,m,\text{train}} = \sqrt{\rho} \mathbf{x}_{m,\text{train}} g_{n,m} + w_{n,m,\text{train}}$$ \hspace{1cm} (47)

where

$$y_{n,m,\text{train}} = [y_{n,m}[0] \cdots y_{n,m}[T-1]]^\ast \in \mathbb{C}^{T \times 1},$$ \hspace{1cm} (48)

$$\mathbf{x}_{m,\text{train}} = [x_m[0] \cdots x_m[T-1]] \in \mathbb{C}^{K \times T},$$ \hspace{1cm} (49)

$$w_{n,m,\text{train}} = [w_{n,m}[0] \cdots w_{n,m}[T-1]]^\ast \in \mathbb{C}^{T \times 1}.$$ \hspace{1cm} (50)

In the training phase, $\mathbf{x}_{m,\text{train}}$ is known to the base station but $g_{n,m}$ must be estimated. While arbitrary training matrices are possible, for simulation purpose in Section VI, we focus on unitary training where $\mathbf{x}_{m,\text{train}}$ satisfies $\{1\}$

$$\mathbf{x}_{m,\text{train}}^\ast \mathbf{x}_{m,\text{train}} = \mathbf{I}_T \text{ if } K \geq T,$$ \hspace{1cm} (51)

$$\mathbf{x}_{m,\text{train}}^\ast \mathbf{x}_{m,\text{train}} = T\mathbf{I}_K \text{ if } K < T.$$ \hspace{1cm} (52)

The normalization term $T$ in the case of $K < T$ ensures the average transmit power equals to $P$ in each channel use.

Similar to the previous sections, we reformulate all expressions into the real domain as

$$\mathbf{y}_{R,n,m,\text{train}} = \sqrt{\rho} \mathbf{x}_{R,m,\text{train}}^T \mathbf{g}_{R,n,m} + \mathbf{w}_{R,n,m,\text{train}}$$ \hspace{1cm} (53)

where

$$\mathbf{y}_{R,n,m,\text{train}} = \begin{bmatrix} \text{Re}(y_{n,m,\text{train}}) \\ \text{Im}(y_{n,m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2T \times 1},$$ \hspace{1cm} (54)

$$\mathbf{x}_{R,m,\text{train}} = \begin{bmatrix} \text{Re}(\mathbf{x}_{m,\text{train}}) \\ \text{Im}(\mathbf{x}_{m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2K \times 2T},$$ \hspace{1cm} (55)

$$\mathbf{g}_{R,n,m} = \begin{bmatrix} \text{Re}(g_{n,m}) \\ \text{Im}(g_{n,m}) \end{bmatrix} \in \mathbb{R}^{2K \times 1},$$ \hspace{1cm} (56)

$$\mathbf{w}_{R,n,m,\text{train}} = \begin{bmatrix} \text{Re}(w_{n,m,\text{train}}) \\ \text{Im}(w_{n,m,\text{train}}) \end{bmatrix} \in \mathbb{R}^{2T \times 1}.$$ \hspace{1cm} (57)

It is important to point out that $\{1\}$ has the same form as $\{1\}$ while the roles of the channel and the transmitted signal are reversed. Therefore, using the same techniques that we exploited for the detectors, we can develop channel estimators based on the one-bit ADC outputs and $\mathbf{x}_{R,m,\text{train}}$.

We define the $i$-th column of $\mathbf{x}_{R,m,\text{train}}$ as $\mathbf{x}_{R,m,\text{train},i}$ and the $i$-th output of the one-bit ADC as

$$\hat{y}_{R,n,m,\text{train},i} = \text{sgn}(\hat{y}_{R,n,m,\text{train},i})$$ \hspace{1cm} (58)

where $\hat{y}_{R,n,m,\text{train},i}$ is the $i$-th element of $\mathbf{y}_{R,n,m,\text{train}}$. Note that there are $2T$ one-bit ADC outputs in total for the $n$-th receive antenna, i.e., $T$ outputs for each of the real and imaginary parts of the received signal. Based on $\hat{y}_{R,k,m,\text{train},i}$, the base station performs the sign-refinement as

$$\tilde{x}_{R,m,\text{train},i} = \hat{y}_{R,n,m,\text{train},i} \mathbf{x}_{R,m,\text{train},i},$$ \hspace{1cm} (59)

and the ML channel estimator is given as

$$\hat{g}_{R,n,m,\text{ML}} = \arg\max_{\mathbf{g} \in \mathbb{R}^{2K \times 1}} \prod_{i=1}^{2T} \Phi \left( \sqrt{2\rho} \tilde{x}_{R,m,\text{train},i}^T \mathbf{g} \right)$$ \hspace{1cm} (60)

$$= \arg\max_{\mathbf{g} \in \mathbb{R}^{2K \times 1}} \sum_{i=1}^{2T} \log \left( \Phi \left( \sqrt{2\rho} \tilde{x}_{R,m,\text{train},i}^T \mathbf{g} \right) \right).$$ \hspace{1cm} (61)
Because \( \Phi(\cdot) \) is a log-concave function, and there is no constraint on \( \hat{\mathbf{g}}_{\text{R}} \), it is possible to solve (61) using standard convex optimization methods (or Algorithm 1 with proper modification). The problem is that this channel estimator easily gives over-estimated norm for \( \hat{\mathbf{g}}_{\text{R},n,m} \), i.e.,

\[
\| \hat{\mathbf{g}}_{\text{R},n,m,\text{ML}} \| > \| \mathbf{g}_{\text{R},n,m} \| \tag{62}
\]
due to the fact that \( \log \Phi(\cdot) \) is an increasing function where \( \mathbf{g}_{\text{R},n,m} \) is the true channel vector.

To overcome this problem, we impose the norm constraint on \( \hat{\mathbf{g}}_{\text{R}} \) and convert (61) to

\[
\hat{\mathbf{g}}_{\text{R},n,m,\text{ML}} = \arg \max_{\mathbf{g}_{\text{R}} \in \mathbb{R}^{2K \times T}} \sum_{i=1}^{2T} \log \left( \Phi \left( \sqrt{2\rho} \mathbf{x}_{\text{R},m,\text{train},i}^{\text{T}} \hat{\mathbf{g}}_{\text{R}} \right) \right) \| \mathbf{g}_{\text{R}} \| \leq K \tag{63}
\]

using the fact that \( \mathbb{E} \left[ \| \mathbf{g}_{\text{R},n,m} \|^2 \right] = K \) for most of channel models. The norm constraint can be further optimized if the base station knows the long-term statistic of the channel norm.

For comparison purpose, we also define a simple ZF-type channel estimator as

\[
\hat{\mathbf{g}}_{\text{R},n,m,\text{ZF}} = \sqrt{K} \left( \mathbf{X}_{\text{R},m,\text{train}} \right)^{\dagger} \hat{\mathbf{y}}_{\text{R},n,m,\text{train}} \tag{64}
\]

which is forced to satisfy \( \| \hat{\mathbf{g}}_{\text{R},n,m,\text{ZF}} \|^2 = K \). We numerically compare our ML channel estimator and the ZF-type channel estimator in Section VI.

VI. SIMULATION RESULTS

We perform Monte-Carlo simulations to evaluate the proposed techniques. The proposed nML detector is based on Algorithm 1 with the termination threshold \( \epsilon = 10^{-3} \) and numerically optimized step size \( \kappa \). We first consider the single cell scenario to compare the performance of detectors. Then we take the multicell scenario into account to consider more practical settings.

For comparison purpose, we also define a simple ZF-type detector, and the GAMP detector from \cite{29} are compared.

A. Single Cell Scenario

We assume IID Rayleigh fading channels, i.e., all elements of \( \mathbf{H} \) are distributed as \( \mathcal{CN}(0,1) \), although the distribution of the channel was not explicitly incorporated into any of the analysis. For the time being, we assume the base station has perfect CSI and evaluate the detectors. Later, we evaluate the detectors with imperfect CSI. We use the average symbol error rate (SER) which is defined as

\[
\text{SER} = \frac{1}{K} \sum_{n=1}^{K} \mathbb{E} \left[ \Pr \left( \hat{x}_n \neq x_n \mid x \text{ sent}, \mathbf{H}, \mathbf{n}, \rho, K, N_r, S \right) \right] \tag{65}
\]

for the performance metric where the expectation is taken over \( x, \mathbf{H}, \) and \( \mathbf{n} \).

We first compare the three detectors: 1) the original ML detector \cite{21} that is based on exhaustive search over all
TABLE I: Average number of iterations for Algorithm 1 to converge with \( M = 16 \).

<table>
<thead>
<tr>
<th>( N_r )</th>
<th>64</th>
<th>128</th>
<th>196</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 4 )</td>
<td>20.8955</td>
<td>18.1177</td>
<td>17.3528</td>
</tr>
<tr>
<td>( K = 8 )</td>
<td>39.5038</td>
<td>27.5954</td>
<td>22.3662</td>
</tr>
</tbody>
</table>

We adopt the same system setup with Fig. 3 except a possible transmitted vectors, 2) the proposed nML detector (28) that is based on convex optimization, and 3) the ZF-type detector. Due to the computational complexity of the original ML detector, we set \( K = 4, M = 8 \) (QPSK) for all users, and \( N_r = 32 \) which may not be considered as massive MIMO. We plot SERs of the three detectors in Fig. 3. The figure shows that the proposed nML detector is suboptimal compared to the original ML detector as discussed in Remark 2 of Section IV-A. The gap, however, will become smaller as \( N_r \) becomes large [38]. Further, the proposed nML detector outperforms the ZF-type detector for all SNR regimes.

In Fig. 3 we plot SERs of the proposed nML detector, the ZF-type detector, and the detector based on the GAMP algorithm from [29] according to SNR with \( K = 8, M = 8 \) (8PSK), and different values of \( N_r \). Note that \( N_r = 64 \) is a very practical antenna number that is considered in practice [48]. We do not consider the original ML detector in this scenario because of its excessive complexity, i.e., the detector needs to compare \( 8^8 = 2^{24} \) possible transmit vectors. The figure shows that the nML and GAMP detectors are comparable (while the nML detector gives better SER than the GAMP detector in the high SNR regime with \( N_r = 128 \)) and outperform the ZF-type detector with the same number of \( N_r \). The ZF-type detector suffers from the error rate floor while other two detectors do not have such floor until \( 10^{-5} \) SER with \( N_r = 128 \). With small SERs, it is possible to use weaker channel coding (with higher code rate or shorter block length) to improve the system throughput [49].

We adopt the same system setup with Fig. 3 except a \( M = 16 \) (16QAM) constellation for data symbol in Fig. 4. The nML and GAMP detector still outperform the ZF-type detector; however, the two detectors also suffer from the error rate floor even with \( N_r = 128 \) because of the excessive number of possible transmit vectors, i.e., there are \( 16^8 = 2^{32} \) possible candidates. With \( N_r = 196 \) antennas, the error rate floor is mitigated, which suggests that the error rate floor for the nML detector will vanish for practical SNR regimes once the base station is deployed with more antennas.

Figs. 3 and 4 both show that the proposed nML detector gives similar SER performance with the ZF-type detector using only half of the receive antennas. Therefore, if the base station has sufficient computation power, it is always beneficial to use the proposed nML detector than the ZF-type detector. Note that the computation power at the base station is related to the digital baseband processing in Fig. 1 and different from having one-bit ADCs. Therefore, the benefit of using one-bit ADCs, e.g., power consumption and cost, still holds for the nML detector although the high computation power is required for the digital baseband processing at the base station.

To evaluate the complexity of the nML detector using Algorithm 1, we compare the average number of iterations for Algorithm 1 to converge with \( M = 16 \) and different values of \( K \) and \( N_r \) in Table I. The table shows that the iteration numbers can be moderate, i.e., 20~40. It is interesting that the average iteration number decreases with the number of antennas, which shows that the nML detector based on Algorithm 1 will become more efficient with large \( N_r \).

Now, we evaluate the ML and ZF-type channel estimators discussed in Section V. We focus on estimating \( \textbf{g}_{n,m} \), i.e., the channel between the \( n \)-th receive antenna and \( K \) users. We define the mean squared error (MSE) of a channel estimator \( x \) as

\[
\text{MSE}_x = \mathbb{E} \left[ \| \textbf{g}_{n,m} - \hat{\textbf{g}}_{n,m,x} \|^2 \right],
\]

(66)
and the normalized MSE (NMSE) as

\[ \text{NMSE}_N = \mathbb{E} \left[ \left\| \frac{g_{n,m}}{\|g_{n,m}\|} - \frac{\tilde{g}_{n,m,x}}{\|\tilde{g}_{n,m,x}\|} \right\|^2 \right], \]

which are used as performance metrics. The expectations are taken over \( g_{n,m} \). In Fig. 5 we compare the proposed ML channel estimator to the ZF-type estimators and the expectation-maximization (EM) method from [34] with different training lengths \( T \) and SNR values with \( K = 8 \). Regarding MSE, the proposed ML estimator outperforms other estimators for both \( \rho = 0 \) and 20dB cases. The EM method performs well only when \( T \) is large with \( \rho = 0 \)dB and fails to estimate the channel norm when \( \rho = 20 \)dB as also shown in [34]. The EM method with the norm fixed to \( K \) gives better performance than the ZF-type estimator when \( \rho = 0 \)dB while the two estimators become exactly the same with \( \rho = 20 \)dB. Regarding NMSE, the proposed ML estimator and the EM method (and the one with the fixed norm as well) are comparable when \( \rho = 0 \)dB while the ML estimator outperforms the EM method with \( \rho = 20 \)dB.

To verify the effect of channel estimations, we plot the SERs of the nML and GAMP detectors with different assumptions on CSI; perfect CSI, perfect normalized CSI, and normalized CSI with MSE that is defined as

\[ g_{n,m} + \sqrt{\text{MSE}_K} e_{n,m} \]

where the element of \( e_{n,m} \) is distributed as \( \mathcal{CN}(0, 1) \). We set \( \text{MSE} = 10^{-1} \), which can be achieved with \( T = 50 \) with most of the detectors as shown in Fig. 5. We set \( K = 8, M = 16, \) and \( N_t = 128 \). It is shown in Fig. 6 that the nML detectors performs well even with the normalized CSI and normalized CSI with MSE. When \( \rho \) is large, the performance of the nML detector with the normalized CSI is comparable to that with the perfect CSI. In this case, the proposed ML channel estimator is better than other estimators because it gives a lower NMSE. On the contrary, the GAMP detector using the normalized CSI experiences performance degradation for all SNR values, which shows a clear benefit of using the nML detector over the GAMP detector.

To compare the detectors in a practical setting, we combine the detectors with a low-density-parity-check (LDPC) code. We assume the base station has perfect CSI for this study. We adopt the rate 1/2 LDPC code with the block length of 672 bits from the IEEE 802.11ad standard [50]. After hard detection by the detectors, the estimated symbols (or bits) are decoded using the bit-flipping decoding algorithm [51]. The coded bit error rates (BERs) of the nML and ZF-type detectors according to SNR with \( K = 4, N = 64, \) and \( M = 8 \) are shown in Fig. 7. Because of its similar performance with the nML detector, we do not consider the GAMP detector in this study. The figure clearly shows that the proposed nML detector outperforms the ZF-type detector even for this practical setting. Further improvements could be expected if further work is put into deriving an appropriate soft decision decoding metric, which requires the probability distribution of \( x_{R,ML}^{(2)} \) in [28]. It is also possible to exploit the approximated soft metric as in [52], but we leave this for future work.

### B. Multicell Scenario

For the last numerical study, we consider the multicell scenario where the detailed simulation parameters are listed in Table II. We consider two different user dropping scenarios. 1) All users except a typical user in the center cell are randomly dropped within corresponding cells. The typical user is located with the distance \( d \) (and a random angle per drop) from the base station in the center cell. 2) All users except the users in the center cell are randomly dropped within corresponding cells. The users in the center cell are randomly dropped within the range \((d - 20m, d + 20m)\) from the center cell base station.
The second scenario can be considered as the case with user scheduling that selects users with similar received signal power while the first scenario corresponds to uncoordinated uplink transmission. We consider coded BER of the typical user for the first scenario while BERs of all $K$ users in the center cell are averaged for the second scenario. The proposed nML detector is performed as explained in Section IV-C.

The BER results for these two scenarios according the distance $d$ are plotted in Fig. 8. We can see that the proposed nML detector outperforms the ZF-type detector for both user dropping scenarios. As $d$ increases, the BER performance of both detectors becomes worse because of the reduced received signal power. Note that the BER performance of the second scenario is much better than that of the first scenario when $d$ is large. For large $d$, the received signal power of the typical user in the first scenario is overwhelmed by other users’ received signals (the near-far effect), resulting in poor BER performance. If all users experience similar SINR as in the second scenario, however, the BER performance is quite good even with large $d$. This shows that the proposed nML detector will perform well with proper user scheduling or uplink power control, which are already common in current cellular systems. Note that the ZF-type detector suffers from the notable error rate floor for the second scenario when $d$ is small (that corresponds to the high SNR regime for the single cell scenario) while the proposed nML detector does not have such floor until $10^{-5}$ BER. In the first scenario, there is no error rate floor even for the ZF-type detector because the received signal of the typical user overwhelms other users’ received signals.

### VII. CONCLUSION

We proposed a unified framework of detection and channel estimation techniques for uplink multiuser massive MIMO systems using a pair of one-bit ADCs at each antenna. The proposed techniques are based on off-the-shelf convex optimization methods, which makes it easy to implement in practice. The proposed nML detector gives far better performance than the ZF-type detector for all range of SNR regimes and number of antennas. Numerical studies showed that the proposed nML detector is able to perform well even with not-so-large number of antennas, robust to inaccurate channel estimation, and outperforms the ZF-type detector for practical channel coded and multicell settings. The proposed ML channel estimator can effectively estimate not only the direction but also the norm of the channel even with one-bit ADCs. Because of the unified structure of the ML detector and channel estimator, same hardware can be used for both tasks, which make the proposed techniques more attractive for uplink massive MIMO systems using one-bit ADCs.

There are several directions for future research. The nML detector can be extended to frequency selective channel considering its possible use in mmWave systems, for example by extending the results in [27], [53]. To make the proposed ML channel estimator more practical, the training overhead should be reduced. As in [54], [55], it may be possible for the proposed ML channel estimator to exploit channel statistics, e.g., the temporal and spatial correlation, to reduce the training overhead. It would be also interesting to consider maximum a posteriori (MAP) detector and channel estimator using one-bit ADCs and compare them with the proposed techniques where MAP detector and channel estimator with low-resolution ADCs have been studied in [53].

### APPENDIX A

**PROOF OF LEMMA 1**

Recall the estimator in (28). When $\rho \to \infty$, 
\[
\| \sqrt{2/\rho} \mathbf{g}_{R,n,i}^{T} \mathbf{x}_{R}^{(2)} \| \to \infty
\]  
(69) 
unless $\mathbf{g}_{R,n,i}^{T} \mathbf{x}_{R}^{(2)} = 0$. Therefore, the estimator (28) finds $\mathbf{x}_{R,ML}^{(2)}$ that satisfies 
\[
\mathbf{g}_{R,n,i}^{T} \mathbf{x}_{R,ML}^{(2)} > 0
\]  
(70) 
for all $1 \leq n \leq N_{t}$ and $1 \leq i \leq 2$ in the high SNR regime because $\Phi(t)$ is an increasing function of $t$ but upper bounded...
Therefore, the requirement of $\hat{y}_1 = \hat{y}_2$ can be decomposed into $2N_r$ independent equations $\text{sgn}(x - y) = \text{sgn}(x + y)$, which gives

$$\Pr(\hat{y}_1 = \hat{y}_2) = \left(\frac{2}{\pi} \arctan \sqrt{K - 1}\right)^{2N_r}.$$  

(87)

REFERENCES


