A New Predictive Approach for Bilateral Teleoperation With Applications to Drive-by-Wire Systems

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Abstract—In this paper, a new predictive approach is proposed for the impedance control of bilateral teleoperation systems. The proposed control structure includes two mirror predictors/observers in both: the master and slave sides. These predictors/observers are used to simultaneously estimate the master and the slave internal dynamics, and thereby to avoid the use of the delayed transmitted information. As a consequence, the influence of the delay on the whole system can be minimized and the performance can be improved. Under a set of suited hypothesis, the proposed control structure is shown to be uniformly ultimate stable even in the presence of time-varying delays. Simulation results are presented to show the effectiveness of the proposed approach. The behavior of the control structure is also experimentally demonstrated while performing remote steering of a small autonomous vehicle.

Index Terms—Bilateral teleoperation systems, Predictive approach, Delayed systems, Network controlled systems, Time varying delays.

I. INTRODUCTION

During the last several decades, many different teleoperation systems have been developed to allow human operators to execute tasks in remote or hazardous environments. They are widely applied in different circumstances such as in the outer space, undersea, nuclear plants, surgical operation, vehicle steering, etc. However, a time delay incurs in a long distance transmission of information. In the case of bilateral teleoperation, signals (i.e. environment contact torques or forces) of the slave system are reflected back to the master side operator providing him with a feel of what is remotely sensed. Similarly, operation commands are sent to the slave system in order to execute the desired operation. According to the used signal transmission mechanism, the delay incurring during such an operation, can be of different nature. For instance, transmission delay may be constant, time varying (slow, or fast), small or large.

Bilateral teleoperation systems are by construction feedback systems with delay transmission in the loop. If no particular attention is paid, the delay at best degrades the closed-loop performance, at worst destabilize the bilaterally controlled teleoperator [1]. As a consequence stability has been the main concern since the beginning of studies on teleoperation [2], [8]. Nonetheless, development of controlled teleoperation system with both stability and performance assessments is still a somewhat open problem.

The aim of this paper is to contribute to the understanding on the control design allowing to improve the teleoperation performance while preserving closed-loop stability.

A. Previous work

In the literature, there are many control schemes proposed for dealing with the time delay in the teleoperation systems: passivity based analysis [14], [32]; pole placement control [2] [3]; observer based design [4]; \( H_\infty \) control [8], [9]; sliding mode control [10] [11]; the master state prediction approach [12], etc. A recent and quite complete overview on the robotic telemanipulation systems can be found in [13].

1) Passivity based control: The passivity control approach has the ability to enforce the passivity of the whole system ensuring closed-loop stability. However, system performance degradation occurs when the delay is large, because enforcing closed-loop passivity results in controllers with low gains [5], [33]. In systems where the transmission delay is time-varying and only an (somewhat conservative) upper bound is known, the passivity approach may result in conservative design (if the maximal delay value is taken into account) or passivity may be lost for large delays. Other problems may also come from the digital implementation of the control law [14] that may break the passivity of closed loop structure, unless some particular care during the implementation (to combat with packet loss) is taken [34], [7].

2) Robust control: In [8], [9], a standard \( H_\infty \) control problem is formulated where the master and slave sides are stabilized locally under the assumption that there is no contact torque. In the case that a contact torque is applied, a third controller is designed to stabilize the closed loop system. The time delays. Simulation results are presented to show the effectiveness of the proposed approach. The behavior of the control structure is also experimentally demonstrated while performing remote steering of a small autonomous vehicle.

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delay is treated as a perturbation to the system, which often results in a conservative approach. The sliding mode control provides another possibility for the time delayed system while many concerns should be on the practical implementation such as the high switching gain and the chattering phenomena. The limitation of the SMC scheme in [11] is that the delays still exist in the impedance control target which means that the stability of the teleoperation system may not be ensured even though the target can be realized by the robust SMC scheme.

3) Finite spectrum assignment: Pole placement approach for constant time delays has also been used in the context of tele-operation. The closed-loop system poles can be assigned arbitrarily according to finite spectrum assignment (FSA) approach [3]. In the FSA, the closed-loop poles are placed using an infinite dimensional controller, resulting in a finite dimensional closed-loop system. Some extension to the case of time-varying delay has been recently reported in [21], [22]. One important drawback of the FSA is that small variations in the estimated time delay, may result in large deviation of the closed-loop solutions.

4) Prediction based control: The prediction method proposed in [12] can only predict the states in the master side while it cannot predict the transmitted data in bilateral channel. The system performance degrades especially when contact torque is applied in the slave side as a result.

B. Facts and contribution

Most of the approaches mentioned previously work well as long as the time delay is constant and not too big when compared to the closed-loop desired bandwidth. However, in many real transmission media and in some application of practical interest, the transmission delays are not only time-varying, but they can also vary in a large enough range so as to create stability and/or performance degradations. Hence it is important to deal with the control problem in the teleoperation systems in the existence of time varying delays.

Since performance has been the bottleneck of the teleoperation problem so far, it seems then natural to search for control methods having the ability to predict the behavior of the master and slave dynamics, so as to compensate for the effects of the transmission delays of important magnitude. In other words, look for less conservative control designs, yielding the best possible predictable performance while preserving the closed-loop stability. Control structures based on prediction represent an interesting alternative to tackle this problem. Nevertheless, readers should note that control prediction methods are model-based control structures requiring precise dynamic models for both the master and the slave systems; a price due by the well known fact that performance and stability can be simultaneously reached only if some system knowledge is available.

In this paper, a new predictive approach is proposed for the impedance control of bilateral drive-by-wire teleoperation systems.

The proposed control structure includes two mirror predictors/observers in both: the master and slave sides. These predictors/observer are used to simultaneously estimate the master and the slave internal dynamics, and thereby to avoid the use of the delayed transmitted information. As a consequence, the influence of the delay on the whole system can be minimized and performance can be improved. Under a set of suitable hypothesis, the proposed control structure is shown to be uniformly ultimate stable even in presence of time-varying delays.

Note that the proposed approach is developed in the case of symmetric time-varying delays in both communication channels, but shows good robustness property in the presence of asymmetric ones. Let us mention that most of studies dedicated to teleoperation assumes symmetric and constant delays while few works are devoted to time-varying delays (see for instance [7]). Future work will focus on the predictive based teleoperation approach with asymmetric random delays and packet dropouts.

Finally, robustness of the proposed control approach with respect to the model uncertainties is partially addressed via simulations, and supported via experimental trials done on a teleoperated small autonomous vehicle platform.

A particularity of the vehicle teleoperation case, is that:

- it is a single degree-of-freedom (1 DOF) system
- the environment model is nonlinear due to the friction caused in the rotational operation of the vehicle wheels [16], [17], [18]
- it is important to have reflected force feedback. The contact friction force is then necessary to be transmitted back from the slave side to the the steering operation side so that the human operator could have better feeling of the environment during the steering. This is necessary for driving comfort, and to avoid vehicle oversteering/understeering, which represents the transparency of the teleoperation control scheme [15], [18].
- the steer-by-wire is a bilateral teleoperation system (see above), while the longitudinal teleoperation is a unilateral teleoperation system (the vehicle speed is sent as a reference to the slave side).

Although the study shown here is particularly addressed to the problem of wheeled ground vehicle drive-by-wire teleoperation, the results are general enough to be applied to systems of different nature.

C. Notation, hypothesis and definitions

1) Notation: For simplicity reasons, the time dependency in the signal will, in general, be excluded, unless we would like to stress such a dependency, as in the case of delayed signals where this dependency will be explicitly indicated. For instance, system of the form \( \dot{x}(t) = Ax(t) + A_dx(t - T(t)) \),
will simply be noted as:

\[ \dot{x} = Ax + A_d x(t - T) \]

where \( x = x(t) \), and \( T = T(t) \).

The notation \( \dot{x} \) will indicate estimated signals, whereas \( \dot{x} \) will denote error signals. \( c \) will indicate a generic constant.

In general, the linear operators are written without its Laplace signal dependence, i.e. the operator \( G(s) : u \mapsto y \), is simply denoted as \( y = Gu \). Alternatively, and when needed, we will use the notation \( y = G \circ u \).

2) Hypothesis: With respect the general teleoperation scheme shown in Figure 1, the following hypotheses are made:

**Assumption 1:** We assume that system knowledge is available, in term of models for:

- the environmental impedance model. In teleoperation of robot manipulator, the model is assumed to be a memoryless linear map described by a spring-like force/displacement model. In vehicular applications, the environmental impedance model. In vehicular applications, the model is assumed to be a memoryless linear map described by a spring-like force/displacement model. In vehicular applications, the model is assumed to be a memoryless linear map described by a spring-like force/displacement model.
- the master and the slave actuators. In our vehicle steering applications, the actuator are of one degree of freedom, therefore actuators can be simply described by single-input single-output double integrators.

**Assumption 2:** The following signals are assumed to be measured by the corresponding sensors:

- The human torque \( F_h \) interacting with the driving wheel is measured by placing a torque sensor in a proper location.
- Driving wheel position angle \( x_m \) (eventually the rotational velocity \( v_m \) may also be assumed to be measurable, by time-derivation of \( x_m \)).
- The aggregate effect of the environment torque \( F_e \) described by the wheel/ground interaction, can be measured at the level of the wheel axis rotation.
- The wheels orientation (with respect to the chassis of the vehicle) \( x_s \) (and eventually its time-derivative, \( v_s \)) are measured by an encoder.

**Assumption 3:** The transmission delay \( T = T(t) \) is bounded as \( 0 \leq T \leq T^* < \infty \) and \( |T| \leq T^+ < 1 \), where \( T^* \) and \( T^+ \) are positive constants.

**Assumption 4:** We consider through the paper that the interaction between the operator and the steer wheel, as well as the one between the vehicle and the environment, is a compliant phenomenon [20]. We thus assume that, for any delay \( T(t) \), the human and contact torques, \( F_h(t) \) and \( F_e(t) \), cannot escape in finite time, i.e. there exist positive constants \( \rho_h \) and \( \rho_e \), such that:

\[ F_i(t - T(t)) - F_i(t) < \rho_i, \quad i = h, e \]

3) Definitions: The following definitions will be used in this paper. \( \| \cdot \| \) is the Euclidean norm and induced matrix norm when apply.

**Definition 1:** Schur Complements: For matrices \( \Omega_1, \Omega_2, \Omega_3 \) where \( \Omega_1 = \Omega_1^T < 0 \) and \( \Omega_2 = \Omega_2^T > 0 \), then \( \Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0 \) if and only if,

\[
\begin{bmatrix}
\Omega_1 & \Omega_3^T \\
\Omega_3 & -\Omega_2
\end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix}
-\Omega_2 & \Omega_3 \\
\Omega_3^T & \Omega_1
\end{bmatrix} < 0.
\]  \hspace{1cm} (1)

**Definition 2:** [24] A linear matrix inequality (LMI) has the form

\[
F(x) = F_0 + \sum_{i=1}^{i=m} x_i F_i > 0,
\]

where \( x \in \mathbb{R}^m \) is the variable and the symmetric matrices \( F_i = F_i^T \in \mathbb{R}^{n \times n}, i = 0, \ldots, m \) are given.

The LMI (2) is equivalent to a set of \( n \) polynomial inequalities in \( x \), i.e. the leading principle minors of \( F(x) \) must be positive. In (2), \( F(x) < 0 \) can also be interpreted in a similar way.

**Definition 3:** \( H_\infty \)-Attenuation property : Given \( z(t) \) a controlled output and \( w(t) \) a disturbance, if there exists some positive scalars \( \gamma \) and \( \beta \) such that:

\[
\int_0^\infty \|z(t)\|^2 dt \leq \gamma^2 \int_0^\infty \|w(t)\|^2 dt + \beta.
\]

then the \( H_\infty \)-attenuation of the disturbance signal \( w \) on the controlled output \( z \) is said to be satisfied. In other words, the system will be finite-gain \( L_2 \) stable w.r.t the disturbance input [25].

Thus, as in [27], the following performance index is then considered throughout the paper:

\[
J(w) = \int_0^\infty (z^T(t)z(t) - \gamma^2 w^T(t)w(t))dt.
\]

Moreover, as we deal with time-delay systems, the following norm will be useful for any signal \( v(t) \).

\[
\|v(t)\|_c = \sup_{\theta \in [-T(t),0]} \|v(t+\theta)\|.
\]

**Corollary 1:** By (5), it then follows:

\[
\lim_{\tau \to \infty} \int_0^\tau \|v(t)\|_c dt = \lim_{\tau \to \infty} \int_0^\tau \sup_{\theta \in [-T(t),0]} \|v(t+\theta)\| dt = \lim_{\tau \to \infty} \int_0^\tau \|v(t)\| dt.
\]

The outline of this paper is as follows.

In section II, we present the considered teleoperation model, the assumption on the transmission delays as well as the target impedance model.

Section III is devoted to the predictor-based control design. First the control structure that allows to reach the target model is given. Then the proposed predictors (with specific gain on the delayed prediction errors) are given in order to estimate some instantaneous variables instead of using measured delayed ones. It is shown that the prediction error satisfies a delayed state equation which has to be stabilized.

The way to design the gains on the delayed prediction errors, such that the prediction error time-delay systems is stable, is presented in section IV using a Lyapunov type method. A three step design method is proposed : a first step to evaluate the
performance on the scheme without prediction, the second one
overcomes some nonconvexity problems and solves the design
of predictor gains (in the form of some LMIs to be solved),
and the third one evaluates the performance of the obtained
solution.

Section V completes the closed-loop stability proof by the
analysis of the error dynamics between the closed-loop system
and the target one. The final performance is then qualified in
relation with the predictor design.

In section VI simulation results are provided and the ex-
perimental framework is described in section VII. Concluding
remarks end the paper.

II. PROBLEM FORMULATION

This section describes the model of the teleoperation system
addressing in particular the problem of vehicle teleoperation,
and the formulation of the control objectives.

A. Teleoperation model

The following dynamics of the single DOF master and slave
system is considered

\[
\begin{align*}
M_m \dot{\theta}_m &= u_m + F_h, \\
M_s \dot{\theta}_s &= u_s - F_e,
\end{align*}
\]

(6)

where \( \theta_i \) and \( u_i \) \((i = m, s)\) are the actuators motor angular
velocities and torques respectively. The subscripts “\( m \)” and
“\( s \)” stand for master and slave respectively. \( M_i \) \((i = m, s)\) is
the mass. \( F_h \) is the torque applied at the master side by the
human operator and \( F_e \) is the torque exerted on the slave by
its environment.

The structure of the teleoperation system is shown in Fig.2,
with \( G_m = \frac{1}{C_m} \) and \( G_s = \frac{1}{C_s} \). \( C_m \) and \( C_s \) are the
controllers to be designed. \( H \) denotes the human operator
impedance, and \( F_{h,ext} \) is external torque applied by the operator.

Fig. 2. Block diagram of the teleoperation controlled system

In vehicle teleoperation, the environment forces are due to
the wheel/ground interaction. These forces can be modelled by
any of the existing tire/road friction models, see for example
[16]. The environment forces are due to:

- the torques produced by the interaction between the
  longitudinal and the lateral motion of the vehicle, known
  as the auto-alignment torque. They are mainly present at
  high speed.
- the torsional torques due to the tire rotational friction.
  They are dominant when the vehicle speed is low.

In our vehicle teleoperation setup, where mainly low speeds
are operated, we will then neglect the auto-alignment torque,
and only consider the torsional torque as the unique source for
\( F_e \). This torque can be approximated as a nonlinear function
of the form [16], [17]:

\[
F_e = \sigma_1 v_s + f_n \arctan \left( \frac{v_s}{\delta} \right),
\]

(7)

where \( \sigma_1, f_n \) and \( \delta \) are constants depending on the tire and
road adhesion, \( v_s \) is the wheel rotational angular speed.

When using (7), it is important to remark that the impedance
map for the environment \( EV \), is not anymore a linear map, as
in many classical formulations. This issue will introduce addi-
tional technical difficulties specific to our vehicle teleopera-
tion problem.

Finally let us note that this paper deals with bilateral
teleoperation problem for the linear plant (6) with nonlinear
environmental force (7): this is not a classical robot teleopera-
tion, as this considers vehicle teleoperation. As well known
in automotive control, the most important nonlinearity is in
that case the tire/road contact friction, modelled here by (7)
[16], [17] (and thus taken into account in the theoretical
developments). Moreover, in a vehicle, all the non linear
forces (as Coriolis ones) are transmitted through the tire/road
interaction. In our scheme, they could be captured by \( F_e \).

B. Transmission delay estimation

In network controlled systems, it is usual to denote as
\( T = T(t) \), the half of the time delay resulting from the
round trip information transmission. In wireless transmission
systems, the incurred delay is more likely to be deterministic
and better known when an ad hoc protocol as UDP is chosen.
Unlike this case, in wire-based TCP transmission media \( T \)
is badly known and need to be estimated. In addition, the time
delay between master-to-slave, is not necessarily the same than
the one from slave-to-master. Although this latter problem is
never really treated specifically, it is worth to be mentioned.

Delay estimation has been addressed in the literature in
various manners. The possibility to get an estimate, \( \hat{T} \) of \( T \),
rates from model-based protocol estimation [21], to the use
of direct measurements. An example of the latter, the method
consists in using time-stamping of the transmitted variable.
That is, in sending a local (master) time signal \( t \) to the remote
(slave) side, and sent it back to the master side. This allows
to get the round-trip time, half of which being denoted \( \hat{T} \).
Of course this delay will be updated at each measurement
sample (so not for all time \( t \)). This method, often used to
evaluate the transmission delays, is interesting because it is
easy to implement and it adds some filtering action on the
delay estimation. The delay is assumed to fulfill Assumption 3.

Remark 1: Note that with this method only the round trip
delay can be measured. To measure the one direction time
delay it is required that both computers (the one in the master
and the slave side) have a synchronized time basis. In this
paper the mean of the round trip time will be considered.
Variations around this value can be considered as noise.

C. Target impedance

The control objective is to design a control law such that,
in presence of transmission delays, the following delay-free
target impedance is obtained,
\[
\begin{align*}
    x_m &= Y_m \circ [(1 - \alpha)F_h + N \circ F_e], \\
    x_s &= Y_s \circ [W \circ x_m - k_f F_e], \quad (8)
\end{align*}
\]
where \(Y_m(s), N(s), Y_s(s),\) and \(W(s)\) are linear design operators for the master and slave side, respectively. \(\alpha\) and \(k_f\) are also design constants. These specifications, can also be written only in terms of velocity and torque variables (see also Fig. 1),
\[
F = [F_h, -F_e]^T \quad \text{and} \quad \bm{v} = [v_m, v_s]^T, \quad \text{as}
\]
\[
\bm{v} = G \circ F 
\]
where
\[
G = G(s) = \begin{bmatrix}
Y_m(s)(1 - \alpha)s & (Y_s(s)W(s)Y_m(s)(1 - \alpha)s) - Y_m(s)N(s)s \\
Y_s(s)W(s)Y_m(s)(1 - \alpha)s & (Y_s(s)W(s)Y_m(s)N(s)s)
\end{bmatrix}
\]

The target impedance matrix \(G(s)\) is chosen according to the user specification subject to the passivity constraint of the map \(F \mapsto \bm{v}\) for the system (9), given by \(\int_0^\infty F^T(t)\bm{v}(t) > 0,\) or equivalently,
\[
\lambda_{\min}(G(j\omega) + [G(j\omega)]^*) \geq 0,
\]
where \(\lambda_{\min}(\cdot)\) denotes the minimum eigenvalue, and \([G(j\omega)]^* = [G(-j\omega)]^T\). In this paper, the specifications are selected as:
\[
\begin{align*}
Y_m(s) &= \frac{1}{(M_m s^2 + B_m s + K_m)} \\
Y_s(s) &= \frac{1}{(M_s s^2 + B_s s + K_s)} \\
W(s) &= B_w s + K_w \\
N(s) &= \beta
\end{align*}
\]
\(\alpha\) and \(k_f\) are all constants.

Note that the previous choice is very similar to those used for most of the well-known impedance control approaches. Also it corresponds in our case to the transparency objective, i.e. the driving comfort [15].

III. PREDICTOR-BASED CONTROL DESIGN

In this section, we introduce the predictor-based control scheme designed to perform the target maps (or reference model) described by (8). To this aim we first assume that both position and velocity (in the master and the salve side) are available for feedback, as described in Assumption 2. In the last part of this section, in this section an extension including a velocity observer as well will be presented.

The schematic diagram is as shown in Fig. 3.

\[\text{Fig. 3. The schematic diagram of the bilateral teleoperation system with the proposed control scheme including the bi-predictor. (Obs: Observer)}\]

Similarly \(\hat{x}_m,\) and \(\hat{v}_m = s \circ \hat{x}_m\) are the predicted master position and velocity, respectively. Note that \(\hat{F}_e,\) and \(\hat{x}_m\) need to be predicted because they are used remotely, and hence they are not locally available\(^2\).

With the control law (10), the closed loop dynamics of the teleoperation system results in
\[
\begin{align*}
    x_m &= Y_m \circ \left\{ (1 - \alpha)F_h + \beta \hat{F}_e \right\}, \quad (11) \\
    x_s &= Y_s \circ \left\{ W \circ \hat{x}_m - k_f F_e \right\}. \quad (12)
\end{align*}
\]

The control goal clearly appears to be the match of the target impedance structure (8). The control design difficulty is thus resumed to the problem of how to predict the contact torque \(F_e(t)\) at the master side, and how to predict the master position \(x_m(t)\) at the slave side. Predictor should only use current and/or delayed measurements.

Since the predictors will form part of the local controllers on each side of the teleoperation system, it becomes necessary to have a mirror reproduction of those predictors. In other words, master and slave controllers will have two predictors at each side as shown below. Predictors are designed on the basis of the closed-loop equations (11), and (12). This choice is motivated for the reasons that will appear clearly when the predictor delayed structure will be presented in forthcoming section.

B. State space representation

As a preliminary to the predictor design, the following state-space representation for the Equations (11)-(12) is introduced. For (11), we have
\[
\begin{align*}
    \dot{z}_m &= A_m z_m + b_{m1} F_h + b_{m2} \hat{F}_e, \\
    y_m &= c_m^T z_m = \begin{bmatrix} 1 & 0 \end{bmatrix} z_m = x_m
\end{align*}
\]
with
\[
A_m = \begin{bmatrix} 0 & 1 \\ -K_m M_m & -B_m M_m \end{bmatrix}, \quad z_m = \begin{bmatrix} x_m \\ v_m \end{bmatrix}
\]
\[
b_{m1} = \begin{bmatrix} 0 \\ (1-\alpha) \end{bmatrix}, \quad b_{m2} = \begin{bmatrix} 0 \\ \beta \end{bmatrix}
\]
and, for (12)
\[
\begin{align*}
    \dot{z}_s &= A_s z_s + b_s \hat{x}_m + b_s F_e, \\
    y_s &= c_s^T z_s = \begin{bmatrix} 0 & 1 \end{bmatrix} z_s = x_s.
\end{align*}
\]
\(^2\text{A signal is locally available when it is measurable at the side (master or slave) that is used.}\)
where
\[ A_s = \begin{bmatrix} 0 & -\frac{K_s}{M_s} \\ 1 & \frac{B_s}{M_s} \end{bmatrix}, \quad z_s = \begin{bmatrix} z_{s1} \\ z_{s2} \end{bmatrix}, \]
\[ b_{s1} = \begin{bmatrix} \frac{K_s}{M_s} \\ \frac{B_s}{M_s} \end{bmatrix}, \quad b_{s2} = \begin{bmatrix} -\frac{K_s}{M_s} \\ 0 \end{bmatrix}. \]

with the transformation:
\[ x_s = z_{s2}, \quad v_s = z_{s1} - \frac{1}{M_s}(B_sz_{s2} - B_w\tilde{x}_m) \]

Remark 2: Note that by construction the \( A_{m}, A_{s} \) matrices have stable eigenvalues. This will be useful later during the stability analysis of the predictors. Note also that the eigenvalues of these matrices, are not completely free; they are stable, but specified by the desired target impedance.

C. Predictor design

We first assume that both positions and velocities sets \((x_m, v_m), (x_s, v_s)\) are measurable at their respective master and slave side. The velocity measurements can be relaxed, as explained later in this section, by the use of additional observers. For simplicity, we first present the predictor design on the basis of the fully measured states \(z_m\), and \(z_s\), which can be computed by a linear transformation from the previously mentioned position and velocity signals. The predictor structure will be designed on the basis of the stable state-space representation given previously.

The main purpose of the prediction construction, is to predict \(F_e\) (or equivalent to predict \(v_s\), since \(F_e\) depends on \(v_s\)) at the master side, and simultaneously to predict \(x_m\) at the slave side. In other words, the problem is to reconstruct \(x_s\) in the master computer, and vise versa to reconstruct \(x_m\) in the slave computer. The difficulties in doing this lay at the following points:

- first note that \(F_h\) (respectively \(F_e\)) is not accessible at current time at the slave side (respectively at the master side). These signals play the role of inputs, and without them a proper predictor can not be constructed. These signals can be measured, and could be sent throughout the communication network. Therefore, their delayed versions \(F_h(t-T), F_e(t-T)\) could be used for control purposes. However, a storage of \(F_e\) and \(F_h\) during \(T\) seconds would be necessary to implement such a control scheme. To avoid a memory buffer we have preferred to artificially delay \(F_h\) and \(F_e\).
- secondly, the state-space equations are coupled with each other through the estimates: \(\hat{F}_e(\hat{v})\), and \(\hat{F}_c\), therefore it is mandatory to have a mirror reproduction of predictors for both equations at each side of the teleoperated system. Thus, we need to build four predictors: two at the master side and two at the slave side. They have however the same structure, and need to be initialized with the same initial conditions at the same time.\(^3\)

The proposed predictors have the following structure:
\[ \hat{z}_m = A_m\hat{z}_m + b_{m1}\hat{F}_h(t-T) + b_{m2}\hat{F}_e(\hat{z}_m, \hat{z}_s) + E_m[\hat{z}_m(t-T) - z_m(t-T)], \]  \(17\)
\[ \hat{z}_s = A_s\hat{z}_s + b_{s1}\hat{F}_c(t-T) + b_{s2}\hat{\tilde{x}}_m + E_s[\hat{z}_s(t-T) - z_s(t-T)], \]  \(18\)

where \(\hat{z}_m = c_m^*\hat{z}_m, \hat{F}_e(\hat{z}_m, \hat{z}_s) = \hat{F}_e(\hat{v}_s)\), with \(\hat{F}_e(\cdot)\) as specified before in (10), and \(\hat{v}_s = \hat{z}_{s1} - \frac{b_{s2}}{M_s}(\hat{z}_{s2} - \hat{z}_{s1})\).

Note that, as \(F_e\) and \(F_h\) are artificially delayed the same prediction equations are implemented in both sides. This structure is then simple and preserves the feasibility of the proposed observer. Thereby, we assume that time delay has been estimated accordingly to the method presented in the previous section\(^4\), i.e. \(\hat{T} = T\). Finally, \(E_m\) and \(E_s\) are the gain matrices to be designed. Note that they are only operating over the delayed error prediction.

Let \(\hat{z}_i(t) = \hat{z}_i(t) - z_i(t)\), be the observation error variables, then from the state-space representations (13)-(14), and the predictor equations (17)-(18), the error dynamics are given by
\[ \hat{\dot{z}}_m = A_m\hat{z}_m + E_m\hat{z}_m(t-T) + b_{m1}[\hat{F}_h(t-T) - F_h(t)], \]  \(19\)
\[ \hat{\dot{z}}_s = A_s\hat{z}_s + E_s\hat{z}_s(t-T) + b_{s1}[\hat{F}_c(t-T) - F_c(t)], \]  \(20\)

which can be casted in the following general form
\[ \hat{\dot{z}}(t) = A\hat{z}(t) + E\hat{z}(t-T) - F(t) + d(t), \]  \(21\)
where \(A, E, \hat{z}\) follow obvious notation from (19), and (20). The vector \(d(t)\) collects all the components of the last vector in the above expression. It can be also rewritten as \(d(t) = b\hat{F}(t)\), with \(\hat{F}(t) = F(t - T(t)) - F(t)\). This vector will be seen here as a disturbance having the following property.

Lemma 1: Assume that the time delay satisfied assumption 3 and that the human and contact torques, \(F_h(t), F_e(t)\) satisfy assumption 4. Then there exists a constant \(c(T^*, \rho_h, \rho_e)\) such that \(d(t)\) has the following bound,
\[ ||d(t)|| \leq c(T^*, \rho_h, \rho_e) \]  \(22\)
with \(c(\cdot)\) being proportional to all its arguments.

Proof: The proof is straightforward using assumptions 3 and 4 and the definition (21) of \(d\), i.e.: 
\[ d(t) = b\hat{F}(t), \text{ with } \hat{F}(t) = F(t - T(t)) - F(t). \]

Remark 3: The assumption on continuity used in the previous lemma should not be understood as an a priori boundedness (circular argument) of the signals involved in the closed-loop. Indeed, this hypothesis only implies that finite escape-time of the future systems closed-loop solutions will not occur. Note that this hypothesis does not avoid signals to diverge. For instance \(F_1(t)\) and \(F_1(t - T)\) may be radially unbounded, while its difference, \(\hat{F}(t)\) is kept bounded during

\(^3\)If both clocks of master and slave computers are not synchronous, this will generate some jitter (noise) in the transmission signals.

\(^4\)As it will be discussed latter, it is possible to account for inaccuracies on \(\hat{T}\), but for reasons of simplicity, the case \(\hat{T}(t) = T(t)\) is now considered.
any receding and finite time-interval.

Remark 4: By Lemma 1, it is also possible to treat the case when $\hat{T} \neq T$. In the error equation (19)-(20), these terms will appear as follows

$$\dot{\tilde{z}}_m = A_m \tilde{z}_m + b_{m1} F_h \tilde{z}_m(t - \hat{T}) + b_{m2} \tilde{F}_e + l_m[y_m - c_m^T \tilde{z}_m].$$

(23)

$$\dot{\tilde{z}}_s = A_s \tilde{z}_s + E_s \tilde{z}_s(t - \hat{T}) + b_{s1} [F_e(t - \hat{T}) - F_e(t)],$$

(24)

and hence they are still in the general form (21), but with $\hat{T}$, instead of $T$.

Remark 5: The robustness issue w.r.t parameter variations can be considered to some stand, as the system description (21) may capture model uncertainties as disturbances (as long as these uncertainties can be embedded in a disturbance term satisfying (22)).

D. Relaxing velocity measurements

Due to the observability form of the chosen state-space representation, it is also possible to derive a predictor on the basis of the position measurement only, $y_m$, and $y_s$. For this, we need to include an additional observer, in the following way.

At the master side, we add the following full-order observer defined with the new variable $\bar{z}_i$,

$$\dot{\bar{z}}_m = A_m \bar{z}_m + b_{m1} F_h \bar{z}_m + b_{m2} \tilde{F}_e + l_m[y_m - c_m^T \bar{z}_m].$$

Defining $e_m = e_m(t) = z_m(t) - \bar{z}_m(t)$, we have

$$\dot{e}_m = (A_m - l_m c_m^T) e_m,$$

where $l_m \in \mathbb{R}^{2 \times 1}$ is designed such that $A_m - l_m c_m^T$ is stable. Hence $e_m \in L_2 \cap L_\infty$, $e_m \in L_\infty$. Which implies that $e_m \to 0$, or equivalently $\bar{z}_m \to z_m$, as $t \to \infty$. The signal $z_m(t)$ is delayed artificially, so that $z_m(t - T)$ can be used locally at the place of $z_m(t)$. Note that $\bar{z}_m(t)$ needs also to be sent to the master side as well, such that when it arrives delayed by the network, $\bar{z}_m(t - T)$ can be also used in the mirror predictor, as described previously.

Similarly, the observer in slave side is defined as

$$\dot{\bar{z}}_s = A_s \bar{z}_s + b_{s1} F_e + b_{s2} \tilde{x}_s + l_s[y_s - c_s^T \bar{z}_s].$$

As before, defining $e_s = e_s(t) = z_s(t) - \bar{z}_s(t)$, we have

$$\dot{e}_s = (A_s - l_s c_s^T) e_s,$$

where $l_s \in \mathbb{R}^{2 \times 1}$ is designed such that $A_s - l_s c_s^T$ is stable. Similar procedure as in master observer is applied for the delayed use, and the transfer of the observed vector $\bar{z}_s(t)$.

Remark 6: Note also that the signals $\tilde{F}_e$, and $\tilde{z}_m$ are obtained from the respective predictors that are still running in parallel to these additional observers. In total, we need to implement two predictors, and one observer at each side of the teleoperated system, in case that only position measurement are used. However this is not mandatory.

Remark 7: It can be easily shown, that the general form of the error equation becomes in this case

$$\dot{z} = A\tilde{z} + E\tilde{z}(t - T) + d + E\epsilon,$$

Being $\epsilon \to 0$ the vector concatenating the components of $e_m$, and $e_s$. It is straightforward to show, that the stability properties of system with $\epsilon = 0$, are fundamentally the same than the system above with $\epsilon$ bounded, and converging to zero exponentially. For these reasons, this case will not be treated anymore in what follows.

IV. Feedback Gain - E Design

In this section, we will focus on the problem of how to design the feedback gain $E$ for the system,

$$\dot{z} = A\tilde{z} + E\tilde{z}(t - T) + d$$

(25)

$$\tilde{z}(t) = \tilde{z}_0(t), \quad t \in [-T(t), 0]$$

(26)

with the disturbance $d$ bounded as described in the previous section and assumed to belong to $L_2(0, \infty)$, and $T$ is the time-varying delay. Convergence and stabilities issues of this equation are next analyzed.

The control design for $E$ for systems of the form $\dot{z} = A\tilde{z} + E\tilde{z}(t - T)$, with possible unstable matrix $A$ do exist. In [26] a sufficient condition by solving a non-convex matrix inequality is derived to design the $E$ matrix. The computation procedure for $E$ involves the use of the algorithm proposed in [28]. However, this algorithm cannot be applied to the system when there are disturbances and the time delay is time varying, as it is the case here.

On the other hand, if $A$ and $E$ are given, then it is possible (as shown in [27]) to verify the stability and attenuation property of the above equation, but when the aim is to design the feedback matrix $E$, this approach cannot be used (due to non convexity).

In this section, we propose a new procedure to solve the problem formulated above.

It is important to note that, by construction, the matrices $A_m$ and $A_s$ are both stable, as discussed in Remark 2. As a consequence matrix $A$ is also stable. Since $d$ is bounded (in the sense described by Equation (22)), the stability of the predictor can be ensured even if $E = 0$. Nevertheless, it is possible to design a delayed feedback gain $E$ such that the performance of the predictor can be improved in the sense of the attenuation property given in Definition 3.

Proposition 1: The proposed procedure for designing $E$ is based on the following 3 steps:

Step 1. Compute the minimum attenuation gain, $\gamma_{min}$, for $E = 0$. This provides a measure of the baseline attenuation case.

Step 2. Design a gain $E$ based on a Lyapunov (delay-free) analysis. This yields attenuation bound that is rather conservative.

Step 3. Test the attenuation bound obtained with gain $E$ designed in Step 2, using less conservative Lyapunov-Krasovskii (delay-dependent) functionals,
until obtain a better attenuation gain, than the one obtained in Step 1. Each of the three steps are described in details next.

A. Step design No.1

Find the minimum disturbance attenuation gain \( \gamma_{\text{min}}^* \) when \( E = 0 \) according to the following theorem.

**Theorem 1:** Under the assumption that \( A \) is stable, there always exist \( P = P^T > 0 \) s.t. \( \gamma > 0 \) satisfying

\[
L_1 = \begin{bmatrix}
   A^TP + PA + I & P \\
   P & -\gamma^2 I
\end{bmatrix} < 0.
\]

Therefore, the estimation signal \( \tilde{z}(t) \) satisfies an \( H_\infty \) attenuation property w.r.t. the disturbance signal \( d(t) \), according to Definition 3, i.e.:

\[
\int_0^\infty \| \tilde{z}(t) \|^2 dt \leq \gamma^2 \int_0^\infty \| d(t) \|^2 dt + \beta.
\]

The minimum disturbance attenuation gain \( \gamma_{\text{min}}^* \) is obtained by solving the convex optimisation problem:

\[
\gamma_{\text{min}}^* = \min_P \gamma \quad \text{s.t.} \quad L_1 < 0, \quad P > 0.
\]

**Proof:** See Appendix A.

B. Step design No.2

Obtain \( E \neq 0 \).

The error dynamics in (21) can be rewritten as

\[
\dot{z} = A\tilde{z} + E\tilde{z}(t-T) + d = (A + E)\tilde{z} + [E^TP^T \psi_1 0 0] \quad \text{s.t.} \quad \tilde{z} \text{ do exist and are uniquely defined. Therefore, they can not escape in finite time, and thereby there exists a positive constant } \nu \text{ such that }
\]

\[
\|g\| \leq \nu
\]

for \( \forall t \in [0, \infty) \).

The design of \( E \) is now done along the following theorem.

**Theorem 2:** Assume that \( A \) is stable. Given \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \), if there exist \( P = P^T > 0 \) and \( S \) such that:

\[
L_2 = \begin{bmatrix}
   A^TP + PA + S + I & S & P \\
   S^T & -\gamma_1 I & 0 \\
   P & 0 & -\gamma_2 I
\end{bmatrix} < 0.
\]

Then the matrix \( E \) obtained as:

\[
E = P^{-1}S,
\]

ensures that system (29) is stable and that the estimation signal \( \tilde{z}(t) \) satisfies an \( H_\infty \) attenuation property w.r.t. the disturbance signals \( g(t), d(t) \), according to Definition 3, i.e.:

\[
\int_0^\infty \| \tilde{z}(t) \|^2 dt \leq \gamma_2^2 \int_0^\infty \| g(t) \|^2 dt + \gamma_1^2 \int_0^\infty \| d(t) \|^2 dt + \beta,
\]

Note that \( \gamma_1 \) and \( \gamma_2 \) are chosen to minimize first the delayed terms, i.e. \( \gamma_1 \) is chosen as small as possible. This step could be further improved by minimizing both \( \gamma_1 \) and \( \gamma_2 \), but possibly leading to a large gain \( E \). Therefore a trade-off has been sought between attenuation level and prediction gain.

**Proof:** See Appendix B.

Note that formally, the above results do not give an estimation of the attenuation ratio compatible (or comparable) with the one obtained in the first step. This is due to conservativeness of the delay-free stability approach used for this step. Therefore, it is advised to use another less conservative sufficient condition (taking into account the delay) to test whether the obtained solution \( E \) can achieve a better attenuation performance. The following step is then proposed to test the performance of the predictor after introducing the gain \( E \) in the system.

C. Step design No.3

Test the performance for the following system

\[
\dot{\hat{z}} = \hat{A}\hat{z} + E\hat{z}(t-T) + d,
\]

with \( E \) designed in Step 2.

**Theorem 3:** Assume that \( A + E \) is stable\(^5\). Given \( \gamma > 0 \), if there exist \( P = P^T > 0, r_1 > 0, r_2 > 0 \) such that

\[
L_3 = \begin{bmatrix}
   \Xi & PE & PE & PE & P \\
   E^T P & \psi_1 & 0 & 0 & 0 \\
   E^T P & 0 & \psi_2 & 0 & 0 \\
   E^T P & 0 & 0 & -\gamma^2 T^{-1} I & 0 \\
   P & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix} < 0,
\]

where

\[
\Xi = (A + E)^TP + P(A + E) + T^r_1 A^T A + T^r_2 E^T E + I, \quad \psi_1 = -r_1 (1 - T^+) T^{-1} I \quad \text{and} \quad \psi_2 = -r_2 (1 - T^+) T^{-1} I
\]

then, system (34) is stable and the estimation signal \( \tilde{z}(t) \) satisfies an \( H_\infty \) attenuation property w.r.t. the disturbance signal \( d(t) \), according to Definition 3, i.e.:

\[
\int_0^\infty \| \tilde{z}(t) \|^2 dt \leq \gamma^2 \int_0^\infty \| d(t) \|^2 dt + \beta.
\]

The minimum disturbance attenuation gain \( \gamma^* \) is obtained by solving the convex optimisation problem:

\[
\gamma^* = \min_{P, r_1, r_2} \gamma \quad \text{s.t.} \quad L_3 < 0, \quad P > 0, \quad r_1 > 0, \quad r_2 > 0
\]

**Proof:** See in Appendix C.

Remark 8: Note that \( L_3 \) could also be written as

\[
L_3 = \begin{bmatrix}
   \Xi & PE & P \\
   E^T P & \mu T^{-1} I & 0 \\
   P & 0 & -\gamma^2 I
\end{bmatrix} < 0,
\]

\(^5\)This is implicitly obtained in the Step 2, due to the first upper left sub-matrix in inequality (31), i.e. If there exists a solution for \( E \) fulfilling (31), it is necessary that this solution satisfy \( (A + E)^TP + P(A + E) + I < 0 \). From the well known necessary and sufficient condition of the Lyapunov inequality, \( (A + E) \) can only be stable.
with
\[ \mu = \left\{ [-r_1(1-T^+)]^{-1} - [r_2(1-T^+)^2]^{-1} - \gamma^{-2}\right\}^{-1}. \]

It is obvious that the LMI in (35) becomes non-convex if \( E \) is not known, due to the term \( E^T E \). A direct design of \( E \) from this step, is therefore more involved. Nevertheless, for a given \( E \), and if (35) has a solution, we can check if the new obtained attenuation level is smaller than the one obtained with \( E = 0 \) in Step 1, i.e. if \( \gamma^* < \gamma^*_{\text{min}} \). If so we keep this solution, else we go back to Step 2 and obtain another \( E \) with different \( \gamma_1 \) and \( \gamma_2 \) values, and repeat the test in Step 3. Alternatively, step 3 may also be done by using smaller bounds on \( T \), and \( \bar{T} \), so as to smooth down the conservativeness of the used analysis.

Although, there is not full warrant to reach a finite number of iterations before get a suited results, it is very likely that the estimation in Step 3, will give a quite refined estimated, because the analysis is based on a delay-dependent Lyapunov-Krasovskii functional, which are less conservative than the Lyapunov functions used in Steps 1 and 2.

V. CLOSED-LOOP STABILITY

In this section we present the closed-loop stability properties resulting from the proposed control law together with the predictors. The analysis is based on the construction of an error dynamics on the variable \( r \) on the form

\[ \dot{r} = F(r) + G(u), \]

resulting from the difference between the closed-loop states \( z_i \), and the states of the desired target (8) dynamics, \( z'_i \), which can be expressed as:

\[ \dot{z}_m = A_m z_m + b_m F_h + b_m F_e, \]
\[ \dot{z'_m} = A'_m z'_m + b'_m x_m + b'_m F_e. \]

In Section II, the passivity of the target system can be ensured by the system specifications, and thereby the stability of the states \( z'_i \). As long as the human and environment dynamics are described by passive operator, their interconnection with the respective ports (\( F_h, v_h \), and \( F_e, v_e \), will inherit closed-loop stability. Equations (39)-(40) can thus be seen as a reference model for the the closed-loop system (13)-(14).

A. Error dynamics

The tracking error between the closed-loop system (13)-(14) and the target system (39)-(40) writes as:

\[ \dot{e}_m = A_m e_m + b_{m2} \dot{F}_e (\dot{v}_s) - F_e \]
\[ \dot{e}_s = A_s e_s + b_{s1} (\dot{x}_m - x'_m), \]

The following relations hold:

\[ \dot{x}_m - x'_m = (\dot{x}_m - x'_m) + (x_m - x'_m) \]
\[ = c_m (\dot{z}_m - z_m) + (z_m - z'_m) \]
\[ = c_m (\dot{z}_m - z_m) + e_m \]
\[ \leq c_m \| e_m \| + \| c_m \| \| \dot{z}_m \| \]

From another hand, we have

\[ \dot{F}_e (\dot{v}_s) - F_e (v_s) = \sigma_1(\dot{v}_s - v_s) + f_n \left[ \arctan(\frac{\dot{v}_s}{\delta}) - \arctan(\frac{v_s}{\delta}) \right] \]

noticing that, from (15)-(16), we have

\[ \dot{v}_s - v_s = \frac{B_{we} c_m T \tilde{z}_m}{M_w} \]

From the Lipschitz characteristics of \( \arctan(\cdot) \), and the mean value theorem, there exist a analytic bounded function \( \Phi(\cdot) \), with bound \( |\Phi(\cdot)| \leq 1, \forall \zeta \in [-\infty, \infty] \), such that

\[ \arctan(\frac{\dot{v}_s}{\delta}) - \arctan(\frac{v_s}{\delta}) = \Phi(\zeta_1) (\dot{v}_s - v_s), \]
\[ \zeta_1 \in [\dot{v}_s, v_s] \]

hence,

\[ \tilde{F}_e (\dot{v}_s) - F_e (v_s) = [\sigma_1 + f_n \Phi(\cdot)] (\dot{v}_s - v_s) \]
\[ = [\sigma_1 + f_n \Phi(\cdot)] \frac{B_{we} c_m T \tilde{z}_m}{M_w} \]
\[ \leq c_1 \| \dot{z}_m \| \]

with \( c_1 = (\sigma_1 + f_n \frac{B_{we}}{M_w}) \| e_m \| \). Here we have used the fact that \( \| \Phi(\cdot) \| \leq 1, \forall \zeta \in [-\infty, \infty] \).

The error dynamics (41) and (42) can be rewritten in compact form, as

\[ \dot{e}(t) = A e(t) + \eta(\dot{z}_m), \]
\[ = A e(t) + D \ddot{z}_m \]

with,

\[ A = \begin{bmatrix} A_m & 0_{2 \times 1} \\ b_{s1} c_m^T & A_s \end{bmatrix}, \]
\[ e(t) = \begin{bmatrix} e_m(t) \\ e_s(t) \end{bmatrix}, \]
\[ \eta(\dot{z}_m) = \begin{bmatrix} b_{m2} [\sigma_1 + f_n \Phi(\cdot)] \frac{B_{we} c_m T \dot{z}_m}{M_w} \\ b_{s1} c_m^T \dot{z}_m \end{bmatrix} \]

Note that the system specifications ensure that \( A \) is a stable matrix. The above system describes an stable linear system perturbed by the bounded signal \( \eta(\dot{z}_m) \).

B. Stability results

The stability of the system is concluded in the following theorem.

Theorem 4: The tracking error norm of the closed loop system (46) with the proposed control law (10), is uniformly bounded and tends, in finite time, to a ball \( B_r \)

\[ B_r = \{ e(t) : \| e(t) \| \leq r(\gamma, \rho_0) \}, \]

where \( \gamma > 0 \) is the constant obtained by the solution of

\[ \begin{bmatrix} A^T P + P A + I & P D \\ D^T P & -\gamma^2 I \end{bmatrix} < 0, \]

with \( P \in \mathbb{R}^{4 \times 4} \), \( P = P^T > 0 \). \( \rho_0 > 0 \) results from the bound of \( \| \dot{z}_m \| \leq \rho_0 \) (as \( \dot{z}_m \) is stable).

Proof: First note that due to the stability of \( A \), there is always a \( P \) and a \( \gamma \) such that the LMI holds. Secondly, from the boundness of observation error \( \dot{z} \) shown in previous theorems, and the bounds (43)-(44), we can conclude the existence of a constant \( \rho_0 \), such that \( \| \dot{z}_m \| \leq \rho_0 \).
Define the Lyapunov candidate $V = e^T P e$. The derivative of $V$ along the solution of the error equation gives,

$$
\dot{V} = e^T (A^T P + P A) e + e^T P D \ddot{x}_m + \ddot{x}_m^T D^T P e
$$

$$
= e^T (A^T P + P A + \gamma^{-2} P D D^T P + I) e
$$

$$
- \left( \gamma \dot{x}_m - \frac{1}{\gamma} D^T P e \right)^T \left( \gamma \dot{x}_m - \frac{1}{\gamma} D^T P e \right)
$$

$$
- e^T e + \gamma^2 \dot{x}_m^T \dot{x}_m
$$

$$
\leq e^T (A^T P + P A + \gamma^{-2} P D D^T P + I) e
$$

$$
- e^T e + \gamma^2 \dot{x}_m^T \dot{x}_m.
$$

If the following inequality, with $P$ being a positive definite symmetric matrix,

$$
A^T P + P A + \gamma^{-2} P D D^T P + I < 0,
$$

or equivalently, the following LMI

$$
\begin{bmatrix}
A^T P + P A + I & PD \\
D^T P & -\gamma^2 I
\end{bmatrix} < 0.
$$

holds, then

$$
\dot{V} \leq -e^T e + \gamma^2 \dot{x}_m^T \dot{x}_m, \leq -||e||^2 + \gamma^2 r_0^2.
$$

That is to say that the error $e(t)$ is bounded, i.e. the estimation error norm $||e(t)||$, tends, in finite time, to a ball $B_r$ defined as

$$
B_r = \{ e(t) : ||e(t)|| \leq \gamma \cdot r_0 \cdot \kappa(P) = r \}.
$$

with $\kappa(P) = \frac{\lambda_{min}(P)}{\lambda_{max}(P)} \geq 1$, being the conditioning number of $P$. Note that by virtue of the stability of matrix $A$, there always exists $P$ and $\gamma$ satisfying the above inequality.

An alternative proof probably yielding less conservative estimates $r$ for the set $B_r$, can be derived by introducing the function (or functional)

$$
V(e, \dot{z}) = V_1(\dot{z}) + e^T P e
$$

with $P$ being the matrix of the LMI of this theorem, and $V_1(\dot{z})$ being, either the function given in Theorem 2, or the function given in Theorem 3.

For instance, if function $V_1$ in Theorem 2 is used, we get a expression for $V$ of the form

$$
\dot{V} \leq -||\dot{z}||^2 + \gamma_1^2 \dot{z}_3^2 + \gamma_2^2 \dot{z}_4^2 - ||e||^2 + \gamma^2 ||\dot{z}||^2
$$

which shows, that if $\gamma < 1$, the new estimate for $r$, is

$$
r = \kappa(P) \sqrt{\gamma_1^2 \gamma_3^2 + \gamma_2^2 \gamma_4^2} / (1 - \gamma)
$$

which may yield less conservative estimates.

VI. ILLUSTRATIVE EXAMPLE: SIMULATION RESULTS

The purpose of the simulation presented in this section is to compare the new proposed predictor design to the case when no prediction is used, and to evaluate the robustness of prediction control scheme with respect to transmission delay uncertainties.

Data used in simulations are given in Appendix D. Design operators and constants: $Y_m, Y_s, N, W, k_f$, are selected so as to fulfill the specifications in term of driveability, and passivity of the operator $G : F(t) \mapsto v(t)$ in the target impedance as defined in (9).

The human force $F_h$ is modeled as [15],

$$
F_h = \frac{M_{hn} \beta^2 + B_{hn} s + K_{hn}}{M_{hd} s^2 + B_{hd} s + K_{hd}} \circ x_m + F_h^{ext},
$$

(48)

where $H$ denotes the human operator impedance. $M_{hn}$, $M_{hd}$, $B_{hn}$, $B_{hd}$, $K_{hn}$, and $K_{hd}$ are constants, and $F_h^{ext}$ is external torque applied by the operator. The model has been experimentally validated in [15].

A. Comparison with the scheme without predictors

Here the proposed scheme is compared with the conventional case when there is no prediction applied, in which the closed loop system is

$$
\begin{cases}
    x_m = Y_m [(1 - \alpha) F_h + NF_c (t - T)], \\
    x_s = Y_s [W x_m (t - T) - k_f F_s].
\end{cases}
$$

with the proposed data given in Appendix D. As expected, for certain values of $T$, the passivity of the map $F(t) \mapsto v(t)$, of the above closed-loop equation does no longer hold (note that this map is different to the target one because the delay presence in some entries of the resulting operator $F \mapsto v$). Hence the system under this conventional scheme may be unstable for certain time delays in the system.

In the proposed scheme, the delayed feedback gain matrices in the predictor, $E_m$ and $E_s$ are designed according to the three steps described in Section IV.

$E_m$ is achieved according to Theorem 2 when $\gamma_1 = 1$ and $\gamma_2 = 18$. From Theorem 3, a better disturbance attenuation level $\gamma^* = 22.6063 < 22.6459 = \gamma_{min}^{\alpha}$ is achieved, where $\gamma_{min}^{\alpha} = 28.645$ is obtained according to Theorem 1. Similarly, $E_s$ is achieved when $\gamma_1 = 0.1$ and $\gamma_2 = 0.7071$ according to Theorem 2. A better disturbance attenuation level $\gamma^* = 0.2905 < 0.6149 = \gamma_{min}^{\epsilon}$ is achieved, where $\gamma_{min}^{\epsilon} = 0.7778$ is obtained according to Theorem 1.

In Fig.4, the profiles of the positions $x_m$ and $x_s$ are shown. $x_m'$ and $x_s'$ are the reference signals from the desired model in (8). Note that the tracking errors of the proposed scheme are smaller than that of the scheme without prediction, in which the system is still stable because the passivity of the system with the delays in this simulation is still not broken. Note that the tracking errors of the master sides are smaller than those of the slave sides. This is because the minimum disturbance attenuation level of the prediction error of the slave states $\gamma = 0.2762$ is smaller than that of the master states $\gamma = 22.5668$, hence the estimation errors of the predictors

---

6This equivalence follows from the Schur complements, as defined in Definition 1.
are different, which are reflected on the tracking errors of the whole system. Fig.5 shows the evolutions of the forces $F_h$ and $F_e$.

As shown in Fig.6, the tracking errors of the system are compared between the two cases: with and without the delayed feedback control $E_i x_i(t - T_i)$ in the predictors. During the steady state, the bound of the tracking errors of the system are around 32% smaller with the incorporation of the delayed feedback gain in the predictors due to the smaller prediction error bounds. This further shows the advantages of applied predictors in the teleoperation system and another design freedom for the time delay controlled system.

B. Robustness with respect to time delay uncertainties

We further study the robustness of the proposed scheme on the time delay uncertainties. Two cases are considered here. In the first case, the measured delays are different as the real ones, i.e. it is artificially set to be $T(t) = 0.5 + 0.1 \sin(t)$ sec, while the real time delay is $T(t) = 0.4 + 0.2 \sin(t)$ sec. As in Fig.7 (a) and (b), the proposed scheme contains the robust property on the time delay estimation. In the second case, the real time delay from slave to the master side is set to be $0.7 + 0.3 \sin(t)$ sec which is different with the time delay from master to slave side. As in Fig.7 (c) and (d), the system is still stable and hence the proposed scheme seems to exhibit a certain degree of robustness with respect to time delay uncertainties (as pointed out in remark 4). As shown in Fig. 7 the robustness property is improved thanks to the predictors.

VII. EXPERIMENTAL APPLICATION

The proposed predictive control scheme is implemented to the teleoperation control of the Pekee small autonomous vehicle in slave side by the Drive-by-Wire system in the masterside as shown in the experimental setup in Fig.8.

A. System description

The Drive-by-Wire system consists of a system that steers the wheels, a device that is controlled by the driver (steering wheel unit) and a controller that interacts between these two systems. The steering wheel unit can be considered as the master side system and consists of a steering wheel which is connected to a torque sensor and a DC motor. With the torque sensor, the human applied torque on the steering wheel can be measured. With the DC motor, the master controller can give force feedback to the driver.

The Pekee is an open robotic platform designed by Wany Robotics company in France. Pekee is driven by two independent motorized wheels, with a free rotating caster wheel at the back. This layout offers substantial mobility - for example the unit can do an on-the-spot U-turn. In Pekee, a video camera
is also installed such that the image in front of it can be transmitted to the computer by wireless transmission. This makes it possible for teleoperation. The wireless communication is applied by the BeWAN Wi-Fi Access Point. There are two odometers (180 impulses/wheel-turn) in the two wheels for measuring the wheel rotational velocity. The settings of the computing is 16-Mhz Mitsubishi micro-controller (16-bit), with 256 KB Flash-ROM and 20KB RAM.

The internet protocol UDP (User Datagram Protocol) is applied in the data transmission in this experiment. By using UDP, the delay will be shorter compared that of TCP/IP though some data may be lost. Here UDP is selected because the delay magnitude is a key point for the application. The UDP server is applied in the Pekoe local computer and the UDP client is applied in the drive-by-wire master side system.

B. Modelling step

The system parameters are: Dive-by-Wire system -
\[ G_m(s) = \frac{1}{M_m s^2 + b_m s} \]
where where \( M_m = 0.02076 \) kg · m² and \( b_m = 0.0011 \) Nm/s are identified. In the experiment, according to the physical structure of the Drive-by-Wire system, the calculation of the \( F_h^{ext} \) in the experiment can be as the follows:

\[ F_h^{ext} = \frac{M_{wheel} (-u_m + F_{meas})}{M_m} - F_{meas} \]

where \( M_{wheel} = 0.01 \) kg · m² is the mass of the steering wheel, and \( F_{meas} = F_h - u_m \) is the measurement torque from the encoder. In the slave side system, \( G_s(s) = \frac{1}{M_s} \), where \( M_s = 0.02 \) kg · m².

The contact torque is as same as in (7) as
\[ F_c = \alpha_1 v_x + \beta_1 \arctan \left( \frac{v_x}{v_y} \right), \]
where \( \alpha_1 = 0.2, \beta_1 = 0.1 \) and \( \delta = 0.2 \) are selected.

C. Control design

The proposed predictive controller is designed according to the theory in Section III, the designed parameters of the master and the slave side is shown as the follows: for the master side, \( B_m = 0.3529 - b_m, K_m = 2.076, \alpha = 0.5, \beta = 1, l_m = [1, 1]^T \) and for the slave side, \( B_s = 0.5, K_s = 4, B_w = 0.2, K_w = 0.5, k_f = 2, l_s = [1, 1]^T \). All the initial values of the predictors and observers are set to be zero.

Hence, the matrixes \( A_{m}, A_{s}, \) and \( A \) matrix are all stable. Also from these data, the desired target results to be passive.

The feedback gains \( E_m \) and \( E_s \) are also designed according to the three steps with three Theorems in the subsection IV.

\[ E_m = \begin{bmatrix} -4.8614 & -2.833 & -0.1649 & -0.0011 \\ -2.8289 & -1.7054 & -0.0968 & -0.0006 \\ -0.1646 & -0.0967 & -0.0066 & 0 \\ -0.0012 & -0.0007 & 0 & 0 \end{bmatrix}, \]

\[ E_s = \begin{bmatrix} -3.1288 & 0.3213 \\ -0.6723 & 0.0701 \end{bmatrix}. \] (49)

By using UDP protocols, the delay is somewhat a constant and not so varied according to the time. So here in this experimental approach \( T(t) \) is treated by a constant, which is estimated by taking the half value of the round trip time delay. As shown in Fig 9, the delay is time varying and with the average to be around 20 ~ 30 milliseconds. So in our application, \( T = 0.01 \) sec, which is the half value of the round trip delay.

D. Experimental Results

In the technical parts and simulation results it is shown that the prediction gives degrees of freedom to improve the
performance of the tracking error between the closed-loop system and the target. We have therefore chosen to implement only the predictive scheme only.

As shown in Fig. 10, the system performance is compared with the desired target. This shows that the slave position is controlled in an efficient way. The experimental results also show that the proposed predictive approach is robust with respect to time delay uncertainties. Note that the scale of the tracking error is not that small as in the simulation results. This may be caused by the model parameters, time delay uncertainties and by the very simple sensors and real-time architecture of the Pekee. Another factor may be caused by the data measurement errors both in the steering wheel and in the small autonomous vehicle motors. Fig. 11 shows the evolutions of the forces $F_h$ and $F_e$. Some further studies may concern the robustness analysis and design w.r.t such modelling errors.

A new predictive approach has been proposed for bilateral torque reflecting teleoperation system. The same predictors in both sides are designed such that the transmitting signals at current time can be estimated according to the knowledge of the teleoperation system model. At the same time, the designed predictor convergence is ensured and a better prediction accuracy can be achieved through a delayed feedback design. Compared with the scheme without predictors, the proposed approach can achieve better system performance while ensuring the system stability in the existence of time varying delays. Simulation results, as well as experimental ones, emphasize the efficiency and applicability of the proposed approach, which is a new framework of teleoperation systems.

Even if the robustness of the proposed approach to uncertainties is not theoretically tackled here, the proposed scheme shows intrinsic robustness properties as described in simulation by changing the delays (leading to non-symmetric delays) and also as the experimental results are satisfactory while the considered model may different from the real Pekee robot. Some further studies may of course concern the robustness analysis and design w.r.t such modelling errors.

This approach can be used for the design of drive-by-wire systems, and also in particular for those applications in which the transmission delays are large and time-varying, i.e. underwater vehicles and space teleoperation.

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In this part we aim to design a matrix $P$ such that system (50) is stable and the following attenuation criterion:
\begin{equation}
\int_0^\infty (\hat{z}^T \hat{z} - \gamma^2 d^T d) dt < \infty.
\end{equation}
which can be written as:
\begin{align*}
J(d) &= \int_0^\infty (\hat{z}^T \hat{z} - \gamma^2 d^T d) dt + \int_0^\infty (V(\hat{z}, t)) dt - V(\hat{z}, 0).
\end{align*}
As $V \geq 0$ (i.e. $V(\hat{z}, t) \geq \lambda_{\text{min}}(P) ||\hat{z}||^2$), then it follows:
\begin{align*}
J(d) &\leq \int_0^\infty (\hat{z}^T \hat{z} - \gamma^2 d^T d + \hat{z}^T (A^T P + P A) \hat{z} + d^T P \hat{z} + \hat{z}^T P d) dt + V(\hat{z}_0(0), 0).
\end{align*}
Noting that
\begin{equation}
\mathcal{L}_1 = \begin{bmatrix} A^T P + P A + I & P \\ P & -\gamma^2 I \end{bmatrix},
\end{equation}
choosing $\beta = \lambda_{\text{max}}(P) ||\hat{z}_0(0)||^2 \geq 0$ and defining $\xi = [\hat{z}, d]^T$, we obtain:
\begin{align*}
J(d) &\leq \int_0^\infty \xi^T(t) \mathcal{L}_1 \xi(t) dt + \beta.
\end{align*}
Note that by virtue of the stability of matrix $A$, there always exists $P$ and $\gamma$ s.t. $\mathcal{L}_1 < 0$, and therefore s.t.
\begin{equation}
J(d) - \beta \leq 0.
\end{equation}
This concludes the proof according to Definition 3.

**APPENDIX B: PROOF OF THEOREM 2**

The error dynamics is
\begin{equation}
\dot{\hat{z}} = (A + E) \hat{z} + E g + d.
\end{equation}
Define the Lyapunov candidate $V = \hat{z}^T P \hat{z}$. The derivative of $V$ is as the followings
\begin{align*}
\dot{V} &= \hat{z}^T [(A + E)^T P + P(A + E)] \hat{z} + \hat{z}^T P E g + g^T E^T P \hat{z} + \hat{z}^T P d + d^T P \hat{z}.
\end{align*}
In this part we aim to design a matrix $E$ such that system (50) is stable and the following attenuation criterion:
\begin{equation}
J(g, d) = \int_0^\infty (\hat{z}^T \hat{z} - \gamma_1^2 g^T g - \gamma_2 d^T d) dt
\end{equation}
is such that $J(\mathbf{g}, \mathbf{d}) < \beta$ for some constant $\beta$.
Using here the same arguments that in Appendix A, and
defining $\xi = [\dot{z}, \mathbf{g}, \mathbf{d}]^T$, we easily obtain:

$$J(\mathbf{g}, \mathbf{d}) \leq \int_0^\infty (\xi(t) \mathcal{L}(t) \xi(t)) dt + V(\dot{z}_0(0), 0)$$

with

$$\mathcal{L} = \begin{bmatrix} (A + E)^T P + P(A + E) + I & PE & P \\ PE & -\gamma_1^2 I & 0 \\ P & 0 & -\gamma_2^2 I \end{bmatrix}.$$

Therefore, if there exist $P, \gamma_1, \gamma_2$ and $E$ such that $\mathcal{L} < 0$ then, the system (50) is asymptotically stable and satisfies the attenuation criterion $J(\mathbf{g}, \mathbf{d}) - \beta < 0$.
However, the previous LMI is non convex when $E$ is to be designed; we then use the following classical linearizing change of variable:

$$S = PE$$

leading to the LMI:

$$\mathcal{L}_2 = \begin{bmatrix} A^T P + PA + SA^T + S + I & S \\ S & -\gamma_1^2 I & 0 \\ S & 0 & -\gamma_2^2 I \end{bmatrix} < 0. \quad (51)$$

Note that, since $A$ is a stable matrix, there always exists $P, \gamma_1 > 0, \gamma_2 > 0$, and $S$ solutions of (51). This concludes the proof according to Definition 3.

**APPENDIX C: PROOF OF THEOREM 3**

Consider the following Lyapunov-Krasovskii functional (see [30], [31]), with $P = P^T > 0, r_1 > 0$ and $r_2 > 0$:

$$V(\dot{z}, t) = \dot{z}^T P \dot{z} + \int_{t-T(t)}^t \int_0^1 r_1 \ddot{z}(s) A^T \dddot{z}(s) ds d\theta$$

$$+ \frac{1}{1 - T^+} \int_{t-T(t)}^t \int_0^1 r_2 \ddot{z}(s) E^T \dddot{z}(s) ds d\theta$$

$$+ \int_{t-T(t)}^t \int_0^1 r_2 \ddot{z}(s) E^T \dddot{z}(s) ds d\theta$$

Note that

$$\dot{z}(t - T(t)) = \dot{z}(t) - \int_{t-T(t)}^t \dot{z}(\theta) d\theta = \dot{z}(t) - \int_{t-T(t)}^t [A\dddot{z}(\theta) + E\dddot{z}(\theta - T(\theta)) + \mathbf{d}(\theta)] d\theta$$

Hence substitute the above equation to the system dynamics and define $\mathbf{d}_1(t) = \int_{t-T(t)}^t \dddot{z}(\theta) d\theta$, we have

$$\dot{z}(t) = (A + E)\dddot{z}(t) + \mathbf{d}(t) - E\mathbf{d}_1(t)$$

$$- E \int_{t-T(t)}^t [A\dddot{z}(\theta) + E\dddot{z}(\theta - T(\theta))] d\theta.$$
and
\[ \ddot{z}^T P\dot{d} + (\dot{z}^T P\dot{d})^T = -((\gamma^{-1} P\dot{z} - \gamma d)^T (\gamma^{-1} P\dot{z} - \gamma d) + \gamma^2 \ddot{z}^T PP\dot{z} + \gamma^2 d^T d. \]

Using \( \frac{1 - \dot{T}}{1 - T^+} \geq 1 \) or \( (1 - \dot{T}) > (1 - T^+) > 0 \), and the derivation above, (52) becomes
\[ \dot{V} \leq \ddot{z}^T [(A + E)^T P + P(A + E)] \dot{z} + T(t)\gamma^2 \ddot{z}^T PEE^T P\dot{z} + \gamma^2 \int_{t-T(t)}^t d(\theta)^T d(\theta) d\theta + \gamma^2 \ddot{z}^T PP\dot{z} + \gamma^2 d^T d + T(t)\gamma^2 \ddot{z}^T PEE^T P\dot{z} + T(t)\gamma^2 \ddot{z}^T PEE^T P\dot{z} + T(t)\gamma^2 \ddot{z}^T PEE^T P\dot{z} - \ddot{z}^T T^* \left( \frac{r_1 A^T A + \left[ \frac{1}{1 - T^+} + 1 \right] r_2 E^T E}{1 - T^+} \right) \dot{z} + \ddot{z}^T T^* \left[ T\gamma^2 (r_1 (1 - T^+)^{-1}) + T^* (r_2 (1 - T^+)^{-1}) \right] PEE^T P\dot{z} - \ddot{z}^T T^* \left( \frac{1}{1 - T^+} + 1 \right) r_2 E^T d + \gamma^2 \int_{t-T(t)}^t d(\theta)^T d(\theta) d\theta + \gamma^2 \ddot{z}^T d + \ddot{z}^T T^* \left( \frac{1}{1 - T^+} + 1 \right) r_2 E^T E < 0. \]

If the following inequality is satisfied
\[ (A + E)^T P + P(A + E) + T^* \gamma^2 PEE^T P + I + \gamma^2 PP + T^* (r_1 (1 - T^+)^{-1}) + T^* (r_2 (1 - T^+)^{-1}) \right] PEE^T P + T^* r_1 A^T A \]
\[ + \left[ \frac{1}{1 - T^+} + 1 \right] r_2 E^T E < 0. \]

then the system (34) is asymptotically stable according to the Lyapunov-Krasovskii theorem.

Now, according to the Schur complement, the inequality (54) is equivalent to the following LMI
\[ \begin{bmatrix} \Xi & PE & PE & PE & P \\ E^T P & \psi_1 & 0 & 0 & 0 \\ E^T P & 0 & \psi_2 & 0 & 0 \\ E^T P & 0 & 0 & -\gamma^2 T^{-1} I & 0 \\ P & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \]

where \( \Xi = (A + E)^T P + P(A + E) + T^* r_1 A^T A + \left( \frac{T^*}{1 - T^+} + T^* \right) r_2 E^T E + I, \psi_1 = -r_1 (1 - T^+) T^{-1} I \) and \( \psi_2 = -r_2 (1 - T^+) T^{-1} I. \) Now, when (55) is satisfied, then we have, from (53):
\[ \dot{V}(\dot{z}, t) \leq -\ddot{z}^T \dot{z} + \gamma^2 \int_{t-T(t)}^t d(\theta)^T d(\theta) d\theta + \gamma^2 \ddot{z}^T d + \gamma^2 \int_{t-T(t)}^t d(\theta)^2 d\theta + \gamma^2 \ddot{z}^T d.
\]

Using the norm definition (5), we get:
\[ \dot{V}(\dot{z}, t) \leq -\parallel \dot{z} \parallel^2 + \gamma^2 T^* \parallel d(t) \parallel^2 + \gamma^2 \parallel d(t) \parallel^2 + \gamma^2 \ddot{z}^T \dot{z} \leq -\parallel \dot{z} \parallel^2 + \gamma^2 (1 + T^*) \parallel d(t) \parallel^2 \]

Therefore, we have:
\[ \parallel \dot{z} \parallel^2 \leq \gamma^2 (1 + T^*) \parallel d(t) \parallel^2 + \dot{V}(\dot{z}, t) \leq 0, \]

which, as \( V(\dot{z}, t) \geq 0 \) leads to, for all \( \tau > 0: \)
\[ \int_0^\tau \parallel \dot{z}(t) \parallel^2 dt \leq \gamma^2 (1 + T^*) \int_0^\tau \parallel \dot{d}(t) \parallel^2 dt + V(\dot{z}(0), 0) \]

Therefore, there exists a positive constant \( \beta, \) such that:
\[ \int_0^\tau \parallel \dot{z}(t) \parallel^2 dt \leq \gamma^2 (1 + T^*) \int_0^\tau \parallel \dot{d}(t) \parallel^2 dt + \beta \]

Then, using Corollary 1, we can conclude that:
\[ \int_0^\tau \parallel \dot{z}(t) \parallel^2 dt \leq \gamma^2 (1 + T^*) \int_0^\tau \parallel \dot{d}(t) \parallel^2 dt + \beta, \]

which ends the proof according to Definition 3.

APPENDIX D: SIMULATION DATA USED IN SECTION VI

In the simulation, the parameters of the system (6) are selected as \( M_m = 0.03 \text{ kg} \cdot \text{m}^2, B_m = 0.5 \text{Nms}, K_m = 2 \text{ Nm}, \alpha = 0.5, M_s = 0.04 \text{ kg} \cdot \text{m}^2, B_s = 0.5 \text{ Nms}, K_s = 4 \text{ Nm}, k_f = 2, B_w = 0.2 \text{ Nms}, K_w = 0.5, M_{hd} = 1, B_{hd} = 3, K_{hd} = 1, M_{hn} = 0.5, B_{hn} = 0.5, K_{hn} = 1, \beta = 1, \sigma_1 = 0.2, \delta = 0.5 \) and \( f_n = 0.1. \) Hence, \( Y_m(s) = 1/(0.03s^2 + 0.5s + 2), Y_s(s) = 1/((0.04s^2 + 0.5s + 4) \text{ and } W(s) = 0.2s + 0.5. \)

The external operator torque is \( F_h^x(t) = 5 \sin(t). \) The initial conditions of the system in state space form are \( x_m(0) = 0.1 \text{ rad}, v_m(0) = 0 \text{ rad/sec}, x_s(0) = 0.1 \text{ rad and } v_s(0) = 0 \text{ rad/sec}. \) The initial conditions of the predictor are \( \dot{z}_m(0) = [0, 0, 0, 0.1]^T \) and \( \dot{z}_s(0) = [0, 0.1]^T. \) The delays are \( T(t) = 0.4 + 0.2 \sin(t) \text{ sec}. \) Hence here \( T^* = 0.6 \) and \( T^* = 0.2. \) With these data, the passivity of the system desired target is satisfied.