Performance of a Sensor Network Aided Cognitive Radio System

Pål Grønsund† and Ole Grøndalen*

*Telenor, Snarøyveien 30, 1331 Fornebu, Norway
†Department of Informatics, University of Oslo, Gausdalallen 23, 0373 Oslo, Norway
Email: *{pal.gronsund, ole.grondalen}@telenor.com, †paalrgr@ifi.uio.no

Abstract—A Sensor Network Aided Cognitive Radio Network uses the concept of a wireless sensor network that assists a cognitive radio network by providing information about the current primary spectrum occupancy. In this paper we study the performance of the cognitive network for various cell sizes and transmit power levels when deployed co-located with a mobile primary system that uses a cellular reuse pattern with seven frequencies. Co-location of secondary and primary base stations has been found to be very important in order to reduce network costs. An expression for the degree of co-location of secondary and primary base stations is derived and used for identifying optimal secondary cell sizes. For these optimal secondary cell sizes, we determine the performance in terms of throughput, packet loss, delay, coverage and connectivity when using spectrum holes in the space, time and frequency domains. Especially, we find that cell size and transmit power levels are crucial for secondary system performance, and that smaller cell sizes and less expensive base stations for the secondary system such as WiFi access points and femto-cells is beneficial.

I. INTRODUCTION

Several measurement campaigns have demonstrated that radio spectrum is underutilized. The main application for cognitive radio (CR) is to exploit the spectrum resources more efficiently by dynamically utilizing radio spectrum not utilized by primary networks, referred to as spectrum holes. A wireless sensor network (WSN), not necessarily embedded in the cognitive radios, can be deployed to detect these spectrum holes and report to the secondary system as proposed in the EU FP7 project SENDORA [1] (Sensor Network Aided Cognitive Radio). Input on real time spectrum monitoring by using a separate low cost WSN was recently also requested by FCC [2, point 50].

In this paper we study and determine the performance of an example SENDORA system, a continuation and complete study of the work initiated in [3, Section 2.2]. Since the SENDORA system is a comprehensive one involving different systems and algorithms, we implement the whole SENDORA system in the system level simulator ns-2. This simulator enable simulation of the performance with a realistic implementation of the complete OSI protocol stack, network topologies and traffic models. All parts of the SENDORA system are implemented; a primary network, a centralized secondary network, a WSN consisting of energy detectors and a centralized fusion centre (FC) which aggregates spectrum usage information from the sensors and allocates channels to the secondary system. In [4] we found that a high degree of co-location of secondary and primary base stations (BSs) is very important in order to achieve a positive business case. Therefore we study the performance of the secondary system when the degree of co-location with primary BSs is optimal, which are the two cases when secondary cell size is equal to and half the primary cell size.

A power control study in CR networks in [5] found that the probability of spectrum opportunity decreases exponentially with the transmission power of secondary users (SUs), where the exponential decay constant is given by the traffic load of primary users (PUs). It was found in [6] that spectrum holes get saturated quite fast due to interference among SUs in a point-to-point communication network. Similar studies were performed in [7] for a single cell and in [8] for a frequency planned network. These works assumed a reduction in primary cell size as a compromise for the primary system to allow SUs and that the SUs can perfectly judge the distance to the PUs. In this paper, we consider a point-to-multipoint network, a realistic channel model and the PUs are mobile such that the considered spectrum holes [9] change dynamically in time and space. An interference requirement to allow secondary operation uses a constraint on maximum experienced interference at the PU [10], [11] which, realistically, is estimated by the FC by querying the WSN for presence of PUs.

The main focus and contribution of this paper is on estimating realistic network performance in a SENDORA system by simulations and on how spectrum holes can be exploited while respecting primary system interference constraints. Impact on the primary system performance will also be studied. The performance metrics for the secondary network is throughput, packet loss, delay, coverage and connectivity. A second contribution is the derivation of an expression for the degree of co-location of secondary and primary BSs. Though the WSN is a core component of the system and is implemented with energy detectors deployed in a rectangular grid, the main focus of this paper is not on WSN performance. It should also be noted that the objective of this paper is not on algorithm optimization.

II. SYSTEM MODEL

To obtain an available channel the secondary system consults the FC which has the total responsibility for communicating with the WSN. The simulation model for the primary and secondary systems uses an ns-2 implementation of WiMAX [12] developed in WiMAX Forum.
A. Network Model

The considered network model is illustrated in Fig. 1. The primary system uses a three-cell reuse pattern with 7 BS cells with frequencies $F_1$ to $F_7$ in the 2 GHz band, each channel of 10 MHz and the system is assumed to be noise limited. The secondary system consists of 1 BS cell co-located with a primary BS as illustrated by the colored BS in Fig. 1. The secondary BS and its SUs are able to use all frequencies, but will use one of the frequencies $F_2$ to $F_7$ if available since the primary nodes in the cell using $F_1$ are always within interference range of the secondary system. The WSN and FC are not illustrated in Fig. 1. For both primary and secondary systems, BS heights are 30 m and subscriber station heights 1.5 m. The distance between primary BSs is 2 km and the cells are hexagonal with radius 1.15 km. There are 65 sensors/km$^2$ [4].

B. Path Loss and Channel Model

The COST Hata model [13] is valid for distances $d$ above 1 km and the Cost Walfish-Ikegami (WI) [13] for distances above 20 m. Both models are valid for the 2 GHz frequency band. A path loss model which agrees with the COST-WI line-of-sight model, $P_{W,L}(d)$, for short distances and with the COST Hata model, $P_{H,L}(d)$, at distance above 1 km is used:

$$a = P_{W,L}(1\,\text{km}),$$

$$b = P_{H,L}(1\,\text{km}),$$

$$P_{L}(d) = \begin{cases} 
    P_{W,L}(d) + d(b-a), & d < 1\,\text{km}, \\
    P_{H,L}(d), & d \geq 1\,\text{km} 
\end{cases}$$  (1)

The channel model implemented in the OFDMA module in the ns-2 simulator is the path loss model described above combined with a Clarke-Gans implementation of Rayleigh Fading. Doppler effects are included to capture effects of node mobility and fast fading is included by modeling the channel as a Rayleigh fading channel with multiple taps as described by the ITU Pedestrian A model [14]. The path loss component is computed during simulation whereas the fast fading is computed prior to simulation.

Interference modelling in the ns-2 simulator is done at the subcarrier level by capturing packets from all transmitters in the system, both from secondary and primary nodes. When the received signal to interference plus noise ratio (SINR) on each subcarrier is calculated for each packet, a decision is made to further process or drop the packet. This is done by first finding the EESM (exponential effective SIR mapping) [15] to get the effective SINR and then extracting the block error rate (BLER) from the SINR, modulation and coding rate and block size. Based on the BLER value a decision is made whether to drop the packet or not. Please refer to [12] for details on the OFDMA implementation and interference modelling.

III. SENSOR NETWORK AIDED COGNITIVE RADIO SYSTEM

A. Primary System

The primary WiMAX system implemented in ns-2 is based on the IEEE 802.16e [16] standard and includes a very detailed model of the MAC and also a good model of the PHY layers. WiMAX uses orthogonal frequency division multiple access (OFDMA) and time division duplex (TDD) with 5 ms OFDMA frames. The main parameters given in Table 1 are common for the primary and secondary systems. Operating frequency is in the 2 GHz band and the channel bandwidth is 10 MHz with a total of 1024 subcarriers. The system uses partially used subcarrier allocation and vertical stripping for mapping of data to OFDMA symbols and subchannels [16].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Band (GHz)</td>
<td>2</td>
</tr>
<tr>
<td>Channel Bandwidth (MHz) / FFT</td>
<td>10 / 1024</td>
</tr>
<tr>
<td>Sampling Frequency $F_s$ (MHz)</td>
<td>11.25</td>
</tr>
<tr>
<td>Sampling Period 1/$F_s$ ($\mu$s)</td>
<td>0.18</td>
</tr>
<tr>
<td>Subcarrier Spacing $\Delta f = F_p/N_{F-F}$ (kHz)</td>
<td>10.94</td>
</tr>
<tr>
<td>Useful Symbol Period $T_S = 1/\Delta f$ ($\mu$s)</td>
<td>91.4</td>
</tr>
<tr>
<td>Guard Time $T_G = T_S / 8$ ($\mu$s)</td>
<td>11.4</td>
</tr>
<tr>
<td>Symbol Duration $T_s = T_S + T_G$ ($\mu$s)</td>
<td>102.9</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>QPSK</td>
</tr>
<tr>
<td>Forward Error Correction (FEC) rate</td>
<td>1/2</td>
</tr>
<tr>
<td>OFDMA Frame Duration (milliseconds)</td>
<td>5</td>
</tr>
<tr>
<td>Downlink/Uplink ratio</td>
<td>2:1</td>
</tr>
<tr>
<td>Cyclic prefix mode</td>
<td>1/4</td>
</tr>
<tr>
<td>Downlink/Uplink</td>
<td></td>
</tr>
<tr>
<td>Number of used subcarriers ($N_{	ext{used}}$)</td>
<td>421</td>
</tr>
<tr>
<td>Number of pilot subcarriers ($N_0$)</td>
<td>120</td>
</tr>
<tr>
<td>Number of data subcarriers ($N_c$)</td>
<td>720</td>
</tr>
<tr>
<td>Number of data subcarriers/subchannel</td>
<td>28</td>
</tr>
<tr>
<td>Number of subchannels ($N_{SC,\text{ch}}$)</td>
<td>30</td>
</tr>
<tr>
<td>Number of symbols (total 43)</td>
<td>28</td>
</tr>
</tbody>
</table>

The required transmit power $P_t$ for the primary BS with antenna gain $G_t$ when transmitting to a PU with antenna gain $G_r$ at cell edge of a cell with radius $r_p$, is given by:

$$P_t = P_r + G_t + G_r - P_L(r_p) - X,$$  (2)

where $P_L(r_p)$ is the estimated path loss given by Eq. (1) between BS and PU at cell edge and $X$ is a gaussian distributed random variable with zero mean and standard deviation of 8 dB representing shadow fading.

The primary system uses QPSK modulation and forward error correction (FEC) coding rate 1/2, which requires a
minimum received signal to noise ratio (SNR) \( \frac{C_r}{N} = 2.46 \text{ dB} \). The primary system is assumed to be designed to have a cell edge coverage of 75%, which approximately corresponds to an area coverage of 90% [17]. The required received power at the PU is then:

\[
P_{r, dBW} = \left( \frac{C_r}{N} \right) dB + N_{dB} + 10 \log_{10}(kT B),
\]

where \( B \) is the bandwidth in Hz, \( k \) the Boltzmann constant and \( T \) the temperature in Kelvin.

The average path loss at cell edge is found by using Eq. (1) with center frequency \( f_c = 2 \text{ GHz} \), BS height \( h_b = 30 \text{ m} \), PU height \( h_p = 1.5 \text{ m} \) and correction factor \( C_M = 0 \) since a medium sized city is considered, to be \( P_L(1.15 \text{ km}) = 139.9 \text{ dB} \). The temperature is \( T = 300 \text{ K} \) and utilized bandwidth is \( B = 9189.6 \text{ kHz} \). Due to limitations in the simulator, only omni-directional BS and user terminal antennas can be used. We therefore assume that both the BS and PU antenna gains are 0 dB. As a result, the calculated transmission powers will be larger than in a real scenario with directional antennas. With \( G_t = 0 \text{ dB} \) and \( G_r = 0 \text{ dB} \), transmit power for the primary BS and the PU is:

\[
P_{t, dBW} = P_L(r_p) dB + X dB + \left( \frac{C_r}{N} \right) dB + 10 \log_{10}(kT B) = 13.5 \text{ dBW}.
\]

### B. Wireless Sensor Network

The WSN provides information about spectrum occupancy in the area where the secondary network is deployed. The WSN is deployed in a rectangular grid and each sensor senses all the primary frequency bands. Furthermore, the sensors are simple energy detectors, thus synchronization of quiet periods between the WSN and secondary network is important. During these periods, the secondary stops all transmissions and the sensors measure primary activity. The sensing period is 30 milliseconds which is repeated with 0.5 second intervals. After each sensing period, the WSN communicates its sensing result to the FC responsible for allocating frequencies to the secondary system. A common control channel is used for reporting sensing measurements.

In [18] it was required that a single sensor must be able to detect a user terminal with a probability of 0.95 (since there are many sensors the total probability of detection will be higher). Assuming shadow fading with a standard deviation of 8 dB, the required shadow fading margin for 95% probability of detection is 13.16 dB. The threshold is then given by:

\[
Threshold_{dBW} = P_{t, dBW} - P_L(r_{ws}) dB - 13.16 dB,
\]

where \( r_{ws} \) is the wireless sensor radius. With 65 sensors/km\(^2 \) [4] each sensor covers an area of 1/65 km\(^2 \). Each side in the rectangle is then \( \frac{\sqrt{2}}{65} \ast 1000 \text{ m} \) and hence each sensor must cover a cell with radius \( r_{ws} = 87.7 \text{ m} \). By symmetry, both the BS and PU transmit power as found in Eq. (4) should be 13.5 dBW, so the threshold is:

\[
Threshold_{dBW} = 13.5 \text{ dBW} - 82.8 \text{ dBW} - 13.16 \text{ dB} = -82.5 \text{ dBW}.
\]

### C. Fusion Centre

The FC collects information from the sensors and will at any time have a near real-time overview of the spectrum occupancy for the area covered by the WSN. Upon spectrum request from the secondary system, the FC uses an algorithm that consults a matrix, representing the sensors in the WSN, that contain spectrum usage measurements for all potential channels from all sensors. Required functions for receiving sensor reports, calculating the spectrum map, managing allocations and communicate with the secondary system are implemented in the FC. The channel allocation algorithm allocates one of the available channels not used by the primary system.

In the simulator, the FC calculates the interference range \( r_{int} \) for the querying SU as the minimum distance beyond which the generated interference is below a given limit [10], [11]. Next, the FC checks the received power for all sensors within \( r_{int} \) of the querying SU to check if the received power is above the threshold found in Eq. (6). If not, the channel is reported as available for the SU. The interference limit used for calculating \( r_{int} \) is determined as follows: the interference generated to the primary system shall correspond to an increase of the noise-floor by less than 0.5 dB with a 90% probability.

This interference requirement corresponds to a shadow fading margin of 10.3 dB. The interference range \( r_{int} \) then satisfies:

\[
P_t - P_L(r_{int}) + 10.3 dB = 10 \log_{10}(kT B) + 10 \log_{10} \left( 10^{\frac{N_{dB}}{10}} - 1 \right).
\]

and by using Eq. 2, the FC is able to find the interference range \( r_{int} \) as a function of transmitted power \( P_t \).

### D. Secondary System

The secondary network is also based on WiMAX and the ns-2 simulator model developed in WiMAX Forum. The simulator model was modified with required functionality for operation as a secondary cognitive radio network. A centralized network with a single cell, a single BS and a set of SUs is considered. Two main cognitive functions are implemented. The first is a cognitive actuation module that communicates with the FC to obtain an available frequency allocation and then actuates this in the secondary network. The second is time synchronization with the WSN for quiet periods during which the WSN can measure primary activity.

To obtain an available channel the secondary system communicates with the FC which communicate with the sensors. First, the secondary BS queries the FC to obtain an available frequency and then informs the SUs about its allocated frequency. Next, since the BS is unaware of the SU coverage area, the SUs also need to query the FC if the frequency allocated
to the BS is available for it. If not, the affected SU notifies the BS which queries the FC for a new frequency.

IV. PERFORMANCE EVALUATION

Motivated by the findings in [4] that a high degree of co-location of secondary and primary BSs is important to achieve a positive business case, we aim to find the network scenarios with a high degree of co-location. In Appendix A we show that for the case when $ISD_s = \frac{r_s}{2} \cdot ISD_p$, where $ISD_s$ and $ISD_p$ are Inter Site Distance for the secondary and primary systems respectively and \( \frac{r_s}{2} \) is an irreducible fraction, one of every \( b^2 \) secondary BS is co-located with a primary BS. Table II presents the degree of co-location of secondary and primary BSs for a set of secondary cell sizes.

<table>
<thead>
<tr>
<th>$r_s$ (km)</th>
<th>1/4</th>
<th>1/3</th>
<th>1/2</th>
<th>2/3</th>
<th>3/4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-location (%)</td>
<td>6.25</td>
<td>11.11</td>
<td>25.00</td>
<td>11.11</td>
<td>6.25</td>
<td>100.00</td>
</tr>
</tbody>
</table>

It is found that the two cases $r_s = 1$ and $1/2$ gives highest degree of co-location with 100% and 25% respectively. Therefore, in order to determine performance of the secondary system for different cell sizes in the considered network scenario we study the two most promising cases:

- Secondary cell size = $1/2$ primary cell size ($r_s = 0.575$ km)
- Secondary cell size = primary cell size ($r_s = 1.15$ km)

Selected values for the simulation scenario are given in Table III. The SUs are offered a nomadic broadband service and are therefore static, whereas the PUs are mobile with a random waypoint mobility pattern moving at random speeds. Constant bit rate (CBR) traffic is transmitted downlink (DL) to both SUs and PUs. Considering the dynamic behavior of the primary network and the random nature of the radio channel, the simulation scenario will be very dynamic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Primary System</th>
<th>Secondary System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell radius (km)</td>
<td>1.15</td>
<td>0.575 and 1.15</td>
</tr>
<tr>
<td>Transmit power (dBW)</td>
<td>13.5</td>
<td>-40, -35, -30</td>
</tr>
<tr>
<td>Traffic per node (Mbps)</td>
<td>0.2 (DL CBR)</td>
<td>1.0 (DL CBR)</td>
</tr>
<tr>
<td>Packet size (Bytes)</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>Nodes per BS</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Nodes location</td>
<td>Random location</td>
<td>Random location</td>
</tr>
<tr>
<td>Nodes mobility</td>
<td>Random waypoint</td>
<td>No</td>
</tr>
</tbody>
</table>

Each simulation is run 15 times, with a duration of 500 s and warm up time of 20 s to ensure that a stable point of network operation is reached. The results are averaged.

A. Secondary System Performance

1) Throughput: The average throughput for the secondary system for both the studied cases is plotted in Fig. 2(a). It can be observed that it is difficult to obtain coverage for low power levels, especially for the $r_s = 1.15$ km case. Also, it can be observed that throughput reduces dramatically for higher power levels of -10 dBW since the probability of obtaining an available channel is lower.

For $r_s = 0.575$ km, maximum throughput, which is 3 Mbps with QPSK 1/2, is reached when the secondary system transmits with power in the range -27 dBW to -14 dBW. For $r_s = 1.15$ km, a throughput of 1.85 Mbps is reached at -14 dBW. When comparing the two cases, a main difference is that the outage probability is lower in the $r_s = 0.575$ km case because the probability that the SU is within coverage of the cell is higher due to the smaller cell size. A second difference is that the SUs are farther away from the PUs in the $r_s = 0.575$ km case, hence the probability of obtaining an available channel is higher.

2) Connectivity: SU outage is not visible in Fig. 2(a) since each SU downloads streams with bitrate 1 Mbps, hence full cell throughput can be achieved with only 3 SUs connected. To illustrate SU outage, average number of connected SUs is given in Fig. 2(b). It can be seen that the number of connected SUs is 53% for $r_s = 1.15$ km when maximum throughput is achieved, before throughput reduces rapidly for higher transmit power levels since there are no available channels.

3) Packet Loss: The number of dropped packets was in general low. Both cases showed higher values for low transmit power levels. This was mainly caused by weaker link budget and not by the secondary system characteristics.

4) Delay: Average delay at the network layer including queuing delay was measured as given in Fig. 2(c), where it can be seen that the delay is slightly higher for the $r_s = 1.15$ km case. The reason for this is first of all because of more channel errors and more retransmissions. It can also be seen that average delay for the two cases approaches each other as power increases. It should be noted that for -5 dBW, few packets are transmitted, and delay was only measured for packets before the primary system was detected.

5) Channel Switches: The average number of channel switches given in Fig. 2(d) increases gradually before a peak is observed for -10 dBW. For higher transmit power levels, the number of channel switches is reduced since there are no available channels. Also for low transmit power levels more channel switches are issued for $r_s = 1.15$ km than for $r_s = 0.575$ km, which is because the probability is higher that a PU is in interference range of a SU.

B. Primary System Performance

1) Throughput: Average throughput for the primary system is plotted in Fig. 3(a). It can be seen that throughput reduces slightly as secondary transmit power increases, and variance in throughput occurs earlier for the $r_s = 1.15$ km case since SUs are closer to the PUs.

2) Connectivity: No PUs experienced outage.

3) Packet Loss: Fig. 3(b) illustrates that packet loss for the primary system is low in general. There seems to be a relation that packet loss increases when amount of channel switches increases. This could be improved with more frequent sensing periods, but this is left for future work.
4) Delay: Delay at the network layer including queuing delay was measured for the PUs and was in average 0.51 ms, which is slightly lower than optimal delay for the SUs. This is mainly because of a better link budget with higher transmit power levels causing less channel errors and retransmissions. Also, there are no channel switches in the primary system.

V. CONCLUSIONS

An example SENDORA system has been studied with focus on secondary and primary system performance. An expression for the degree of co-location of secondary and primary BSs was derived and used to find the most promising cases for the performance study, which where (i) the secondary system having half the primary cell size and (ii) the two systems having equal cell size. First, it was found that equal cell size for the secondary and primary systems with a cellular reuse pattern with seven frequencies is difficult to achieve. This contradicts to some extent with results found in a prior business case analysis where co-location of secondary and primary BSs was shown to be very important in order to achieve a positive business case. Second, it was found that a good service could be offered with secondary cell size set...
to half the primary cell size and with restricted transmit power levels. The number of BSs installed will then be quadrupled and at least 75% of these would not be co-located with primary BSs leading to increased costs. This point in the direction of shorter range, smaller and less expensive BSs for the secondary system such as WiFi access points and femtocells. Third, it was found that the impact on primary system performance was low, but that optimal tuning of the sensor network is important.

**ACKNOWLEDGMENT**

The research leading to these results was performed in the SENDORA project (FP7/2007-2013, under grant agreement no 216076).

**APPENDIX A**

**DEGREE OF CO-LOCATION OF SECONDARY AND PRIMARY BASE STATIONS**

Consider a regular layout of hexagonal cells extending infinitely in all directions with a primary BS in the centre of each cell as shown in Fig. 4. The distance between neighbouring BSs is denoted as the ISD$_p$ (Inter Site Distance for the primary system).

The coordinates for the primary BSs is then given by:

$$x = m \cdot ISD_p \cdot \frac{\sqrt{3}}{2},$$
$$y = ISD_p \cdot (n + \frac{m}{2}),$$

(8)

where $n, m \in \mathbb{Z}$.

Consider a geometry of hexagonal cells with the same orientation and a BS in (0,0) used for the secondary system with an Inter Site Distance ISD$_s$ = $\frac{a}{2} \cdot ISD_p$, where $a, b \in \mathbb{Z}^+$, $b < a$ and $\gcd(a, b) = 1$. Then a secondary BS will be co-located with a primary BS if there are integers $n_1$, $m_1$, $n_2$, $m_2$ satisfying the equations:

$$m_1 \cdot ISD_p \cdot \frac{\sqrt{3}}{2} = m_2 \cdot \frac{a}{b} \cdot ISD_p \cdot \frac{\sqrt{3}}{2},$$

(9)

and

$$ISD_p \cdot (n_1 + \frac{m_1}{2}) = \frac{a}{b} \cdot ISD_p \cdot (n_2 + \frac{m_2}{2}).$$

(10)

The first equation can be simplified to $m_1 = m_2 \cdot \frac{a}{b}$, which can be substituted into the second equation simplifying this to $n_1 = n_2 \cdot \frac{b}{a}$. Since $m_1$ and $n_1$ are integers and $\gcd(a, b) = 1$, $b$ must be a factor in both $m_2$ and $n_2$. Hence, secondary BSs are co-located with primary BSs only when $m_2$ and $n_2$ are whole multiples of $b$. The reverse is also true, since there will always be integers $n_1$ and $n_1$ satisfying the equations above whenever $m_2$ and $n_2$ are whole multiples of $b$.

There is one-to-one mapping between the secondary system index pairs $(m_2, n_2)$ and the $(x, y)$ coordinates of the secondary BSs. Let $\{\Phi_{i,j} | i, j \in \mathbb{Z}\}$ be the partition of the set of index pairs $\{(m_2, n_2) | m_2, n_2 \in \mathbb{Z}\}$ given by:

$$\Phi_{i,j} = \{(m_2, n_2) | m_2 \in [i \cdot b, \ldots, (i + 1) \cdot b - 1],$$
$$n_2 \in [j \cdot b, \ldots, (j + 1) \cdot b - 1]\}.$$  

(11)

Then each $\Phi_{i,j}$ contains $b^2$ index pairs and only one of these corresponds to a co-located BS. Hence, one of every $b^2$ secondary BS is co-located with a primary BS.

**REFERENCES**


