\(H_\infty\) Filtering of Lipschitz Nonlinear Systems with Network-Induced Uncertain Delays

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Abstract—In this paper, a filter design scheme is proposed for Lipschitz nonlinear systems with delays. It is assumed that the measurements are transmitted through a communication network and therefore reach the filter with an uncertain delay. The design methodology is formulated as an optimization problem with LMI constraints, which are derived using a less conservative Lyapunov-Krasovskii function and establish an \(H_\infty\) attenuation level on the effects of the unwanted exogenous inputs on the estimation error. Simulation results are given to illustrate the effectiveness of the proposed filter.

I. INTRODUCTION

Networked control systems (NCSs) have received a great deal of attention over the last decade. What characterizes a network control system is that the information (such as plant input, output, etc.) between various subsystems (such as plant, controller, observer, etc.) is exchanged using a network or communication channel, possibly wirelessly. Primary advantages of NCS include reduced system wiring and installation, easy system diagnosis and maintenance, and increased flexibility. The insertion of the communication network in the feedback loop, on the other hand, makes the analysis and design of an NCS far more complex than a network-free control system. The main difficulties encountered are the following:

- Limited channel capacity and quantization effects: data rates used throughout networks are typically constrained, thus limiting the sampling rates used in control design. Quantization effects therefore become important because the number of quantization levels used in the transmission, affects the communication flow and the capacity required to transmit information.
- Network-induced delays: time-varying delays originated and dependent upon network transmission delays.
- Communication constraints: the presence of more than one sensor on the network asks for a network-access scheduling scheme. This leads to the unavailability of the sensor data to the filter at every sample.
- Packet-dropouts: information transmitted through the network is broken into a stream of packets. Depending on the characteristics of the network, packets can not only suffer delays but they can also be lost during transmission.

Networked control systems have extensively been studied during the last decade and many important results have been obtained. Pioneer work on NCS include, among others, [14], [23], [21], [7] and [28]. See also the recent survey [15] covering the subject up to 2007. In this paper our interest is in the state estimation or filtering problem over communication networks. A filter is, in fact, a state observer with the capacity to limit the effects of exogenous inputs on the estimation error. Perhaps the most celebrated result in estimation theory is the classical Kalman filter [17], which has been applied in numerous applications ranging from guidance, navigation and control, to biological systems [13], [27]. A standard assumption in Kalman filtering is that the process dynamics and measurements are affected by additive white noise with known covariance properties, which is not always easy to obtain. An alternative to the Kalman filtering problem is the \(H_\infty\) filtering which provides a guaranteed noise attenuation level in the presence of noise with unknown statistics. \(H_\infty\) filtering has received much attention. See, for example, [10], [6], [27] and the references therein.

In this paper our interest is in the \(H_\infty\) filtering problem in a network setting. Our goal is to explicitly discuss the effects of the uncertain delay induced by the network on the filter stability and performance and to propose a design methodology that can limit those effects with an arbitrary attenuation level. Most references dealing with the filtering problem have concentrated on linear systems. In [20] filtering of discrete-time linear systems over wireless fading channels is discussed where a mobile sensor observes a dynamical system and sends its observation to a remote estimation unit. [10] considers the problem of \(H_\infty\) estimation for continuous-time linear uncertain systems when network-induced problems such as quantization, delay and packet dropout are present. In [2] an optimal Kalman filter is designed for linear continuous-time systems with known state and observation delays. In [4] an optimal filter dual to Smith predictor is proposed for linear systems with fixed delays and then a sliding mode compensation scheme is used for disturbance rejection. In [8] authors use the descriptor model transformation and Park’s inequality to design a robust delay-dependent \(H_\infty\) filter for linear continuous-time systems with time varying delay and uncertainty in the parameters. In [18] an stochastic approach is taken to estimate the states of a Lipschitz nonlinear system with time-varying delays in the states. For more references on filtering of systems with delayed states and measurements, the reader is referred to [3], [5], [12], [22], and the references therein. For filter design under other problems induced by the network, see for

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In this paper, a method for designing filters for discrete-time nonlinear systems with network-induced delays is proposed. The nonlinear system is modeled as a linear system plus a nonlinear function, that satisfies a Lipschitz continuity condition, and is disturbed by unwanted exogenous inputs. Lipschitz systems are important because they provide a mechanism that can account for the effect of, at least, mild nonlinearities and because of their generality, most nonlinear system models can be represented as a linear system plus a Lipschitz nonlinearity, at least locally around an equilibrium point. It is assumed that the data sent by the sensors is subject to uncertain variable delays. Using a Lyapunov-Krasovskii function, an optimization problem with linear matrix inequalities is proposed to guarantee filter stability as well as an $H_{\infty}$ performance bound on the error system. To the best of authors' knowledge, this design methodology has never been reported for Lipschitz nonlinear systems in a NCS setup. This work is different from [18] in the sense that although in that work too, discrete-time Lipschitz nonlinear systems are discussed, the delay is assumed to be in the plant states, the approach is stochastic, and a batch of estimators are designed instead of one, whereas, the approach in this paper is not stochastic and the uncertain delay is in the transmitted measurements rather than the plant model.

The rest of the paper is organized as follows. In section II, we introduce the notation used in this paper along with the plant and filter models. Section III discusses the effects of the network-induced delays on the filter design and proposes the design methodology. In section IV the proposed filter is tested via simulation and section V concludes the paper.

II. PLANT AND FILTER MODELS

In this section we will first introduce the notation used throughout the paper and then we will move on to the plant and filter models.

The notation used in this paper is as follows: $\mathbb{R}$ represents the field of real numbers and $\mathbb{Z}$ the set of integers. $\mathbb{R}^{n \times m}$ represent the field of real matrices with $n$-rows and $m$-columns and $\mathbb{R}^n$ the set of real vectors of length $n$. $A^T$ and $A^{-1}$ represent, respectively, the transpose and inverse of the matrix $A \in \mathbb{R}^{n \times n}$. Given $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ is the euclidean norm of $x$. The energy of a causal discrete-time signal $v(k) : \mathbb{Z} \rightarrow \mathbb{R}^n$ is given by

$$||v||^2 = \sum_{n=0}^{\infty} |v(n)|^2.$$  

(1)

The space of causal sequences with finite energy will be denoted $\ell_2(0, \infty)$, or simply $\ell_2$; i.e. $v \in \ell_2$ provided that $||v||^2 < \infty$.

Consider the following discrete-time nonlinear plant model,

$$x(k+1) = Ax(k) + Bw(k) + \phi(x,u)$$
$$y(k) = Cx(k) + v(k)$$
$$z(k) = Hx(k)$$  \hspace{1cm} (2)

where $x \in \mathbb{R}^n$ is the state vector; $y \in \mathbb{R}^p$ represents the measured outputs; $u$ is the control input; $z \in \mathbb{R}^q$ is the signal to be estimated, and $w \in \mathbb{R}^m$ and $v \in \mathbb{R}^p$ denote state and measurement noises, respectively. Both $w$ and $v$ are assumed to be in $\ell_2$. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ are the state space matrices of the linear part of the model; and $\phi$ is a Lipschitz function with Lipschitz constant $l$, as defined in Definition 2.1.

Definition 2.1: The nonlinear function $\phi(x,u)$ with $\phi(0,u) = 0$ is said to be locally Lipschitz in the region $D$ with respect to $x$, uniformly in $u$, if there exists a constant $l > 0$ satisfying

$$||\phi(x_1,u^*) - \phi(x_2,u^*)|| \leq l||x_1 - x_2||$$  \hspace{1cm} (3)

where $u^*$ is any admissible control signal. The smallest $l > 0$ satisfying (3) is known as the Lipschitz constant, (see [19]).

It should be noted that the model description (2) is very general and can provide an accurate description of a large number of systems of interest, at least locally in a neighbourhood of an equilibrium point.

The discrete-time nonlinear filter is given by

$$x_F(k+1) = Ax_F(k) + L(y(k - d_k) - y_F(k)) + \phi(x_F,u)$$
$$y_F(k) = Cx_F(k)$$
$$z_F(k) = Hx_F(k)$$  \hspace{1cm} (4)

where $x_F \in \mathbb{R}^n$, $y_F \in \mathbb{R}^p$ and $z_F \in \mathbb{R}^r$ are, respectively, the state vector, output vector, and estimate vector of the filter. $L$ is the filter parameter to be designed, and $y(k - d_k) \in \mathbb{R}^p$ is the delayed feedback term.

III. FILTER DESIGN

In this section, we intend to design a stable filter of the form (4) for the plant (2), when the measurements are sent via a communication channel and therefore subject to uncertain transmission delays. We assume that the uncertain delay $d_k$ satisfies $0 \leq d_k \leq d_M$. Defining the estimation error as $e(k) = x(k) - x_F(k)$ leads to the following error system:

$$e(k) = (A - LC)e(k) + LCx(k) - LCx(k - d_k)$$
$$+ Bw(k) + Lv(k - d_k) + \Delta \phi(x,F,k)$$
$$\varepsilon(k) = z(k) - z_F(k) = He(k)$$  \hspace{1cm} (5)

where $\varepsilon$ is the estimation error, and $\Delta \phi(x,F,k) = \phi(x,F,k) - \phi(x,F,u)$. Our interest is in designing a filter with the following properties:

- (Stability) In the absence of external disturbances the estimation error $\varepsilon$ converges to zero asymptotically.
- (Filtering) The region of attraction is maximized for an arbitrary attenuation level $\mu$ on the effects of exogenous disturbances on the estimation error; i.e. we find the maximum Lipschitz constant $l$ such that

$$||\varepsilon|| < \mu||\omega||$$

where $\omega$ is the vector of the exogenous disturbances.
Our solution is based on the use of Linear Matrix Inequalities (LMIs) and is therefore free of the stringent existence requirements encountered in the Riccati approach. Our design procedure is effective in the sense that it renders a stable filter, if one exists and can be solved efficiently using commercially available softwares.

Before investigating the effects of network-induced delay on the filter design process, we need to introduce the following lemma.

**Lemma 3.1:** [29]

For any \( x, y \in \mathbb{R}^n \) and any positive definite matrix \( T \in \mathbb{R}^{n \times n} \), we have:

\[
2x^T y \leq x^T Tx + y^T T^{-1} y
\]

Since the state error equations given in (5) have the term \( x(k-d_k) \), the stability analysis needs to be done by augmenting the plant and the error models. Thus,

\[
X(k+1) = A X(k) + A_d X(k-d_k) + B \omega(k) + \Omega(X,u)
\]

\[
\varepsilon(k) = C X(k)
\]

where

\[
X(k) = \begin{bmatrix} x(k) \\ \varepsilon(k) \end{bmatrix}, \quad \omega = \begin{bmatrix} w(k) \\ v(k-d_k) \end{bmatrix}
\]

\[
\Omega(X,u) = \begin{bmatrix} \phi(x,u) \\ \Delta \phi(x,x_F,u) \end{bmatrix}
\]

and

\[
A = \begin{bmatrix} A & 0 \\ LC & A - LC \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ -LC & 0 \end{bmatrix}
\]

\[
B = \begin{bmatrix} B \\ B - L \end{bmatrix}, \quad C = \begin{bmatrix} 0 & H \end{bmatrix}
\]

The augmented system is also Lipschitz and its Lipschitz constant is calculated as follows,

\[
\Omega^T \Omega = \phi^T \phi + \Delta \phi^T \Delta \phi \leq l^2 (x^T x + e^T e)
\]

\[
\Rightarrow \| \Omega \| \leq l \| X \|
\]

The following theorem establishes a filter design methodology for Lipschitz nonlinear systems affected by variable delays.

**Theorem 3.1:** Given the plant (2) with measurements transmitted with an uncertain delay \( d_k \) satisfying \( 0 \leq d_k \leq d_M \). Then the filter given in (4) is optimal with an \( H_\infty \) bound \( \mu \) on the effects of the unwanted exogenous inputs on the estimation error, if there exist scalars \( \alpha, \eta_1, \eta_2 > 0 \) and matrices \( P = P' + P'' \), \( P' = \text{diag} \{ P_1', P_2' \} \), \( P'' = \text{diag} \{ P_1'', P_2'' \} \) > 0, and \( Q, R \geq 0 \), and \( G = (P_2' + P_2'') L \), and \( M, S, N \) for which the following optimization problem has a solution:

\[
\min \{ \bar{w} \alpha + \eta_1 + d_M \eta_2 \}
\]

s.t.

\[
\begin{bmatrix} \Gamma_3 + \Gamma_J + \Gamma_4 + \Gamma_4^T & \Gamma_1^T & \Gamma_2^T & \Gamma_7 \\ \star & -\Gamma_5 & 0 & 0 \\ \star & \star & -\Gamma_6 & 0 \\ \star & \star & \star & -\Gamma_8 \end{bmatrix} < 0
\]

where \( \bar{w} > 0 \) is an optimization weight for the Lipschitz constant, and

\[
\begin{align*}
\Gamma_1 &= \begin{bmatrix} P \hat{A} & P \hat{A}_d & 0 & P \hat{B} \end{bmatrix} \\
\Gamma_2 &= \sqrt{d_M} \begin{bmatrix} P(\hat{A} - I) & P \hat{A}_d & 0 & P \hat{B} \end{bmatrix} \\
\Gamma_3 &= \text{diag} \{ \alpha^{-1} I - P + C^T C + (d_M - d_m + 1) Q + R, -Q, -R, 0 \} \\
\Gamma_4 &= \begin{bmatrix} M + N & S - M & -S - N & 0 \end{bmatrix} \\
\Gamma_5 &= P - \eta_1^{-1} P^2, \quad \Gamma_6 = P - \eta_2^{-1} P^2 \\
\Gamma_7 &= \begin{bmatrix} \sqrt{d_M} M & \sqrt{d_M - d_m} S & \sqrt{d_M} \end{bmatrix} \\
\Gamma_8 &= \text{diag} \{ P', P'', P'' \}
\end{align*}
\]

Also the optimal filter gain \( L \) and Lipschitz constant \( l \) can be calculated as follows,

\[
L = (P'_p + P''_p)^{-1} G
\]

\[
l = 1/\sqrt{\alpha (\eta_1 + d_M \eta_2)}
\]

**Proof:** Consider the following discrete-time Lyapunov-Krasovskii function \([11]\):

\[
V(k) = X(k)^T P X(k) + \sum_{i=k-d_k}^{k-1} X(i)^T Q X(i) + \sum_{i=k-d_M}^{k-1} X(i)^T R X(i) + \sum_{j=-d_M+1}^{k-1} \tau(i)^T (P' + P'')^T \tau(i)
\]

where \( P', P'', P = P' + P'', Q, R \geq 0 \), and

\[
\tau(k) = X(k+1) - X(k) = (A - I) X(k) + A_d X(k-d_k) + B \omega(k) + \Omega(X,u)
\]

The forward difference of (13) can be written as

\[
\Delta V = \xi^T \Gamma'_1 T P \Gamma'_1 \xi + 2 \xi^T \Gamma_1 T P Q \xi + \xi^T \Gamma_2 \xi
\]

\[
+ \sum_{i=k-d_M+1}^{k-d_k} X(i)^T Q X(i) + \sum_{i=k-d_M+1}^{k-d_k} X(i)^T R X(i) + \sum_{i=k-d_M}^{k-1} \tau(i)^T (P' + P'')^T \tau(i)
\]

\[
- \sum_{i=k-d_k}^{k-1} \tau(i)^T P' \tau(i) - \sum_{i=k-d_M}^{k-1} \tau(i)^T P'' \tau(i)
\]

where

\[
\xi = \begin{bmatrix} X(k)^T X(k-d_k)^T X(k-d_M)^T \omega(k)^T \end{bmatrix}^T
\]

\[
\Gamma'_1 = \begin{bmatrix} A & A_d & 0 & \hat{B} \end{bmatrix} \\
\Gamma'_2 = \begin{bmatrix} A - I & A_d & 0 & \hat{B} \end{bmatrix} \\
\Gamma'_3 = \text{diag} \{ -P + (d_M + 1) Q + R, -Q, -R, 0 \}
\]
Now, using lemma 3.1, we can establish the following inequalities:
\[ 2\tilde{\Gamma}^T P \Omega \leq \Omega^T W_1 \Omega + (\tilde{\Gamma}_1^T P W_{-1}^1 P \tilde{\Gamma}_1^T) \] (16)
\[ 2\tilde{\Gamma}_2^T P \Omega \leq \Omega^T W_2 \Omega + (\tilde{\Gamma}_2^T P W_{-1}^2 P \tilde{\Gamma}_2^T) \] (17)
\[ 2\tilde{\xi}^T M \left( X(k) - X(k - d_k) - \sum_{i=k-d_k}^{k-1} \tau(i) \right) \leq \sum_{i=k-d_k}^{k-1} \tau(i)^T P \tau(i) + d_k \tilde{\xi}^T M P_{\mu-1}^1 M \tilde{\xi} + 2\tilde{\xi}^T M \left[ \begin{array}{ccc} I & -I & 0 \end{array} \right] \frac{\partial}{\partial x} \] (18)
\[ 2\tilde{\xi}^T S \left( X(k - d_k) - X(k - d_M) - \sum_{i=k-d_M}^{k-d_k-1} \tau(i) \right) \leq \sum_{i=k-d_M}^{k-d_k-1} \tau(i)^T P \tau(i) + (d_M - d_k) \tilde{\xi}^T S \tau^T S \tau + 2\tilde{\xi}^T S \left[ \begin{array}{ccc} 0 & I & -I \end{array} \right] \frac{\partial}{\partial x} \] (19)
\[ 2\tilde{\xi}^T N \left( X(k) - X(k - d_M) - \sum_{i=k-d_M}^{k-1} \tau(i) \right) \leq \sum_{i=k-d_M}^{k-1} \tau(i)^T P \tau(i) + d_M \tilde{\xi}^T N P_{\mu-1}^1 N \tilde{\xi} + 2\tilde{\xi}^T N \left[ \begin{array}{ccc} 0 & 0 & I \end{array} \right] \frac{\partial}{\partial x} \] (20)
where \( W_i = \eta_i I - P > 0 \) \( i = 1, 2, \) and
\[ M = \left[ \begin{array}{ccc} M_1^T & M_2^T & M_3^T \end{array} \right]^T \]
\[ S = \left[ \begin{array}{ccc} S_1^T & S_2^T & S_3^T \end{array} \right]^T \]
\[ N = \left[ \begin{array}{ccc} N_1^T & N_2^T & N_3^T \end{array} \right]^T \]
are matrices with appropriate dimensions. The first two inequalities help us get rid of the terms involving the nonlinear function \( \Omega \), while the last three provide a more conservative approach to eliminate the summation terms in (15). It should be noted that in the last three inequalities, the left-hand side of the inequality is equal to zero. Using the above inequalities and also the Lipschitz property of \( \Omega \), we get
\[ \Delta V \leq \xi^T \Gamma_3 \xi + \xi^T \Gamma_4 + (\Gamma_4 + \Gamma_4^T) \xi + \xi^T \Gamma_4^T (P^{-1} + W_i^{-1}) \Gamma_1 \xi + \xi^T \Gamma_4^T (P^{-1} + W_i^{-1}) \Gamma_2 \xi + d_k \xi^T M P_{\mu-1}^1 M \tilde{\xi} + d_M \xi^T N P_{\mu-1}^1 N \tilde{\xi} + (d_M - d_k) \xi^T S \tau^T S \tau \] (21)
where
\[ \Gamma_1 = PT_1^T, \quad \Gamma_2 = \sqrt{d_M} PT_2 \]
\[ \Gamma_3 = \Gamma_3^T + diag\{\eta_1 + d_M \eta_2\}, \quad 0, 0, 0 \}
\[ \Gamma_4 = \left[ \begin{array}{ccc} M + N & S - M & -S - N \end{array} \right] \]
Since \( P \) and \( W_i \) are copendent, we follow a procedure similar to the one introduced in [1] to simplify the term \( P^{-1} + W_i^{-1} \).
\[ P^{-1} + W_i^{-1} = P^{-1} + (\eta_i I - P)^{-1} \]
\[ = (\eta_i I - P)^{-1}((\eta_i I - P)P^{-1} + I) \]
\[ = (\eta_i I - P)^{-1} \eta_i P^{-1} \]
\[ = (P - \eta_i^{-1} P^2)^{-1} \quad i = 1, 2 \] (22)
Now defining
\[ \Gamma_7 = diag\{\sqrt{d_M} M, \sqrt{d_M} - d_m S, \sqrt{d_M} N \} \]
and using (22), one can simplify \( \Delta V \) as given by
\[ \Delta V \leq \xi^T \Gamma_3 + \Gamma_4 + \Gamma_4^T \Gamma_5 \Gamma_5^{-1} \Gamma_1 + \Gamma_4^T \Gamma_6 \Gamma_6^{-1} \Gamma_2 + \Gamma_7 \Gamma_8 \Gamma_8^{-1} \Gamma_7 \xi \] (23)
where \( \Gamma_5 = P - \eta_1^{-1} P^2 \) and \( \Gamma_6 = P - \eta_2^{-1} P^2 \). It should be noted that \( d_k \) is an uncertain variable with upper and lower bounds, and thus it cannot be used in the design formulation. As a result, it needs to be replaced with its bounds such that the inequality (21) still holds, as done in formulating (23). Since our problem is \( H_\infty \) filtering (not just observation), we need to limit the sensitivity of the estimation error to unwanted exogenous inputs. To this end, we define
\[ J \triangleq \sum_{k=0}^{\infty} \left\{ \varepsilon(k)^T \varepsilon(k) - \mu^2 \omega(k)^T \omega(k) \right\} \] (24)
Adding (13) to the right hand side of (24), we get \( J \leq \sum_{k=0}^{\infty} J_k \), where
\[ J_k = J = \xi^T (k) \xi(k) - \mu^2 \omega(k)^T \omega(k) + \Delta V_k \] (25)
Now, if we design our filter such that \( J_k \leq 0 \), we can conclude that \( J \leq 0 \), which is equivalent to \( \|\varepsilon\|^2 < \mu^2 \|\omega\|^2 \). This implies that the second norm of the estimation error is bounded by a factor of the second norm of the exogenous input. In other words, this establishes an \( H_\infty \) bound on the estimation error system.

Now, substituting (23) in (25), we have
\[ J_k \leq \xi^T (\Gamma_3 + J_4 + \Gamma_4^T + \Gamma_4^T \Gamma_5 \Gamma_5^{-1} \Gamma_1 + \Gamma_4^T \Gamma_6 \Gamma_6^{-1} \Gamma_2 + \Gamma_7 \Gamma_8 \Gamma_8^{-1} \Gamma_7) \xi \] (26)
where \( \Gamma_4 = diag\{\varepsilon^T C, 0, 0, -\mu^2 I\} \). To avoid running into bilinear matrix inequalities, the following variable changes need to be performed:
\[ P^T = diag\{P_1', P_2'\}, \quad P^{\mu} = diag\{P_1', P_2'\} \quad G = (P_2' + P_2')L, \quad \alpha^{-1} = (\eta_1 + d_M \eta_2)^2 \] (27)
Now for stability we should have \( J_k < 0 \), which by using Schur’s complement is the same as the LMI given in (11).

IV. SIMULATION RESULTS
In this section, we will use the proposed approach to design an optimal filter for an example system. We assume that the discrete model of the plant is obtained using a \( t = 0.01 \text{s} \) sampling period. Assume the system given in (2) with the following parameters.
\[ A = \left[ \begin{array}{ccc} 0.9388 & 0.0655 \end{array} \right], \quad B = \left[ \begin{array}{ccc} 0.01 \end{array} \right], \quad C = \left[ \begin{array}{ccc} 1 \end{array} \right], \quad H = \left[ \begin{array}{ccc} 0 \end{array} \right] \]
\[ \phi(x, u) = \left[ \begin{array}{ccc} -0.1(1 - \cos(x_2)) \end{array} \right] \]
Choosing $\mu = 0.5$ the maximum Lipschitz constant and the corresponding optimal observer gain are calculated to be as follows:

$$L = \begin{bmatrix} 0.0049 & -0.0064 \end{bmatrix}^T \quad l = 0.0343$$

The value of $l$ in each case determines a neighborhood of $x_2$, in which the filter stability is guaranteed. In other words, the filter with uncertain delay is locally stable as long as $|x_2| \leq 0.65$. Figure 1 illustrates the estimated variable $z_F$.

V. CONCLUSION

In this paper, the filtering problem of discrete-time Lipschitz nonlinear systems with network-induced uncertain delay was addressed. It was assumed that the induced transmission delay was bounded by a known upper limit. The filter design problem was formulated into an optimization problem, which maximized the Lipschitz constant and therefore the region of attraction of the proposed filter. Using a less conservative Lyapunov-Krasovskii functional, and also assuming an arbitrary attenuation level on the effects of the unwanted inputs on the estimation error, a design LMI was derived. Employing the proposed method, a filter was designed and simulated for an example system. It was shown through simulation that the proposed filter tracks the system states despite the delayed measurements and unwanted inputs.

REFERENCES