Total Restrained Domination in Trees

J. H. Hattingh, GSU, E. Jonck, UJ, E. J. Joubert, UJ and A. R. Plummer∗, GSU

Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is a total restrained dominating set if every vertex is adjacent to a vertex in $S$ and every vertex of $V - S$ is adjacent to a vertex in $V - S$. The total restrained domination number of $G$, denoted by $\gamma_{tr}(G)$, is the smallest cardinality of a total restrained dominating set of $G$. We show that if $T$ is a tree of order $n$, then $\gamma_{tr}(T) \geq \lceil (n + 2)/2 \rceil$. Moreover, we show that if $T$ is a tree of order $n \equiv 0 \mod 4$, then $\gamma_{tr}(T) \geq \lceil n/2 \rceil + 1$. We then constructively characterize the extremal trees $T$ of order $n$ achieving these lower bounds.

Keywords: restrained, total, domination, trees