A Dependent-Chance Programming Model for Fuzzy Time-Cost Trade-off Problem

Hua Ke and Weimin Ma, Member, IEEE

Abstract—In real projects, both the trade-off between the project cost and the project completion time, and the uncertainty of the environment are considerable aspects for decision-makers. However, the research on the time-cost trade-off problem seldom concerns fuzzy environments. In this paper, a new fuzzy time-cost trade-off model with the philosophy of dependent-chance programming is proposed, in which credibility theory is applied to describe the uncertainty of activity durations. A searching method as a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm is produced to search the optimal schedule under the given decision-making rule. The purpose of the paper is to reveal how to obtain the optimal balance of the project completion time and the project cost in a fuzzy environment.

I. INTRODUCTION

The time-cost trade-off problem considers the trade-off between the project cost and the project completion time, which is a particular type of project scheduling problem. For project decision-makers, the analysis of the time-cost trade-off is one of the most important aspects of project scheduling and control. In 1961, Kelly [16] first studied the special type of the project scheduling problem. In the following 40 years, the research on the time-cost trade-off problem mainly focused on the problem with deterministic environments [28] [30]. For solving the deterministic time-cost trade-off problem, the common analytical methods are linear programming and dynamic programming [2] [31]. Besides, some heuristic algorithms, such as genetic algorithm [1] [6] [7], are also introduced.

Although most research work on the time-cost trade-off problem assumes that the problem is always in some deterministic environment, the real world is full of non-deterministic factors. The project completion time may vary due to many external factors, such as the change of weather, the increase of productivity level, etc. Besides, Goldratt [9] questioned the validity of deterministic environments in the project scheduling problem. The readers may refer to [5] [8] [10] [15] to see different types of project scheduling problem with stochastic activity durations. In 1985, Wollmer [32] discussed a stochastic version of deterministic linear time-cost trade-off problem, in which some discrete random variables were introduced to depict the uncertainty in the problem. Gutjahr et al. [11] designed a modified stochastic branch-and-bound approach and applied it to a specific stochastic discrete time-cost trade-off problem. The interested readers may consult the survey by Herroelen and Leus [13].

Probability theory can be regarded as a tool for describing objective uncertainty, while credibility theory, a new theory dealing with fuzziness, is a powerful instrument for treating with subjective uncertainty. In 1965, Zadeh [33] originally introduced the concept of fuzziness to describe fuzzy phenomena via membership function. Then, in 1978, Zadeh [34] proposed the concept of possibility measure for measuring a fuzzy event. In 2002, Liu and Liu [20] presented a new self-dual credibility measure as possibility measure has no self-duality property, which is a very important property for real-life applications. The readers interested in credibility theory may refer to [22] [24]. With the development of the research on fuzziness, the fuzzy set theory was also applied into the project scheduling problem, originally by Prade [29] in 1979. Furthermore, many other authors discussed the fuzzy project scheduling problem, such as [3] [12] [14]. However, to the knowledge of the authors, the only work on the fuzzy time-cost trade-off problem was done by Leu et al. [17]. Though the literature on the uncertain time-cost trade-off problem is little, the trade-off between the project completion time and the project cost is an important issue which decision-makers should necessarily consider in real projects. In [17], the activity durations were characterized by fuzzy numbers, and the fuzzy relationship between the activity time and the activity cost was demonstrated by membership function. Furthermore, the philosophy of chance-constrained programming, which was initiated by Charnes and Cooper [4], was introduced to be the decision-making rule. However, as we mentioned above, possibility measure does not have self-duality property, which is necessary for many applications, for instance, for well defining the concept of expected value of stochastic event or fuzzy event.

In this paper, we introduce some concepts of credibility theory into the fuzzy time-cost trade-off problem. The activity durations of a project are described by fuzzy variables. With the philosophy of dependent-chance programming, a new fuzzy time-cost trade-off model is established. A hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm (GA) is designed to deal with the above fuzzy time-cost trade-off model. And finally the algorithm is revealed to be an effective tool to solve this problem by some numerical experiment.

The remainder of the paper is organized as follows: In Section 2, the fuzzy time-cost trade-off problem is described, in which some parameters are given to deduce the project completion time and the project cost. In Section 3, some
basic concepts of credibility theory are introduced and based on the concepts a fuzzy model is built. Section 4 briefly introduces a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm. A numerical example is illustrated to reveal the effectiveness of the hybrid intelligent algorithm in Section 5. Finally, Section 6 draws some conclusions on the fuzzy time-cost trade-off problem.

II. PROBLEM DESCRIPTION

The research [27] showed that the activity duration time and the corresponding activity cost are related with each other. Actually, in most real projects, decision-makers always need to take into account the trade-off between the total project cost and the project completion time. Naturally it is desirable for decision-makers to find the most effective way to complete a project in some predetermined completion time limit and meanwhile with the minimal cost in some sense, which is just what the time-cost trade-off problem wants to solve.

Generally, a project can be described by an activity-on-the-arc network $G = (V, A)$ like Figure 1, where $V = \{1, 2, \cdots, n\}$ is the set of nodes and $A$ is the set of arcs representing the activities.

![Fig. 1. Project Construction Network](image)

First we introduce the parameter $\xi_{ij}$ as a fuzzy variable representing the normal duration time of activity $(i, j)$, whose uncertainty attributes to the variation of the external environment, and $c_{ij}$ as the normal cost per day of activity $(i, j)$, which is a constant. That is, $\xi_{ij}$ represents the duration time of activity $(i, j)$ without the influence of the decision made by the decision-maker. The decision variable $x_{ij}$, which is assumed to be an integer, represents the change of the duration time of activity $(i, j)$, which can be controlled by decisions of the decision-maker, such as determining the number of workers, determining the quality of instruments, etc. Apparently, the variable $x_{ij}$ is bounded by some interval $[l_{ij}, u_{ij}]$ owing to some practical conditions, where $l_{ij}$ and $u_{ij}$ are assumed to be integers. Accordingly, for each activity $(i, j)$, there exists another associated cost $d_{ij}$, which is the additional cost of per unit change of $x_{ij}$ and is also assumed to be a constant. Then the goal is to find the optimal vector $x = \{x_{ij} : (i, j) \in A\}$ to meet some scheduling requirements.

The fuzzy normal activity duration times are concisely written as $\xi = \{\xi_{ij} : (i, j) \in A\}$. We denote $T_{ij}(x, \xi)$ as the starting time of activity $(i, j)$ and the starting time of activity $(1, j) \in A$ is defined as $T_{1j}(x, \xi) = 0$. For simplicity, it is assumed that each activity can be processed only if all the foregoing activities are finished, and it should be processed without interruption. Then, the starting time of activity $(i, j)$, $i = 2, 3, \cdots, n - 1$, can be decided by

$$T_{ij}(x, \xi) = \max_{(k,j) \in A} \{T_{ki}(x, \xi) + \xi_{ki} + x_{ki}\},$$

and the completion time of the project can be calculated by

$$T(x, \xi) = \max_{(k,n) \in A} \{T_{kn}(x, \xi) + \xi_{kn} + x_{kn}\}. \quad (1)$$

The total cost of the project is composed of the normal costs and the additional costs, which can be simply written as

$$C(x, \xi) = \sum_{(i,j) \in A} (c_{ij}\xi_{ij} - d_{ij}x_{ij}). \quad (2)$$

III. FUZZY DEPENDENT-CHANCE PROGRAMMING MODEL

A. Credibility Theory

Credibility theory, founded by Liu [22] in 2004, is a branch of mathematics for studying the behavior of fuzzy phenomena. Let $\Theta$ be a nonempty set, and $\mathcal{P}$ the power set of $\Theta$. In order to define the concept of credibility measure, we introduce the following four axioms:

Axiom 1. (Normality) $\operatorname{Cr}\{\Theta\} = 1$.

Axiom 2. (Monotonicity) $\operatorname{Cr}\{A\} \leq \operatorname{Cr}\{B\}$ whenever $A \subseteq B$.

Axiom 3. (Self-Duality) $\operatorname{Cr}\{A\} + \operatorname{Cr}\{A^c\} = 1$ for any event $A$.

Axiom 4. (Maximality) $\operatorname{Cr}\{\bigcup_{i=1}^{n} A_i\} = \sup_{i} \operatorname{Cr}\{A_i\}$ for any events $\{A_i\}$ with $\sup_{i} \operatorname{Cr}\{A_i\} < 0.5$.

Definition 1: (Liu and Liu [20]) The set function $\operatorname{Cr}$ is called a credibility measure if it satisfies the normality, monotonicity, self-duality, and maximality axioms.

Next, we will introduce the concept of a credibility space, which will be used to define a fuzzy variable.

Definition 2: (Liu [22]) Let $\Theta$ be a nonempty set, $\mathcal{P}$ the power set of $\Theta$, and $\operatorname{Cr}$ a credibility measure. Then the triplet $(\Theta, \mathcal{P}, \operatorname{Cr})$ is called a credibility space.

Based on the above definition, the concept of a fuzzy variable can be given as follows:

Definition 3: (Liu [22]) A fuzzy variable is a function from a credibility space $(\Theta, \mathcal{P}, \operatorname{Cr})$ to the set of real numbers.

Actually, there exists some relationship between the credibility measure and the membership function of a fuzzy variable, which is called the credibility inversion theorem.

Theorem 1: (Liu [23]) Let $\xi$ be a fuzzy variable with membership function $\mu$. Then for any set $B$ of real numbers, we have

$$\operatorname{Cr}\{\xi \in B\} = \frac{1}{2} \left( \sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x) \right).$$

2184
B. Credibility Maximization Model

In this subsection, we will apply some new type of decision-making rule into the fuzzy time-cost trade-off problem as the philosophy of dependent-chance programming introduced by Liu [18]. In practice, some project scheduling goals can not be obtained exactly due to environmental uncertainty. Hence, to decision-makers a realistic approach may be to maximize the credibility of achieving the optimization goals. Let $\xi$ and $\eta$ be two fuzzy variables defined on some credibility space $(\Theta, \mathcal{P}, C_r)$. To compare two fuzzy variables, the basic criterion is that $\xi > \eta$ if and only if $\xi(\theta) > \eta(\theta)$ for all $\theta \in \Theta$. However, in practice, this way of comparison is obviously not appreciated. Here, a practical way is to compare the chance of two fuzzy events with some predetermined level. In detail, we say $\xi > \eta$ if and only if $\text{Cr}\{\xi > \tau\} > \text{Cr}\{\eta > \tau\}$ for some predetermined level $\tau$. With this ranking method, we can compare project costs or project completion times as they are functions of fuzzy variables. The details about fuzzy dependent-chance programming may be found in Liu [19]. To maximize the credibility of controlling the project cost within the budget with some chance constraint of the project completion time, we can build the following credibility maximization model:

$$\begin{align*}
\max C_r \{ C(x, \xi) \leq C^0 \} \\
\text{subject to:} \\
\text{Cr}\{T(x, \xi) \leq T^0\} \geq \alpha \\
x_{ij} \in [l_{ij}, u_{ij}], \forall (i, j) \in A \\
x_{ij}, \forall (i, j) \in A, \text{ integers}
\end{align*}$$

where $\alpha$ is a predetermined confidence level, $T^0$ is the due date of the project, $C^0$ is the budget, $l_{ij}$ and $u_{ij}$ are integers given in advance, and $T(x, \xi)$ and $C(x, \xi)$ are defined by (1) and (2), respectively.

IV. HYBRID INTELLIGENT ALGORITHM

In this section, we briefly introduce the design of the hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm for the credibility maximization model.

In the model, there exists one type of fuzzy function $\text{Cr}\{T(x, \xi) \leq T^0\}$. With the relationship between credibility measure and membership function shown in the credibility inversion theorem, the fuzzy function can be estimated by the form of membership function, which can be easily simulated. The detailed procedure of fuzzy simulation can be referred to the process given by [21] [25]. The theory and the application of fuzzy simulation, including some convergent results about the use of fuzzy simulation, can be found in [21] [26].

Successively, we embed the fuzzy simulation into genetic algorithm to search the optimal solution for the credibility maximization model.

The procedure can be summarized briefly as follows. The first step is to initialize $\text{pop.size}$ chromosomes, where the fuzzy function can be calculated and the feasibility can be checked by the proposed fuzzy simulation. The second step is the update process of the chromosomes by crossover and mutation operations. In the next step, we compute the objective values for all chromosomes and accordingly calculate the fitness of each chromosome. The final step is to select the chromosomes by spinning the roulette wheel. After running the above steps for a given number of cycles we report the best chromosome as the optimal solution.

V. NUMERICAL EXPERIMENT

<table>
<thead>
<tr>
<th>Activity</th>
<th>Normal Duration $\xi_{ij}$</th>
<th>Normal Cost $c_{ij}$</th>
<th>Additional Cost $d_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2)</td>
<td>(7, 9, 12)</td>
<td>170</td>
<td>200</td>
</tr>
<tr>
<td>(1,3)</td>
<td>(4, 6, 8)</td>
<td>300</td>
<td>280</td>
</tr>
<tr>
<td>(1,4)</td>
<td>(7, 10, 12)</td>
<td>45</td>
<td>70</td>
</tr>
<tr>
<td>(2,5)</td>
<td>(4, 6, 9)</td>
<td>270</td>
<td>300</td>
</tr>
<tr>
<td>(3,5)</td>
<td>(8, 10, 13)</td>
<td>35</td>
<td>50</td>
</tr>
<tr>
<td>(3,6)</td>
<td>(7, 8, 10)</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>(3,7)</td>
<td>(6, 8, 11)</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>(4,7)</td>
<td>(5, 6, 8)</td>
<td>600</td>
<td>400</td>
</tr>
<tr>
<td>(5,8)</td>
<td>(6, 8, 11)</td>
<td>55</td>
<td>100</td>
</tr>
<tr>
<td>(6,8)</td>
<td>(7, 10, 12)</td>
<td>200</td>
<td>180</td>
</tr>
<tr>
<td>(6,9)</td>
<td>(5, 7, 9)</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>(6,10)</td>
<td>(9, 11, 14)</td>
<td>320</td>
<td>380</td>
</tr>
<tr>
<td>(7,10)</td>
<td>(7, 10, 13)</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td>(8,11)</td>
<td>(6, 8, 10)</td>
<td>70</td>
<td>50</td>
</tr>
<tr>
<td>(9,11)</td>
<td>(9, 11, 13)</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>(10,11)</td>
<td>(5, 7, 9)</td>
<td>90</td>
<td>120</td>
</tr>
</tbody>
</table>

Now let us consider a project shown in Figure 1. The duration times, which are assumed as triangular fuzzy variables, the normal costs and the additional costs of the activities in the project are presented in Table I, respectively. And the decision variables $x_{ij}$ are assumed to be integers and be limited in the interval $[-3, 3]$ for all $(i, j) \in A$. The budget of the project is assumed to be 17800 and the project completion time limit is 36. With the philosophy of dependent-chance programming, the credibility maximization model can be established as follows:

$$\begin{align*}
\max C_r \{ C(x, \xi) \leq 17800 \} \\
\text{subject to:} \\
\text{Cr}\{T(x, \xi) \leq 36\} \geq 0.9 \\
x_{ij} \in [-3, 3], \forall (i, j) \in A \\
x_{ij}, \forall (i, j) \in A, \text{ integers}
\end{align*}$$

The results of the credibility maximization model are shown in Table II. The designed hybrid intelligent algorithm is proved to be effective as all the errors do not exceed 1.18%.

VI. CONCLUSIONS

The trade-off between the project cost and the project completion time is an important issue for decision-makers in real projects. In this paper, we established the credibility maximization model, in which the uncertainty of the activity durations was described by credibility theory, for solving the fuzzy time-cost trade-off problem, which is seldom studied in uncertain environments. To solve the model, a hybrid
intelligent algorithm integrating the fuzzy simulation and genetic algorithm was built. And through the numerical example, the effectiveness of the hybrid intelligent algorithm was illustrated. The main work of the paper is that the paper adopted credibility theory to establish a framework for the time-cost trade-off problem with fuzzy factors, which can be studied more deeply in the future research of the time-cost trade-off problem.

ACKNOWLEDGMENT

The work was partly supported by the National Natural Science Foundation of China (70671004), Program for New Century Excellent Talents in University (NCET-06-0172), A Foundation for the Author of National ExcellentDoctoral Dissertation of PR China (200782), the Shuguang Plan of Shanghai Education Development Foundation and Shanghai Education Committee (08SG21), and China Postdoctoral Science Foundation (20080440091).

REFERENCES


TABLE II

<table>
<thead>
<tr>
<th>pop_size</th>
<th>$P_{0}$</th>
<th>$P_{c}$</th>
<th>Credibility</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.2</td>
<td>0.4</td>
<td>0.9201</td>
<td>1.05</td>
</tr>
<tr>
<td>40</td>
<td>0.3</td>
<td>0.2</td>
<td>0.9195</td>
<td>1.18</td>
</tr>
<tr>
<td>50</td>
<td>0.4</td>
<td>0.2</td>
<td>0.9283</td>
<td>0.17</td>
</tr>
<tr>
<td>50</td>
<td>0.2</td>
<td>0.3</td>
<td>0.9204</td>
<td>1.02</td>
</tr>
<tr>
<td>60</td>
<td>0.3</td>
<td>0.4</td>
<td>0.9270</td>
<td>0.31</td>
</tr>
<tr>
<td>60</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9299</td>
<td>0.00</td>
</tr>
</tbody>
</table>