On the Complete Pivoting Conjecture for Hadamard Matrices of Small Orders

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In this paper we study explicitly the pivot structure of Hadamard matrices of small orders 16, 20 and 32. An algorithm computing the \((n-j) \times (n-j)\) minors of Hadamard matrices is presented and its implementation for \(n = 12\) is described. Analytical tables summarising the pivot patterns attained are given.

Keywords: Gaussian elimination, pivot size, complete pivoting, minors, Hadamard matrices.

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1. INTRODUCTION
Let \(A\) be an \(n \times n\) real matrix, and let \(b\) be a real \(n\)-vector. In his fundamental work on backward error analysis Wilkinson(1963) proved that when the linear system \(A \cdot \hat{x} = b\) is solved in floating point arithmetic by Gaussian elimination (GE) with either partial or complete pivoting the computed solution \(\hat{x}\) satisfies

\[(A+E) \cdot \hat{x} = b\]

where the norm of the perturbation matrix \(E\) can be bounded from above as follows:

\[||E||_\infty \leq g(n,A) \cdot f(n) \cdot u \cdot ||A||_\infty\]

where \(u\) is the unit roundoff, \(f(n)\) is a cubic polynomial of \(n\), and \(g(n,A)\) is the growth factor defined by

\[g(n, A) = \max_{i,j,k} \frac{|a_{i,j}^{(k)}|}{|a_{11}^{(0)}|}\]

where \(a_{i,j}^{(k)}\), \(k = 1, 2, ..., n-1\) denotes the \((i,j)\) th element that occurs at the \(k\)-th step of elimination. The elements \(a_{i,j}^{(n-1)}\) are called pivots.