Abstract—Dynamic spectrum management makes it possible for the owner of usage rights on some frequency blocks to sublet each of them in real time and for a limited period of time. As a softer implementation with respect to the spot market a two stage assignment is here proposed through the use of options, which give buyers the right to purchase the usage right on a single block and for a timeslot. In the sale of options the primary owner may accomplish an overbooking strategy, which consists in selling more blocks than the available ones and acts as hedging against the risk of unsold blocks. A model for the overbooking strategy is described and evaluated, which takes into account both the value of the option, the correlated decisions taken by the prospective purchasers, and the penalty to be paid to the unsatisfied customers. The dependence of the economical convenience of the overbooking strategy on the relevant parameters (among which the penalty value and the overbooking factor) is shown for a significant range of cases.

I. INTRODUCTION

Usage rights on wireless spectrum are primarily assigned through long term licenses and with several limitations, e.g. on coverage and quality performances. Transfer of usage rights is typically prohibited and takes place just when severe difficulties emerge for the completion of the network infrastructure needed to provide the service, as recent cases in the launch of the UMTS service testify [1][2][3]. However, the possibility to freely transfer usage rights, so to create a secondary market for spectrum (a.k.a. spectrum trading) has been strongly advocated and starts being a reality in many countries (see, e.g., [4] and [5]). A market is therefore envisaged where any owner of primary usage rights on spectrum can sell them to spectrum consumers (we employ this term to indicate operators who in turn use that spectrum to provide service to end consumers). The transfer may be semi-permanent or temporary. An assessment of the value of semi-permanent transfers has been conducted in [6]. In this paper we focus on temporary transfers. Dynamic spectrum management is a well established research area, for which a tentative roadmap, considering technical as well economical and regulatory issues, has been sketched in [7]. Several technical solutions have been proposed to assign spectrum on a short term basis. In [8] the Fast Command and Control (FCCM) model has been proposed, where an underlying ad hoc network is employed to coordinate spectrum use, allowing short term licenses with enforcement. Here we do not deal with the technical solutions that make spectrum secondary assignment possible, but rather consider the secondary assignment mechanism, i.e. how the primary owner of usage rights resells them. In the straightforward implementation of a trading mechanism the owner of usage rights may reassign them by selling them on the spot. This is of course the hardest implementation of a real time market, since the seller doesn’t know the offers in advance and the ownership rights are to be switched in a very short time. On the other hand we advocate here that the owner of usage rights may opt for a less demanding intermediate solution, by adopting a selling arrangement based on two stages through the use of options. In this arrangement, a potential spectrum consumer first buys an option, sold by the primary owner of usage rights, to use a specified block of spectrum at a specified time in the future. The option, just like financial options, gives the spectrum consumer the right, rather than the obligation, to buy those usage rights. When the time comes, the spectrum consumer may decide (most likely on the basis of its service demand) whether to actually exercise the option and buy the usage rights or to let the option expire. This two stage mechanism allows the selling side (the owner of usage rights) to carry on a minimal planning activity while cashing in on the option sale. It is envisaged that just a fraction of the options will be exercised. This spread between bookings and purchases may leave the primary owner with unused spectrum blocks though all the available blocks had been booked. Since this is highly undesirable for the primary owner, it may lead it to adopt an overbooking strategy, i.e., to sell more options than the number of available spectrum blocks. The main con of such strategy is that the number of option owners willing to exercise their option may result to be really larger than the number of available blocks. The option owners in excess won’t be able to receive their spectrum blocks and should receive an appropriate compensation. Notwithstanding the resulting outpayments due to compensate the unsatisfied consumers, such strategy may result to be economically viable for the primary owner. Selling options in the overbooking model is equivalent to what is known in finance as a naked or uncovered option [9]. The aim of this paper is to evaluate the economic feasibility of the overbooking strategy. We introduce for this purpose two metrics and adopt a model for consumers’ decisions that incorporates correlation.

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the viewpoint of the primary owner. The overbooking strategy and the associated economic results are detailed in Section IV, after a brief review of usage rights in Section II. Since the overbooking strategy is based on the use of options, we describe this tool in Section III and review its price setting in Section V. In Section VII we evaluate the economic feasibility of the overbooking strategy. We envisage a decision method based on the choice of proper values for the overbooking factor (the ratio between the number of options that are sold and the number of available spectrum blocks) and on the use of tolerable thresholds for the penalty associated to unsatisfied spectrum consumers. We show the following results: 1) the average gain from overbooking increases for small values of the overbooking factor and decreases for larger values; 2) the maximum usable overbooking factor decays as the correlation among consumers’ decisions grows; 3) the average gain from overbooking decays linearly as the penalty grows; 4) the impact of penalty is larger the larger the correlation.

II. USAGE RIGHTS ON WIRELESS SPECTRUM

Usage rights for spectrum are assigned through telecommunications licenses, which authorise an entity to provide telecommunications services or operate telecommunications facilities. In this section we briefly review the main ways in which licenses are awarded and highlight some shortcomings of current assignment procedures, which advocate for the introduction of transferability rights.

Licenses also generally define the terms and conditions of such authorisation, and describe the major rights and obligations of a telecommunications operator.

Several mechanisms have been put into place to assign telecommunications licenses. They can be summed up in the following three classes: 1) General authorization (requires compliance with basic terms and obligations); 2) Beauty contest (where assignments are based on the evaluation of technical quality of applications); 3) Auctions.

The first mechanism is non-competitive and applies to cases where the resource to be awarded is not scarce (e.g., fixed wireline infrastructures), so that it is typically not of interest in the context of wireless spectrum. The latter two ones are instead competitive mechanisms, where the number of licenses to be awarded is limited and typically much less than the number of competitors (though some degenerate cases may happen where the number of licenses to be awarded is equal to or even larger than the number of competitors). Basically, all wireless services (such as UMTS, WiFi and WiMax) should be awarded with competitive mechanisms, since they assign varying degrees of rights on a scarce resource such as the radio spectrum. In particular licenses for 3G mobile services in Europe were assigned either by beauty contest or by auction [1].

In addition to their institutional role in the assignment of operating authorization, licenses have also a business role, since they provide certainty for investors and lenders, and with it the confidence that is required to invest the millions or billions of dollars needed to install or upgrade a telecommunications infrastructure. On the other hand, licenses should also provide certainty for consumers and the regulator, namely that the licensed operator would be able to fulfil its obligations and bring to completion the infrastructure building. However, licences don’t provide just rights, but come with a set of obligations, e.g. the non transferability of licence. The rationale behind the imposition of such constraints is related to the willingness to compel service providers to make full use of the license they have been awarded. Without these constraints, it is thought, customers could be left without service when none of the service providers actually goes through the investment plan associated to the license. Actually, these constraints may turn out to be counterproductive. If the market conditions are good, the service providers will need no spur to carry on their projects. But, if the operator is in dire straits, the failure to comply with obligations will lead to licence revocation, and consequently to a delay in infrastructure realization and service offer, which in the end represents a negative consequence for customers. Actually, in 3G spectrum assignments in Western countries 27% of the licenses were still idle at the end of 2005 [10]. Even if licenses are not idle, their spectrum occupancy can be very low. If we turn to the measure of spectrum occupancy, it is claimed that, in some densely populated areas, spectrum occupancy is at most 25% [11].

Such figures constitute a clear evidence of the limitations deriving from forbidding license transferrals. Including in the auction conditions the possibility to transfer the license (transferring at the same time the associated rights and obligations to the buyer) would expedite crisis resolution and the entry into service. Relaxing the constraint on license transferability would add value to the license itself. The possibility, deployed on a wide scale, for license assignees to resell their license creates a secondary market for licenses. In the literature such possibility is usually named Spectrum Trading. Its opportunity is justified in [12] with mainly qualitative arguments. However, in the same paper a simple quantitative argument in favour of spectrum trading is reported, where the social welfare increases when the license holder sells the licence at a price higher than the original licence cost but lower than the present licence value for the prospective buyer. In that case the license holder can make a profit by selling the license. However, during the license’s life, its value may move downward as well as upward for a number of reasons:

- Entry of a new operator in the market;
- Changes in customers’ preferences;
- Availability of new technologies.

But the previous simple line of reasoning applies also when the licence evaluation has lowered. In fact, if the licence holder cannot sell its license and its business prospects are doomed, the license value is certainly below its acquisition cost and may be zero for all practical purposes. The tradability of the licence provides an added value to its ownership. Such added value is to be reflected in the license acquisition cost, which should be higher than that applicable when the transferability.
option doesn’t hold. The government can therefore benefit of this added value by earning more in the auctioning event, a welcome opportunity since the maximization of revenues for the government is considered as one of the main objectives in the licensing process [1]. Spectrum tradability is generally meant to imply that all the rights and obligations originally associated to the license are transferred jointly with the licence itself, among which the most relevant ones are:

- set of frequencies assigned;
- geographical area of validity of license;
- time period of validity;
- set of applications allowed by license;
- degree of protection granted by license assigners with respect to other users (in the same or adjoining frequency band);
- obligation not to interfere with other spectrum users (again in the same or adjoining frequency band).

At present spectrum trading, put forward e.g. in [4] and [13], has already been introduced in the national legislations of many countries [14] [15] [16] [5] [17] [18] [19]. In [6] the relaxation of transferability limitations on a semi-permanent basis has been shown to amount to add a real option to the set of rights and obligations associated to the license, so to allow to evaluate its economical benefits for the auctioneer (i.e., the government).

Though the removal of transferability limitations has so far been dealt with on a semi-permanent basis, nothing forbids to think of a secondary market acting on smaller timescales and with a strictly limited duration. That calls for a more precise definition of the resource whose usage rights are transferred.

An early attempt has been accomplished by De Vany, who introduced the concept of TAS (Time-Area-Spectrum) packets, whereby the owner of rights on a TAS has the exclusive right to produce electromagnetic waves for a specified period of time over a specified geographic area and a specified range of frequencies [20] [21]. In [22] that concept has been refined into that of Frequency-Space-Time (FST) block, where the notion of space, rather than area, is used to take into account the three-dimensional nature of electromagnetic radiation. The use of the time dimension allows for temporary transfers of rights. The discretization of usage rights according to the FST model leads to a representation of block assignments through a Rubik cube like image, where each block in the cube is a unique assignment of an FST block and each color represent a unique spectrum consumer (see Fig. 1). In this paper we rely on this view of complete disaggregation of resources, so that each FST may be traded separately.

III. OPTIONS ON USAGE RIGHTS

In Section II we have introduced the notion of FST block as the basic resource that can be traded in the spectrum market. In this section we build on that notion and describe a trading strategy that includes the temporary transfer of usage rights through the use of options.

We assume that an operator owns usage rights on N FST blocks, which differ just for the F components, i.e., they refer to different spectrum blocks but at the same time and over the same space. We also assume that these N blocks are perfect and complete substitutes for one another, so that any of them is equivalent for the spectrum consumer (it can run the same applications in any of them). The operator is entitled to transfer usage rights for any of these blocks, i.e, to sublet them. If that’s not the case, i.e., if there are applications that require the use of a number of contiguous blocks, the owner of usage rights may run several sales, one for single blocks and others for bundles of adjacent blocks. Rather than running a spot market the operator decides to perform a preliminary arrangement by selling options to spectrum consumers on the use of those blocks. An option gives the spectrum consumer the right to buy the usage right on an FST block at the specified time (the T of FST) in the future and at a specified price (the exercise price). The spectrum consumer pays immediately a price for the option; if, when the time T comes (or, equivalently a short time in advance to allow for switching of ownership rights), it decides to exercise the option (i.e., buy the usage right on the FST block), it will then pay the prespecified price for the block.

We assume also that there are M spectrum consumers, each one buying an option for exclusive usage rights on one FST block. The operator sells each of the consumers the option right at the price V, with an exercise price set to P. The number of spectrum consumers that can in the end get a block is limited by the number of available blocks.

IV. THE OVERBOOKING STRATEGY

However, the operator may decide to apply an overbooking strategy, i.e., selling a number of options larger than the number of available blocks. Overbooking is currently employed in a number of industry sectors, namely in the air transportation sector [23] [24], where it is applied despite...
the real-time limitation with respect to the management of telecommunications services. The rationale for this behaviour is that not all the option buyers will actually end up buying the FST block, so that the operator may take some risk in order to saturate its offer and increase its revenues. We then indicate by \( B \leq M \) the number of spectrum consumers that actually exercise their option. The flip side of the coin is that, should the number of exercised options exceed the number of available blocks (i.e., \( B > N \)), the operator will be compelled to pay a penalty to the \( B - N \) option owners who don’t get their block.

We may therefore identify two cases:

1) The number of consumers asking to exercise the option is null or not larger than the number of FST blocks available, \( 0 \leq B \leq N \);

2) The number of consumers asking to exercise the option is larger than the number of available blocks \( B > N \).

In the first case the operator gets both the option price and the exercise price (the latter just if \( B > 0 \)), namely the option price from \( M \) spectrum consumers and the exercise price from \( B \) spectrum consumers. In the second case the operator gets the option price from all the \( M \) consumers, the exercise price from the \( N \) consumers who actually get their FST block, but has to pay a penalty \( C \) to each the \( B - N \) consumers who don’t get the FST block they had requested. The net revenues \( R \) in the two cases are then respectively

\[
R = M \cdot V + B \cdot P, \quad (1)
\]

\[
R = M \cdot V + N \cdot P - (B - N)C, \quad (2)
\]

which can be written in the compact form

\[
R = MV + \min(B, N)P - \max(B - N, 0)C. \quad (3)
\]

V. OPTION PRICING

The expression obtained for the revenues in Section IV incorporate the price \( V \) paid for the option. This variable affects positively the revenues, since it is paid upfront by all the spectrum consumers interested in getting an FST block. We expect that price to be directly related to the economic benefits the consumers expect to derive from the actual use of the FST block. Though other mechanisms may be thought of to set the option price, we rely on the use of the theory of financial and real options to put a value on it. The most basic financial options are the call and put options, which respectively provide the right to buy and sell a financial security. In our case we are interested in an option concerning an investment in a project (namely the purchase of usage rights on a frequency block in order to use it to provide telecommunications services and cash in on the resulting usage by end customers), hence we rely on the theory of real options. The real options framework has been adopted in the telecommunications sector for a number of investment decisions, see, e.g., [25] [26] [27] [28] [29]. In our case we are considering an investment on a quite smaller scale than those so far considered in the literature, since the lifetime of the investment is limited to the time for which the frequency block is let and the expense is limited to the price of the block itself plus the operating and capital expenses needed to run the frequency block for that period of time. The suitable option model is a call option, since the primary owner sells the right to buy the FST block. For this option we want to set the correct price in order to use it in the evaluation of the overbooking strategy. In fact, the value of the option depends on the actual possibility of exercising it, which in turn depends on the value of the item (the FST block) at the exercise time and on the price pre-set for the FST block. But the value of the item will fluctuate as we progress towards the option deadline, since estimates of the end demand for spectrum will change and get more accurate as time progresses. Since the option is to be exercised at a precise time in the future (the \( T \) in the FST block), the option model that applies is the European call option. For this model the Black-Scholes formula is widely established as a means to assess the value of the option [30]. This formula is based on the following assumptions (which we rephrase in the context of real options, though they were formulated for financial options):

1) The value \( S_0 \) of the FST block at the time of option purchase is the Net Present Value (NPV) of the investment in the FST block;

2) The risk-free rate of interest \( r \) is constant;

3) The value \( S \) of the FST block follows a lognormal random walk, with a mean drift \( \mu \) and a volatility \( \sigma \), i.e., \( dS = \mu S dt + \sigma S dz \), where \( z \) is a Wiener process;

4) There are no transaction costs associated to the block purchase.

Assumption 1 is a standard assumption in real options. The NPV is the discounted value of all the cash flows incurred during the investment lifetime. In our case discounting has to be accomplished back to the time at which the option is bought, and the cash flows consist of the revenues obtained during the time over which the FST block is managed by the spectrum consumer, subtracted of the operational and capital expenses incurred during that time. Assumption 2 is quite straightforward, given the very small time frame over which the action takes place. Assumption 3 may appear controversial, since it is not self-evident that the value of the FST blocks actually follows the lognormal random walk model. However, such assumption is well validated in the literature, and is a basic assumption in most works on real options, namely for the wireless industry [31] [28]. Assumption 4 relies on the use of electronic transactions.

In addition we can assume that the exercise price \( P \) is a fraction of the NPV of the block, since it can be assumed that the spectrum consumer will incorporate an expected rate of return on that investment, i.e., \( P = \gamma S_0 \), with \( 0 < \gamma < 1 \). Under these hypotheses the correct price for the option is the well known Black-Scholes formula

\[
V = S_0 G(d_1) - \gamma S_0 \exp(-rT)G(d_2), \quad (4)
\]
where $G(\cdot)$ is the cumulative standard normal distribution, and $d_1$ and $d_2$ are the following quantities

\[ d_1 = -\frac{\ln(\gamma) + (r + \sigma^2/2)T}{\sigma \sqrt{T}}, \]
\[ d_2 = -\frac{\ln(\gamma) + (r - \sigma^2/2)T}{\sigma \sqrt{T}}. \]

VI. The Copula Model for Users’ Exercise Decisions

In the previous sections we have described the cases that may take place in the overbooking strategy. Each case leads to different revenues. The revenues depend both on the economic quantities that are exchanged between the operator and the consumers (i.e., the price of the option, the price of a block, and the penalty) and on the decisions of the spectrum consumers (the number of consumers purchasing an option and the number of consumers purchasing a block). A key role is played by the number $B$ of consumers purchasing a block (or, rather, asking to purchase a block, since they will get a block just up to the number of available blocks). In fact, if $B \leq N$ the available blocks are enough to satisfy the demand. In that case the overbooking strategy results to be advantageous, since the operator has sold a number of options larger than what it would sell in the no-overbooking strategy (since $M > N$) but has no penalty to pay. On the other hand, when $B > N$ the operator has to pay the penalty to $B - N$ consumers, and those expenses may end up reducing the revenues below those of the no-overbooking strategy or even turn into a straight loss. In order to evaluate the overbooking strategy we need therefore to adopt a model for the number of block purchasing consumers $B$. In this section we propose a model for that quantity.

A simple model may be employed that assumes that the decisions taken by the spectrum consumers are independent of one another, and that each spectrum consumer decides to purchase a block with the same probability as all the other spectrum consumers. This leads to a binomial probability distribution for the number $B$ of purchasing consumers, namely, if we indicate the purchase probability for each individual consumer by $q$:

\[ \mathbb{P}[B = k] = \binom{M}{k} q^k (1 - q)^{M-k}. \]

However, we have to note that the decision by the consumer is influenced by a number of factors, among which the most relevant is the estimated demand, which in turn depends on the overall state of the wireless market and on the specific position of that consumer in that market. Since some of the influencing factors are common to all the consumers, a model based on the independence of the consumers’ decisions is probably not adequate.

We assume instead that the decision for each spectrum consumer is influenced partly by a common factor and partly by the individual position of that spectrum consumer, and then model the decision by the consumer as a function of two influencing factors, where the correlation between the decisions taken by different consumers is due to the presence of the common factor. A model suitable for this purpose is the normal copula model, described in [32] in the context of models for the credit risk in financial markets. Though the model allows for a number of different common factors, we restrict here to its simplest formulation where just a single common factor is present. We indicate the decision taken by the $i$-th consumer, $i = 1, \ldots, M$, by the binary variable $Y_i$, which takes the value 1 if the spectrum consumer decides to buy the block, and the value 0 otherwise. We associate to each $Y_i$ the latent variable $X_i$, through the relationship

\[ Y_i = \mathbb{I}(X_i > b_i), \]

where the indicator function $\mathbb{I}(\cdot)$ takes the value 1 if its logical argument is true, and 0 otherwise. The threshold $b_i$ is set so to meet the conditions on the probability $q_i$ that the $i$-th consumer buys the block (probability that may be different for each consumer), i.e., $b_i = F_{X_i}^{-1}(1 - q_i)$, where $F_{X_i}(\cdot)$ is the cumulative distribution function of $X_i$. The latent variable is in turn composed of the two contributions related to the two influencing factors ($Z$ for the common factor and $W_i$ for the individual one)

\[ X_i = \alpha_i Z + \sqrt{(1 - \alpha_i^2)} W_i, \]

where $Z$ and $W_i$, $i = 1, \ldots, M$, are i.i.d. standard normal random variables. The weight $\alpha_i$ (hereafter referred to as the dependence factor) measures the degree of dependence of the decisions taken by the consumers on the common factor (hence the degree of correlation among the consumers’ decisions): the larger it is the larger the correlation among the purchasing consumers. The latent variables themselves follow then a standard normal distribution. The threshold $b_i$ is then $b_i = G^{-1}(1 - q_i)$, where $G(\cdot)$ is the standard normal distribution function.

VII. Effectiveness of Overbooking

The overbooking strategy has to be evaluated against the standard approach to sell a number of options not larger than the number of available blocks. In this section we employ the models described in the previous sections to accomplish this evaluation.

In general we consider a strategy better than the other if it provides larger revenues. However, revenues depend on the consumers’ decisions (for which we have put forward a tentative model in Section VI) and are then random. As evaluation metrics we consider then:

1) the probability $P_{ob}$ that the overbooking strategy provides lower revenues than the no-overbooking one;
2) the expected differential revenues $\mathbb{E}[R_{ob}]$, which we define in this Section.

We can consider the overbooking to be better than its standard alternative if $P_{ob} < 0.5$ and $\mathbb{E}[R_{ob}] > 0$. The two metrics may provide different indications, since $P_{ob}$ doesn’t take into account the amount of difference in the revenues, and the expected differential revenues provide an average measure. We consider first separately the two cases depicted in Section...
IV. In Case 1 the number $B$ of purchasing users is not larger than the number of blocks ($B \leq N$). The overbooking is practically harmless and provides larger revenues, since it allows to cash the prices of the options sold over the limit. The difference between the revenues in the overbooking strategy and those with no overbooking is $V$ times the number of options sold in excess.

$$R_{ob} = (M - N)V.$$  \hfill (10)

In Case 2, where the spectrum owner sells options beyond its availability ($B > N$), it goes on cashing the price of the options sold over the limit, but has also to pay the penalties to the unsatisfied consumers. The differential revenues now are

$$R_{ob} = (M - N)V - C(B - N).$$  \hfill (11)

Notwithstanding the penalties to be paid in Case 2, the overbooking strategy is still winning if the number of purchasing consumers is not too large, namely

$$B < N \left(1 + \frac{M/N - 1}{C/V}\right),$$  \hfill (12)

where $M/N$ is the overbooking factor. Since the probability of Case 1 is

$$P_{lb} = \sum_{k=0}^{N} \binom{M}{k} q^k (1 - q)^{M-k},$$  \hfill (13)

where for simplicity we assume that all the consumers exhibit the same purchase probability $q_i = q$, $i = 1, 2, \ldots, M$, the expected value of the differential revenues is

$$\mathbb{E}[R_{ob}] = (M - N)V - C(1 - P_{lb}) \mathbb{E}[B|B > N] - N.$$  \hfill (14)

On the other hand the overbooking strategy is losing only in the subcase of Case 2 where the condition 12 doesn’t hold, i.e.

$$P_{ob} = P\left[B > N + \frac{M - N}{C/V}\right].$$  \hfill (15)

We report in the following some results for the expected differential revenues for the relevant ranges of values for the several parameters involved. In particular we describe the dependence on the following parameters:

- Overbooking factor $M/N$;
- Ratio of penalty to the option price $C/V$;
- Block purchase probability $q$;
- Dependence factor $\alpha$ (we assume that the dependence factor is equal for all the consumers).

Due to the presence of the copula model we have to resort to MonteCarlo simulation. We have accomplished 100000 simulation runs for each case. The impact of the overbooking factor is illustrated in Fig. 2; for that case the values of the other parameters are

- Purchase probability $q = 0.25$;
- Number of blocks $N = 5$;
- Penalty/Option price $C/V = 10$.

These values are indicative (the actual values depend on the particular technological and economical context), but representative of significant situations. The expected impact of the overbooking factor is not monotonous, since a massive use of overbooking bring along both larger cash volumes (due to the purchase of options) and larger outpayments for the penalties. Actually we get a parabolic-like curve with the positive effects dominating for the smaller values of the overbooking factor. The presence of correlation between the decisions of the spectrum consumers significantly reduces the range of usable values for the overbooking factor. While the overbooking strategy results to be feasible even with overbooking factor as large as 5.6 when the spectrum consumers act independently of one another, the range of suitable overbooking factor shrinks to roughly 2.6 when the correlation between the consumers is large.

![Fig. 2. Impact of the overbooking factor](image)

Similarly we report the impact of the penalty/option price ratio on the expected differential revenues in Fig. 3, when the other parameters are

- Purchase probability $q = 0.25$;
- Number of blocks $N = 5$;
- Overbooking factor=2.

Large penalties act as negative incentives to apply the overbooking strategy and hence we expect the differential revenues to decay as the $C/V$ ratio grows. Here the dependence we find is practically linear, with a slope extremely sensitive to the dependence factor. Again, when the correlation among the spectrum consumers is null or moderate, very large values are tolerable for the penalty. When the dependence factor is $\alpha = 0.7$, the maximum tolerable penalty is 13 times the price of the option. Finally we consider the effect of the purchase probability in Fig. 4, when the other parameters are

- Penalty/Option price $C/V = 10$;
- Number of blocks $N = 5$;
- Overbooking factor=2.

When the purchase probability grows, the payments by the spectrum consumers grow as well, but just till the limit of...
salable blocks $N$ is reached. From that point on the incomes stop growing while the outpayments due to penalties start growing. Hence we expect the expected differential revenues to be a decaying function of the purchase probability. Actually we see that the decaying slope grows with the purchase probability and, again, the dependence factor may significantly limit the feasibility of the overbooking strategy.

VIII. CONCLUSION

We propose a two-stage subletting mechanism for usage rights on wireless spectrum. We consider that the primary owner of usage rights may sell options on frequency blocks, which spectrum consumers may decide to exercise in the future. In this two-stage mechanism the primary owner may apply an overbooking strategy, i.e., selling more blocks than those it owns, as a hedging protection against the risk of ending up with unsold blocks. The overbooking strategy is economical convenient if the penalties to be paid to the unsatisfied customers do not zero the extra revenues due to the options sale. We have compared the overbooking strategy to the case where no overbooking is applied. The gain due to overbooking decays linearly with the penalty value. Instead, a threshold may be located for the overbooking factor, such that for factors lower than the threshold the overbooking gain increases significantly, decaying fast as the threshold is surpassed. For the cases examines the threshold corresponds to not so small overbooking factors (roughly 2 to 3), so that significant margins exist for the application of the overbooking strategy. Finally, the model incorporates the correlation between the different spectrum consumers. It is shown that this correlation negatively affects the gain due to the overbooking strategy even for small values of the purchase probability.

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