Hybrid Local Search Polynomial-Expanded Linear Multiuser Detector for DS/CDMA Systems

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Abstract This work proposed a new multiuser detector for DS/CDMA systems constituted by the polynomial MMSE detector followed by a local search algorithm 1-adapt LS (one-adaptive local search), namely the Hybrid 1adapt-LS-MuD. In order to reduce computational complexity inherent to recurrent computation of the cross-correlation matrix inverse in DS/CDMA multiuser detection (MuD), this work introduces for the first time a hybrid multiuser detector based on polynomial expansion (PE-MuD) with α-estimation aided by Gerschgorin circles (GC), followed by a low complexity local search procedure, aiming at obtaining a near-optimum multiuser bit-error-rate (BER) performance, but with a considerable saving in computational complexity. The proposed hybrid PE-MuD receiver topology is analyzed under realistic wireless mobile channels, as well as useful system operation scenarios. Numerical results obtained via Monte-Carlo simulations (MCS) have indicated a remarkable improvement in performance-complexity trade-off regarding the classical linear multiuser detectors (LMuD) performance, particularly, the mean square error minimization-based detector (MMSE-MuD).

Keywords: Near-optimum search algorithms, polynomial-expanded multiuser detection, Gerschgorin circles, DS/CDMA, complexity reduction, local search.


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1 Introduction

In code division multiple access (CDMA) systems, the total use of the transmission channel capacity, regardless of the channel adopted, depends on the features of the detector utilized, which prevent the effects generated by the multiple access through non orthogonal code division, mainly, in the effectiveness of the receptor to mitigate the effects of multiple access interference (MAI), as well as to deal with the near-far ratio (NFR) effect.

Multiuser detection (MuD) algorithms usually have very high computational complexity, which greatly limits their adoption. The optimal solution for the multiuser detection problem lies in the deployment of maximum likelihood (ML) detector, proposed by ?. However, ML detector complexity is impractical in almost scenarios of interest. Hence, linear near-optimal MuDs, such as the Decorrelator and MMSE were proposed in (??). Basically, these detectors utilize the inverse cross-correlation matrix of signature waveforms of the active users in the system ($\mathbb{R}^{-1}$) to decouple the desired user’s signal.

Aiming at more efficient linear detectors implementation, a multiple stage detection scheme, which approximately implements the inverse cross-correlation matrix through polynomial expansion in $\mathbb{R}$, has been presented in (?). The resulting detection scheme is namely polynomial expansion (PE) detector and may be applied to approximate both the Decorrelator and minimum mean-squared error (MMSE) MuD algorithms. Polynomial expansion multiuser detector (PE-MuD) can be viewed as an iterative approach in order to approximate the linear multiuser detectors with low complexity quadratic order dependence regarding the number of users, $O(K^2)$. In general, PE approach approximates the cross-correlation matrix inversion via Neumann iterative series expansion, with its coefficients estimated by the Gerschgorin circles method (??).

Other concept widely adopted in this current study is the local search (LS) based on neighborhood with signal detection application. The LS detection method, which sometimes is classified as heuristic, in fact constitutes a deterministic optimization mechanism which implement low-complexity local search solutions into a previously established neighborhood (??). The main advantage of this method lies on its inexpensive very reduced complexity. According to ?, the LS-MuD have similar performance when comparable to the classical heuristic methods such as particle swarm optimization (PSO) and genetic algorithm (GA) algorithms, but with a convergence more accentuated which results in a smallest computational complexity. However, when the modulation order increases, such as deploying $M$–QAM with $M \geq 16$, the LS-MuD suffers with a lack of diversity in the search space, and the near-optimum performance achieved under low order modulation formats is deteriorated.

Several linear PE-MuDs algorithms aided or not by low complexity LS mechanisms in order to improve the performance-complexity trade-off of the linear MuDs have been proposed in the last decade, for instance in (?), (??), (??) and (??). A PE-MuD with low complexity $O(K^2)$ is presented in (?). It approximates the computational intense ($O(K^3)$) matrix inversion necessary for the Decorrelating and MMSE MuDs. In order to accelerate convergence, a normalized PE detector’s matrix with respect to its smallest and largest eigenvalues is deployed. An efficient method to accurately estimate these eigenvalues is proposed for the first time.

In (?), the same low complexity $O(K^2)$ iterative approximation for the MMSE MuD based on polynomial expansion is used. A new method based on the estimation of the eigenvalues of the channel correlation matrix by the implicitly restarted Lanczos method (IRLM) was suggested. As a result, the tight estimate of the eigenvalues has propitiated a better near-far resistance conjugated with a faster convergence when compared to the MMSE-MuD.

In (?), an iterative PE detector with faster convergence and better performance when compared to the MMSE-MuD is obtained even under high mobility scenarios.

A structure formed by the polynomial expansion detector as the first stage followed by a local search algorithm has been presented in (?). This structure is able to offer performance improvements under DSP implementable low-complexity perspective. In a same perspective, ? investigate a new local search algorithm, which maintains the same convergence shape but with a smaller quantity of operations at the expense of a marginal and acceptable increasing in the BER. ? introduces for the first time a hybrid detector that consists in the polynomial MMSE detector followed by the new hybrid local search strategy algorithm. In fact, herein the current study is an extended version of that author’s conference paper.

A comparative analysis for the performance-complexity tradeoff of several hybrid group-blind MuDs based on Gershgorin circles PE under channel errors estimates is carried out in (?).

Besides this introductory section, this work is divided in the following sections. The system model is established in Section ??, in which a review on classic linear single-user (SuD) and multiuser detectors (MuDs) is presented. The polynomial expansion method, used in the inverse cross-correlation matrix approximation, is discussed in Section ??, The application of the local search method in the MuD DS/CDMA problem is addressed in Section ??, The performance and comparison of relevant MuD methods is revealed in Section ??, while the main conclusions are summarized in Section ??.

2 System Model

Herein, a discrete-time baseband system model is adopted, with transmission through a channel with a single antenna in the transmitter and receptor (SISO –
single-input single-output) subjected to additive white Gaussian noise (AWGN) and flat Rayleigh fading. The same channel is simultaneously shared by \( K \) users, which operate under a synchronous DS/CDMA system with binary phase shift keying modulation (BPSK). In the transmission, the \( i \)-th information bit with period \( T_b \), generated by the \( k \)-th user, at a ratio of \( R_b = 1/T_b \) bits per second is denoted by \( b_k \in \{ \pm 1 \} \). At each \( i \)-th bit interval, \( b_k[i] \) is modulated by a spread sequence with pseudo-noise (PN) distribution and length \( L \), at the ratio of \( R_c = L/T_b = LR_b \) chips per second. The spreading code can be represented by the vector:

\[
s_k[i] = [s_{k,1}[i], s_{k,2}[i], \ldots, s_{k,L}[i]]^\top,
\]

with \( s_{k,L}[i] \in \{ \pm 1/\sqrt{L} \} \) and \( L \) denoting the system’s processing gain; \((\cdot)^\top \) denotes the matrix transpose operator.

In the base radio station (BRS), the received signal vector is represented by:

\[
r[i] = \sum_{k=1}^{K} s_k[i] c_k[i] A_k b_k[i] + n[i],
\]

where \( A_k \) is the amplitude of the signal transmitted by the \( k \)-th user; \( n[i] \) is the complex AWGN vector of mean zero and variance \( \sigma_n^2 = N_0 \), with bilateral power spectral density of AWGN noise given by \( N_0/2 \) W/Hz.

The term \( c_k[i] \) denotes the complex coefficient of the channel inherent to the \( k \)-th user, at the \( i \)-th bit interval, perfectly known by the receptor, but not at the transmitter side. In statistical terms, \( c_k[i] \) may be represented by a circularly symmetric complex Gaussian random variable, with mean zero and variance \( \sigma_c^2 \), in the form \( \mathcal{CN}(0, \sigma_c^2) \). In the polar form, the channel’s complex coefficient is described by:

\[
c_k[i] = |c_k[i]| e^{j \theta_k[i]},
\]

where phase \( \theta_k[i] \) is uniform over the range \([0, 2\pi]\), i.e., omnidirectional BS receive antenna, and independent of the magnitude \( |c_k[i]| \). Herein, a non-line-of-sight (NLOS) communication has been assumed; hence, the magnitude of the channel coefficients are suitably characterized by a Rayleigh random variable \((\cdot)\) with probability density function given by:

\[
f(r) = \frac{r}{\sqrt{\pi} \epsilon} e^{-r^2/(2\epsilon^2)}, \quad r \geq 0.
\]

In the notation of matrices, with bold capital letters representing matrices and bold lower case letters representing vectors, and suppressing the bit interval index \( i \) for the sake of convenience, Eq. (2) could be rewritten as follows:

\[
r = \mathbf{S}\mathbf{C}\mathbf{A}\mathbf{b} + \mathbf{n},
\]

with \( \mathbf{A} = \text{diag}(A_1, A_2, \ldots, A_K) \) being the diagonal matrix of the transmit signal amplitudes, \( \mathbf{S} = [s_1, s_2, \ldots, s_K] \) is the spreading code matrix with dimensions \( L \times K \) and \( \mathbf{C} = \text{diag}(c_1, c_2, \ldots, c_K) = \text{diag}([c_1, c_2, \ldots, c_K]) \) corresponding to the channel complex coefficients matrix, where \( \mathbf{F} \) and \( \mathbf{P} \) are, respectively, the diagonal matrices of magnitudes and phases of the channel. Vector \( \mathbf{b} = [b_1, b_2, \ldots, b_K]^\top \) contains bit information transmitted by the \( K \) users and \( \mathbf{n} = [n_1, n_2, \ldots, n_K]^\top \) is the complex noise vector with distribution \( \mathcal{N}(0, \sigma_n^2) \).

The output signal of the conventional matched filters bank (MFB) is described taking account the channel phases:

\[
y_{\text{MFB}} = \mathbf{P}^\top \mathbf{y} = \mathbf{P}^\top \mathbf{S}^\top \mathbf{r} = \mathbf{S}^\top \mathbf{S}^|\mathbf{C}\mathbf{A}\mathbf{b} + \mathbf{P}^\top \mathbf{S}^\top \mathbf{n} = \mathbf{RF}\mathbf{A}\mathbf{b} + \mathbf{w},
\]

where the vector \( \mathbf{y} = [y_1, y_2, \ldots, y_K]^\top \) represents the despread baseband-received signal, whose components are given by \( y_k = s_k^\top \mathbf{r} \) and \( y_{\text{MFB}} = [y_{1,\text{MFB}}, y_{2,\text{MFB}}, \ldots, y_{K,\text{MFB}}]^\top \) is the MFB output information vector; the cross-correlation matrix of the signature waveforms is obtained via \( \mathbf{R} = \mathbf{S}\mathbf{S}^\top \); vector \( \mathbf{w} = \mathbf{P}^\top \mathbf{S}^\top \mathbf{n} \) corresponds to the filtered noise with variance \( \sigma_w^2 \mathbf{R} \); the conjugate operator is denoted by \((\cdot)^\ast\).

Finally, the \( K \) users’ information bits vector is estimated through:

\[
\hat{\mathbf{b}}_{\text{MFB}} = \text{sgn} (\mathbb{R}\{\mathbf{y}_{\text{MFB}}\}),
\]

where \( \text{sgn}(\cdot) \) represents the signum function and \( \mathbb{R}\{\cdot\} \) is the real part operator. As a result, the estimated information bits vector is obtained as \( \hat{\mathbf{b}}_{\text{MFB}} = [\hat{b}_{1,\text{MFB}}, \hat{b}_{2,\text{MFB}}, \ldots, \hat{b}_{K,\text{MFB}}]^\top \).

However, as well known, the conventional detector’s performance decreases remarkably when the system loading \( \mathcal{L} = K/L \) grows, i.e., due to the MAI level increasing as a function of the number of active users.

### 2.1 Optimum Detection

The optimum performance is obtained with the use of the ML detector, presented in (7). ML detector performs the joint information detection of the \( K \) users in the system, maximizing the following cost function:

\[
\Omega (\mathbf{b}) = 2\mathbb{R}\{\mathbf{b}^\top \mathbf{F}\mathbf{A} y_{\text{MFB}}\} - \mathbf{b}^\top \mathbf{C}\mathbf{A}\mathbf{R}\mathbf{A}\mathbf{C}^\dagger\mathbf{b}.
\]

which is based on the Euclidean distance between the received signal and the signal reconstructed in the receptor from the information candidate vector, \( \mathbf{b} \); the matricial operator \((\cdot)^\dagger\) holds for conjugation and transposition.

The optimum multiuser detection (OMuD) criterion yields the best information bits estimated vector \( \hat{\mathbf{b}}_{\text{ML}} \):

\[
\hat{\mathbf{b}}_{\text{ML}} = \arg \{ \max_{\mathbf{b} \in \mathcal{M}^{K}} \{ \Omega (\mathbf{b}) \} \},
\]

where \( \mathcal{G} \) is the transmitted message length and \( \mathcal{M} \) is the symbol alphabet dimension. For the binary modulation
adopted in this work, \( M = 2 \). Although the optimum ML detector achieves the best performance, however, its computational complexity is exponential with the number of user \( K \), modulation order \( m \) and message length \( G \), i.e., it is of the order of \( \mathcal{O} \left( 2^{mGK} \right) \). As a result, huge amount of suboptimum and near-optimum multiuser detectors have been proposed in the last two decades (??).

### 2.2 Linear Methods of Multiuser Detection

In (??), linear methods of detection were discussed, including the Decorrelator detector. This one operates from multiplication of the discrete signals at the matched filters output by the inverse cross-correlation matrix \( R^{-1} \). Considering the coherent reception model, the information bits vector which is estimated after the application of the Decorrelator multiuser filter, can be conveniently described as follows:

\[
\hat{b}_{\text{DEC}} = \text{sgn} \left( \Re \{ R^{-1} y_{\text{MFB}} \} \right)
\]

\[
= \text{sgn} \left( \Re \{ R^{-1} R F A b + R^{-1} w \} \right)
\]

\[
= \text{sgn} \left( \Re \{ F A b + w \} \right),
\]

where \( \hat{b}_{\text{DEC}} = [b_1^{\text{DEC}}, b_2^{\text{DEC}}, \ldots, b_K^{\text{DEC}}]^T \) is the Decorrelator output information vector. The Decorrelator detector presents a gain in the performance regarding the Conventional detector, although the power associated to the noise term \( \bar{w} = R^{-1} P S^T n \), obtained at the Decorrelator output, is always higher or equal to the noise term obtained at the Conventional output (??).

Another classical linear detection method known in literature is the MMSE detector, proposed for CDMA systems in (??). This method is based on the appropriate choice of a linear transformation vector, \( t_k = [t_1, t_2, \ldots, t_K] \), that minimizes the mean square error between the \( k \)th user’s information bit and the \( k \)th linear transformation output, \( t_k y_{\text{MFB}} \), resulting in:

\[
\min_{t_k} \mathbb{E} \left\{ (b_k - t_k y_{\text{MFB}})^2 \right\}.
\]

The vector that minimizes (??) involves the covariance of colored noise \( w \) and the estimated users’ amplitude matrix at the receiver side, \( B = \text{diag}(B_1, B_2, \ldots, B_K) = FA \). By applying this MMSE solution to the joint detection of the \( K \) users, the transformation matrix \( T_{\text{MMSE}} = [t_1^{\text{MMSE}}, \ldots, t_k^{\text{MMSE}}, \ldots, t_K^{\text{MMSE}}] \) with dimension \( K \times K \) is given by:

\[
T_{\text{MMSE}} = (R + \sigma_n^2 B^{-2})^{-1}.
\]

Therefore, the estimated information vector, obtained after the application of the MMSE multiuser filter, is described by:

\[
\hat{b}_{\text{MMSE}} = \text{sgn} \left( \Re \{ T_{\text{MMSE}} y_{\text{MFB}} \} \right).
\]

### 3 Polynomial-Expanded Multiuser Detectors

The computational complexity of the linear MuDs, which originates in the operations associated to the cross-correlation matrix inversion, grows with the third order of the matrix size, i.e., \( \mathcal{O} ((mGK)^3) \). However, any linear transformation matrix, represented by \( T \), can be approximated through the iterative polynomial expansion method with complexity of \( \mathcal{O} ((mGK)^2) \).

#### 3.1 General Result for PE Matrix Approximation

The general result for the \( K \times K \) polynomial expanded transformation matrix \( T_{\text{PE}} \), which is able to implement a PE multiuser detector (by approximating a matrix inversion), is given by (??):

\[
T_{\text{PE}} = \sum_{i=0}^{N_i} w_i Q^i,
\]

where \( N_i \) denotes the number of terms of the polynomial expansion. The matrix \( Q \) and the weights \( w_i \), interpreted as the coefficients for the series convergence rate, have to be chosen such that they suitably approximate the desired multiuser detector.

As a result, the polynomial expansion transformation matrix \( T_{\text{PE}} \rightarrow Q^{-1} \) when the number of expansion terms \( N_i \rightarrow \infty \).

#### 3.2 Polynomial Expansion via Neumann Series

By using the Neumann series expansion method (??), the inverse cross-correlation matrix \( R^{-1} \), for the case of Decorrelator, may be approximated as:

\[
R^{-1} \approx T_{\text{PE}}^{\text{DEC}} = \alpha \sum_{i=0}^{N_i} (I_K - \alpha R)^i, \quad ||I_K - \alpha R|| < 1
\]

where \( I_K \) is an identity matrix of size \( K \), and the associated residual error matrix is given by:

\[
\varepsilon_{\text{PE}}^{\text{DEC}} = \alpha \sum_{i=N_i+1}^{\infty} (I_K - \alpha R)^i,
\]

such that the equality \( R^{-1} = T_{\text{PE}}^{\text{DEC}} + \varepsilon_{\text{PE}}^{\text{DEC}} \) holds.

In the same way, the PE expansion matrix transformation for the linear MMSE detector can be approximated in:

\[
T_{\text{PE}}^{\text{MMSE}} \approx (R + \sigma_n^2 B^{-2})^{-1}
\]

\[
= \alpha \sum_{i=0}^{N_i} \left[ I_K - \alpha (R + \sigma_n^2 B^{-2}) \right]^i.
\]

Finally, the polynomial-expanded multiuser detector (PE-MuD) hard decisions for BPSK DS/CDMA systems are obtained as:

\[
\hat{b}_{\text{PE}}^k = \text{sgn} \left( \Re \{ z_{k}^{\text{PE}} \} \right),
\]

where \( z_{\text{PE}}^k = [z_{1}^{\text{PE}}, z_{2}^{\text{PE}}, \ldots, z_{K}^{\text{PE}}]^T = T_{\text{PE}}^{\text{DEC}} y_{\text{MFB}} \) for the Decorrelator approximation or \( z_{\text{PE}}^k = T_{\text{PE}}^{\text{MMSE}} y_{\text{MFB}} \) for the MMSE approximation.
3.3 Convergence Factor

In Eq. (18), the convergence factor of the Neumann series is equal to the spectral radius\(^a\) of the matricial operator, \(\rho(I_K - \alpha R)\). Therefore, the series converges if the spectral radius’ value is less than one (\(\rho < 1\)). Assuming that the eigenvalues of the \(K \times K\) matrix \(R\) are \(\lambda_k\), \(k = 1, 2, \ldots, K\), all real and limited to the interval:

\[
\lambda_{\min} \leq \lambda_k \leq \lambda_{\max},
\]

the eigenvalues of \((I_K - \alpha R)\), namely \(\mu_k\), with \(k = 1, 2, \ldots, K\), will be laid in the interval:

\[
1 - \alpha \lambda_{\max} \leq \mu_k \leq 1 - \alpha \lambda_{\min}.
\]

Also assuming that \(\lambda_{\min} > 0\), the convergence for the Neumann series depends on the following conditions:

\[
1 - \alpha \lambda_{\min} < 1; \\
1 - \alpha \lambda_{\max} > 1.
\]

As a consequence, the series converges with any scalar \(\alpha\) which satisfies:

\[
0 < \alpha < \frac{2}{|\lambda_{\max}|}.
\]

For the case of linear MMSE detector, the convergence factor is defined herein with the premise that the diagonal amplitude matrix is \(B = I_K\), as follows:

\[
||I_K - \alpha (R + \sigma_n^2 I_K)|| < 1.
\]

Therefore, the parameter \(\alpha\) that determines the convergence of the linear MMSE detector approximation is found on the interval:

\[
0 < \alpha < \frac{2}{|\lambda_{\max} + \sigma_n^2|}. 
\] (20)

3.4 Optimum Value of the Parameter \(\alpha\)

Since the convergence factor of an iterative method can be associated with the matricial operator’ radius, the convergence ratio is related to the dimension of this radius (\(\rho\)). The spectral radius for the matricial operator which approximates the Decorrelator filter is given by:

\[
\rho(I_K - \alpha R) = \max \{ |1 - \alpha \lambda_{\max}|, |1 - \alpha \lambda_{\min}| \}. 
\]

Therefore, the best convergence rate is obtained with the proper choice of the scalar \(\alpha\), in order to optimize the spectral radius. Fig. ?? shows the behavior of the spectral radius as a function of \(\alpha\). Hence, it follows that the optimum value of \(\alpha\) is found between the zeros of the bounds of the intervals in (\(\rho\)), at the point defined by the intersection between the positive slope of the lower bound curve with the negative slope of the upper bound curve (\(\rho\)):

\[
-1 + \alpha \lambda_{\max} = 1 - \alpha \lambda_{\min}.
\]

This operation results in the optimum parameter for linear Decorrelator detector in the PE approximation defined by:

\[
\alpha_{opt}^{\text{DEC}} = \frac{2}{\lambda_{\min} + \lambda_{\max}}.
\] (22)

In turn, for the linear MMSE detector approximation, the optimum value of \(\alpha\) is given by:

\[
\alpha_{opt}^{\text{MMSE}} = \frac{2}{\lambda_{\min} + \lambda_{\max} + 2 \sigma_n^2}.
\] (23)

Important to point out that the deterministic choice of \(\alpha_{opt}\) through the cross-correlation matrix’ eigenvalues calculation is prohibitively complex for the implementation of the polynomial expansion method using practical digital signal processing hardware platforms. As a result, the complexity of only one eigenvalue computation, as well as of all eigenvalues calculation from a \(K\) squared-dimension matrix results in \(O(K^3)\). Thus, it is necessary to estimate the optimum value of the parameter \(\alpha\). Next, the estimation of \(\alpha_{opt}\) is suggested by using the Gerschgorin circles Theorem (\(\rho\)).

3.5 Gerschgorin Circles Theorem

According to Gerschgorin Theorem, any eigenvalue \(\lambda_i\) of a matrix \(R\), which has elements \(r_{i,j}\), \(\forall i, j\), is situated in one of the complex plan’ circles that are centered in \(r_{i,i}\), with radius \(\sum_{j \neq i} |r_{i,j}|\), i.e.,

\[
|\lambda_i - r_{i,i}| \leq \sum_{j \neq i} |r_{i,j}|.
\] (24)

Thus, through a simple calculation, by using the elements of \(R\), the approximated values of \(\lambda_{\min}\) and \(\lambda_{\max}\), which are denoted by \(\hat{\lambda}_{\min}\) and \(\hat{\lambda}_{\max}\), respectively, can be achieved by:

\[
\hat{\lambda}_{\min} \approx \min \left\{ r_{i,i} + \sum_{j \neq i} |r_{i,j}| \right\}, \; \forall i;
\] (25)

\[
\hat{\lambda}_{\max} \approx \max \left\{ r_{i,i} + \sum_{j \neq i} |r_{i,j}| \right\}, \; \forall i.
\] (26)
The Gerschgorin circles (GC) Theorem allows a considerable reduction in the complexity of the minimum and maximum eigenvalues’ computation, being therefore deployed herein on the $\alpha$ estimation.

4 Local Search Methods Applied to the Multiuser Detection

Local search methods propitate the attainment of near-optimum solutions from searches guided in subspaces of the optimization problem’s dimension. The local search algorithm 1-opt LS (??) is described in the following; after that, an adaptation for the 1-opt LS algorithm is proposed in the Subsection ?? and a new algorithm is formed.

4.1 Local Search Algorithm 1-opt LS

The deterministic algorithm 1-opt LS (one-optimum local search) performs guided searches for the vector that maximizes the cost function posed by (??), selecting candidate-vectors located inside the unitary Hamming distance$^b$ from the output vector of MFB. The number of local search’ iterations is defined by $N_{\Delta t}$. Herein, the users’ power profile is considered for the classification at the beginning of the guided search process. The estimated users’ amplitudes matrix $\hat{B}$, as described in Section ?? is utilized in the users’ power classification, setting up the estimated users’ amplitudes vector $\delta = \text{diag}(\hat{B}) = [B_1, B_2, \ldots, B_K]$很好。The pseudo-code for the local search algorithm 1-opt LS is described in the Algorithm ??.

Algorithm 1 1-opt LS

Input: $\hat{B}^{\text{mfb}}; N_{\Delta t}; \delta$; Output: $\hat{b}$:

begin $t = 0$;
1. Classify signals (increasing amplitude order), given $B_k[t]$, $k = 1, 2, \ldots, K$, with $B_k[t] \leq B_{k+1}[t]$;
2. Initialize the local search: $t = 1$;
3. for $t = 1, 2, \ldots, N_{\Delta t}$
   a. Generate candidate vectors with unitary Hamming distance denoted by $\hat{b}_i[t], i = 1, 2, \ldots, K$;
   b. Calculate $\Omega (\hat{b}_i[t])$;
   c. if $\exists \hat{b}_i[t], (j \neq i)$:
      $[\Omega (\hat{b}_i[t]) > \Omega (\hat{b}_{\text{best}}[t])] \wedge [\Omega (\hat{b}_j[t]) > \Omega (\hat{b}_{\text{best}}[t])]$, $\hat{b}_{\text{best}}[t+1] \leftarrow \hat{b}_i[t]$;
   else, go to step 4
end if
end for
4. $\hat{b} = \hat{b}_{\text{best}}$;
end

$^b$Hamming distance between two vectors, e.g., $b_1$ and $b_2$, is defined by $d_H(b_1, b_2) = \|b_1 - b_2\|$, which corresponds to the amount of elements that differ between the vectors.

4.2 Local Search Algorithm 1-adapt LS

In order to reduce the complexity of the LS-MuD, the quantity of calculations of the cost functions during the search for the best candidate vector can be constrained by using a given threshold. $\delta$ establishes a threshold criterion based on channel measurement informations, by selecting a fixed number of the lowest confidence bits to be changed. Differently of Chase search stop criterion, herein for the proposed 1-opt LS algorithm, a dynamic threshold is used in order to create adaptation and reduce complexity. This new algorithm, namely 1-adapt LS, classifies the received signals in order of increasing amplitude. Then, candidate vectors with unitary Hamming distance are generated, following the ordering of the signals (from the weakest to the strongest), and their respective cost functions are evaluated. In case of the cost function value is not increased following a pre-established quantity of consecutive evaluations, denoted by $\kappa$, the search process is interrupted and a new search is initiated. The pseudo-code for the algorithm 1-adapt LS is described in the Algorithm ??.

Algorithm 2 1-adapt LS

Input: $\hat{B}^{\text{mfb}}; N_{\Delta t}; \delta; \kappa$; Output: $\hat{b}$:

begin $t = 0$;
1. Classify signals (increasing amplitude order), given $B_k[t]$, $k = 1, 2, \ldots, K$, with $B_k[t] \leq B_{k+1}[t]$;
2. Initialize the local search: $t = 1$; $\ell = 0$;
   $\hat{b}_{\text{best}}[1] = \hat{B}^{\text{mfb}}$;
   $g_{\text{best}}[1] = \Omega (\hat{b}_{\text{best}}[1])$;
3. for $t = 1, 2, \ldots, N_{\Delta t}$
   a. Generate candidate vectors with unitary Hamming distance denoted by $\hat{b}_i[t], i = 1, 2, \ldots, K$;
   b. Calculate $g_i[t] = \Omega (\hat{b}_i[t])$;
   if $g_i[t] > g_{\text{best}}[t]$,
      $g_{\text{best}}[t+1] \leftarrow g_i[t]$;
      $\hat{b}_{\text{best}}[t+1] \leftarrow \hat{b}_i[t]$;
   else
      $\ell = \ell + 1$;
   end if
end for
4. $\hat{b} = \hat{b}_{\text{best}}$;
end

When the Algorithm 1-adapt LS prioritizes the inversion of the weakest signals’ information vector, assuming that these signals have a greater error probability in the reception, the effectiveness of the search for the best candidate vector is potentiated. Thus, it is possible to reduce the detector’ complexity by limiting the number of signals processed, when the relative increasing in SNR values across the iterations indicates the stagnation in the cost function gain.
4.3 Hybrid 1opt-LS-MuD and 1adapt-LS-MuD Detectors

A detection structure formed by a suboptimal local search algorithm in conjunction with a primary stage of polynomial-expanded linear multiuser detector was presented in (7). This structure has been reproduced herein, by deploying in the first stage, the polynomial MMSE detector with $\alpha$ estimated via the Gerschgorin circles method, and in the second stage, the 1-opt LS algorithm described in Algorithm ??.

In a same way, combining the polynomial-expanded MMSE MuD followed by the new adaptive local search algorithm discussed in section ??, we introduce for the first time the Hybrid 1adapt-LS-MuD, in which the 1-adapt LS strategy is described in Algorithm ??.

It is worth noting that this work introduces for the first time a multiuser detector constituted by the polynomial MMSE detector followed by the new local search algorithm 1-adapt LS, namely the Hybrid 1adapt-LS-MuD. A short version of this work was presented in (??).

The performance and complexity comparisons including both hybrid suboptimal local search PE-MuDs are carried out in Section ??.

5 Performance-Complexity Analysis

In this section, the performances of the suboptimal MuDs are evaluated by means of Monte Carlo simulation method. The flat Rayleigh fading channels, which magnitude and phase coefficients are perfectly estimated at the receptor side, have been adopted. In all numerical results presented in this section, the average SNR, denoted by $\text{SNR}_{\text{avg}}$, is deployed in the context of the near-far effect, i.e., there are two interfering groups of users with near-far ratio:

$$ \text{NFR} = P_{\text{interf}} - P_{\text{interest}} = +5 \text{ dB (K/3 users)}; $$

$$ \text{NFR} = P_{\text{interf}} - P_{\text{interest}} = -5 \text{ dB (K/3 users)}, $$

where $P_{\text{interf}} = B^2_{i,\text{interf}}$ and $P_{\text{interest}} = B^2_{j,\text{interest}}$ is the received power for the $i$th interfering signal and $j$th interested signal, respectively.

A variant of the near-far effect is used in the simulation presented in Fig. ??, where in this case there is only one interfering group of users with:

$$ \text{NFR} = [-15; 30] \text{ dB (K/2 users)}. \quad (27) $$

Hence, the average SNR and bit-error-rate ($\text{BER}_{\text{avg}}$) presented in this section is taken over the interest users group only (K/3 or K/2 users). The $\text{SNR}_{\text{avg}}$ considered in simulations ranges from 0 to 40 dB. Random spreading codes with processing gain length of $L = 36$ have been adopted in a single-rate DS/CDMA system. Furthermore, the number of terms in polynomial expansion is limited to $N_1 = [1; 7]$ terms, while the number of local search algorithm’ iterations is limited to $N_0 = [0; 10]$ iterations. Besides, in the algorithm 1-adapt LS, a good performance-complexity trade-off was achieved with $\kappa = [0.6 \cdot K]$.

5.1 1opt-LS-MuD and 1adapt-LS-MuD

Fig. ?? and ?? show the average BER and the average quantity of cost function calculations by iteration ($\zeta_{\text{avg}}$), respectively, for the 1opt-LS-MuD and 1adapt-LS-MuD, as a function of an increasing number of users (system loading robustness). Both figures were obtained from the same MCS setup, considering the same point of system operation and $\text{SNR}_{\text{avg}} = 14$ dB. In this scenario, the quantity of active users in the system ranges from $K = [9; 36]$ users, i.e., system loading lies on the range $\mathcal{L} = 100 \cdot K/L = [25\%; 100\%]$.

In the region of low system loading, i.e., $\mathcal{L} \leq 33\%$, the value of $\zeta_{\text{avg}}$ is insufficient to ensure a reasonable performance for the 1adapt-LS-MuD. In this region, the detector could be switched for the 1opt-LS-MuD. However, the performance of 1opt-LS-MuD and 1adapt-LS-MuD detectors with 1 iteration are practically identical at high system loading. Taking into account $K = 30$ users in the system ($\mathcal{L} = 83.3\%$), the BER$_{\text{avg}}$ performances of the evaluated detectors are very close. Nevertheless, the complexity of the second detector is smaller, according to Fig. ??, by loading the system in 75%, the value of $\zeta_{\text{avg}}$ accomplished for 1adapt-LS-MuD detector with three iterations is 11% smaller in relation to 1opt-LS-MuD detector, although with a relative increase of 7.1% in the BER$_{\text{avg}}$ of the detector with lower complexity. However, as one can conclude from Fig. ??, neither the proposed Hybrid local search PE multiuser detectors (1opt-LS-MuD and 1adapt-LS-MuD) nor the linear MMSE are completely robust against the system loading (MAI) increasing.
Figure 3  Average quantity of cost function calculations as a function of system loading for 1opt-LS-MuD and 1adapt-LS-MuD; SNR\textsubscript{avg} = 14 dB.

5.2 Hybrid 1adapt-LS-MuD Detector

As shown in Fig. ??, the Hybrid 1adapt-LS-MuD detector with 1 term in the polynomial expansion does not converge to the ML detector; while with 5 or 7 terms and $\kappa = [0.6 \cdot K]$, the convergence is guaranteed within 2 iterations. On the other hand, with $N_t = 3$ terms, the performance of the Hybrid 1adapt-LS-MuD detector is nearly optimum within $N_{\text{it}} = 3$ iterations.

Fig. ?? shows the near-far robustness of the Hybrid 1adapt-LS-MuD detector with $N_t = 1$ term keeps its performance close to that achieved by the linear MMSE-MuD up to SNR\textsubscript{avg} = 12 dB, i.e., in the low-medium SNR\textsubscript{avg} region. However, with $N_t = 3$ terms, this performance is extended up to SNR\textsubscript{avg} = 32 dB. These results represent an excellent performance-complexity trade-off for the proposed hybrid adaptive local search multiuser detector.

6 Conclusions

The proposed local search algorithm 1-adapt LS promotes a remarkable gain in the DS/CDMA system performance equipped with polynomial expansion-based hybrid multiuser detectors. When associated to low-complexity PE-MuD detectors, it provides reliability to
the detection process, without an excessive increasing in its implementation cost, been able to offer a good performance-complexity trade-offs.

Simulation results have shown that the proposed 1-adapt LS is able to provide a considerable level of robustness against the near-far effect when combined to the PE-MuD. Furthermore, this hybrid detector achieves fast convergence by using only three terms in the polynomial expansion, with a remarkable trade-off between near-optimum performance and reduced complexity, specially when the detector operates in scenarios with high system loading and moderate NFR.

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