Closed-Loop Supply Chain Models with Product Remanufacturing

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The importance of remanufacturing used products into new ones has been widely recognized in the literature and in practice. In this paper, we address the problem of choosing the appropriate reverse channel structure for the collection of used products from customers. Specifically, we consider a manufacturer who has three options for collecting such products: (1) she can collect them herself directly from the customers, (2) she can provide suitable incentives to an existing retailer (who already has a distribution channel) to induce the collection, or (3) she can subcontract the collection activity to a third party. Based on our observations in the industry, we model the three options described above as decentralized decision-making systems with the manufacturer being the Stackelberg leader. When considering decentralized channels, we find that ceteris paribus, the agent, who is closer to the customer (i.e., the retailer), is the most effective undertaker of product collection activity for the manufacturer. In addition, we show that simple coordination mechanisms can be designed such that the collection effort of the retailer and the supply chain profits are attained at the same level as in a centrally coordinated system.

Key words: supply chain management; reverse logistics; remanufacturing; channel structure

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1. Introduction
The importance of the environmental performance of products and processes for sustainable manufacturing and service operations increasingly is being recognized. While legislation introduced in Europe, North America, and Japan encourages this awareness, many corporations have proactively taken measures in anticipation of evolving environmental performance requirements. Increasingly, manufacturers of durable and nondurable goods are establishing economically viable production and distribution systems that enable remanufacturing of used products in parallel with the manufacturing of new units. Remanufactured products are typically upgraded to the quality standards of new products, so that they can be sold in new product markets. In this paper, we consider product categories in which there is no distinction between a remanufactured and a manufactured product, and we refer to the distribution systems, which use a combination of manufacturing and remanufacturing, as closed-loop supply chains.

The prospect of remanufacturing poses interesting questions with respect to the design and management of closed-loop supply chains. The goal of this paper is to develop a detailed understanding of the implications that a manufacturer’s reverse channel choice has on forward channel decisions and the used-product return rate from the customers.

In current practice, we find a variety of reverse channel formats deployed by manufacturers. In some cases, manufacturers collect their used products directly from the customers. For instance, Xerox Corporation provides prepaid mailboxes so that customers can easily return their used copy or print cartridges to Xerox without incurring any costs. The company also remanufactures high-value, end-of-lease copiers (Xerox Corporation 2001). The used copiers are collected directly by Xerox as new ones are installed. Overall, the green remanufacturing program saves the company 40%–65% in manufacturing costs through the reuse of parts and materials (Ginsburg 2001). Similar activities are undertaken by Hewlett Packard Corporation for computers and peripherals, and by Canon for print and copy cartridges.

Manufacturers of consumer products such as single-use cameras and mobile phones utilize retailers for collecting their used products. For instance, Eastman Kodak Company receives single-use cameras from large retailers that also develop film for customers. On average, 76% of the weight of a disposed
product remanufacturing. Based on observations return rates, we consider a two-echelon supply chain customers (Model M), (2) manufacturer contracting the manufacturer collecting used products directly from the cus-
tomers, (3) manufacturer contracting the collection of used products to a third party (Model 3P). We con-
trast the results for the decentralized channels with the centrally coordinated system (Model C) to illus-
rate potential sources of inefficiencies in closed-loop supply chains.

More specifically, we address the following research questions: (1) How are the wholesale price, the retail price, and the total channel profits affected by the choice of the reverse channel structure? (2) How do the closed-loop supply chain structures influence the incentives to invest in used-product collection and the product return rates?

Some of the key results of this paper demonstrate that in a closed-loop supply chain, the retailer has an important dual role due to his proximity to the market. The analysis shows that when prices are sensitive to changes in unit production costs, by being closer to the final demand, the retailer can efficiently reflect unit cost savings from remanufacturing to the final price of the product, and jointly optimize the investment in used-product collection. The manufacturer is at a disadvantage in coordinating pricing and used-product return rates, because she faces double marginalization in the forward channel. The third-party model is the least-preferred option because the payments made to the third party for undertaking collection become a direct cost to the supply chain, do not induce incentives to increase the final demand and, therefore, reduce the profitability of product remanufacturing. The model also demonstrates that the manufacturer can further improve the profits of the R Model to the level of a centrally coordinated system by using a single two-part tariff. Because we define the two-part tariff contingent on the product return rate, it not only coordinates the pricing decision of the retailer in the forward channel but also the collection effort in the reverse channel.

On a broader level, this paper contributes to our understanding about the interactions between reverse and forward channel decisions, as well as the incentives of the agents to invest in used-product collection in different reverse channel structures.

The rest of this paper is organized as follows. In the following section, we briefly discuss the current literature and the contribution of this paper. Section 3 is devoted to the conceptualization and formulation of the model. Following the development of the model, the analytical results for the optimal closed-loop supply chain structures are presented in §4. Section 5 examines channel coordination mechanisms with used product collection. We outline the limitations of this work and possible directions for future research in §6.

2. Literature

This paper draws on and contributes to several streams of literature, each of which we review below. A growing literature in operations management addresses reverse logistics management issues for remanufacturable products. We refer the reader to Fleischmann et al. (1997) and Guide et al. (2000) for complete literature reviews. The basic underlying assumption in these papers is that the planning of closed-loop supply chain operations, such as network design (Krikke 1998), shop-floor control (Guide et al. 1997), and inventory control (van der Laan 1997) is done by a central decision maker to optimize total system performance. By adapting a game-theoretic approach, we relax the centralized planner assumption and model the independent decision-making process of each supply chain member. Specifically, we examine the interaction between pricing decisions in the forward channel and the incentives to collect used products under different reverse channel structures.

There is a growing number of research papers on reverse logistics that use game theory to model remanufacturing decisions. Majumder and Groenevelt (2001) examine how third-party remanufacturing can induce competitive behavior when the recycled products cannibalize the demand for the original product. Debo et al. (2002) model the durability choice

Throughout this paper, we adopt the convention of using the pronoun "she" to refer to the manufacturer. All other supply chain members are referred to by the pronoun "he."

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of the (re)manufacturer when consumers act strategically. While we specifically model the reverse channel design decisions, in the cited papers, such decisions are assumed exogenous to the model structure.

In the supply chain management literature, Pasternack (1985), Emmons and Gilbert (1998), and Donohue (2000) determine optimal product return contracts for short life-cycle products. The returns considered in these studies occur at the end of the selling season due to demand uncertainty and the retailer’s overstocking of inventory. Related to this group of work, Padmanabhan and Png (1997) explore how ordering flexibility from buy-back contracts affects the retail-level competition. In contrast, we consider used products, which are returned from consumers for remanufacturing, and we discuss contract forms, which would jointly coordinate the reverse and the forward channels.

Recent research also shows a strong link between supply chain management and environmental improvement. Bierma and Waterstraat (2000) and Reiskin et al. (2000) discuss contractual mechanisms to reduce resource use. Corbett and DeCroix (2001) examine shared-savings contracts to overcome incentive conflicts between a supplier and a buyer to reduce the use of indirect materials. This recent stream of work focuses on cost savings derived from reduction of consumption (ex ante), while we consider cost savings derived from product reuse (ex post). In the marketing literature, Stern et al. (1996) describe the role and the function of each channel member in different closed-loop supply chain structures without providing a quantitative basis for comparing these different channel formats. In contrast, in this paper, we try to provide such a comparison between the roles of the different channel parties. The quantitative models of channel design in marketing have only looked at forward distribution systems (Jeuland and Shugan 1983, McGuire and Staelin 1983, Coughlan 1985). While the model structure presented in this paper is consistent with this stream of work, we expand these models to also include reverse product and financial flows from consumers to manufacturers.

Last, but not least, there is also growing research interest in decision models for used-product acquisition when there is quality uncertainty in the return flows. See Guide and Van Wassenhove (2001) for a detailed discussion of this issue. In this paper, we focus on the channel choice decision and assume homogenous quality of returned products.

Next, we present our modeling assumptions and the three closed-loop supply chain models.

3. Model Assumptions and Notation

We use the following notation throughout the paper: \( c_m \) will denote the unit cost of manufacturing a new product, and \( c_r \) the unit cost of remanufacturing a returned product into a new one, \( p \) is the retail price of the product, \( w \) is the unit wholesale price, and \( b \) will denote the unit transfer price of a returned product from the retailer/third party to the manufacturer. \( D(p) \) is the demand for the new product in the market as a function of product price, and \( \Pi_j \) denotes the profit function for channel member \( j \) in supply chain model \( i \). Superscript \( i \) will take values \( C, M, R, \) and \( 3P \), which will denote the centrally coordinated, manufacturer collecting, retailer collecting, and the third-party collecting models, respectively. The subscript \( j \) will take values \( M, R, \) and \( 3P \), which will denote the manufacturer, the retailer, and the third party, respectively.

The primary goal of this paper is to understand the implications of different closed-loop supply chain structures on incentives to invest in used-product collection and on supply chain profits. Hence, we consider the following scenario and make the following modeling assumptions.

Suppose that the manufacturer has incorporated a remanufacturing process for used products into her original production system, so that she can manufacture a new product directly from raw materials, or remanufacture part or whole of a returned unit into a new product.

ASSUMPTION 1. Producing a new product by using a used product is less costly than manufacturing a new one, i.e., \( c_r < c_m \) and \( c_r \) is the same for all remanufactured products.

This assumption states that savings from materials and assembly of subsystems within the new product dominate the additional costs of disassembly, inspection for reusability, and the cost of remanufacturing. Therefore, ceteris paribus, the manufacturer strictly prefers a higher product return rate to a lower product return rate from the market because, through remanufacturing, she can lower her production costs. The last part of Assumption 1 can easily be relaxed by incorporating a yield rate on returned product quality. The yield rate models the uncertainty in the reusability of used products due to different usage patterns.

ASSUMPTION 2. We characterize the reverse channel performance by \( \tau \), the return rate of used products from the customers. \( \tau \) denotes the fraction of current generation products remanufactured from returned units, i.e., \( 0 \leq \tau \leq 1 \).

2 Kodak incorporates considerations such as part reusability, ease of disassembly, and recoverability into the product design process for its single-use camera line. This enables them to easily disassemble returned cameras and, thus, manufacture new ones at lower unit costs by only replacing parts such as the lens and the battery.

3 It can easily be shown that the uncertainty in the returned product quality reduces the incentives to invest in used product collection, because the benefit from the investment will be lower.
\( \tau \leq 1 \). We model \( \tau \) as a function of the product collection effort, which is denoted by \( I \), the investment in collection activities. Such investments can be considered as promotional expenditures undertaken by the collecting agent.

Hence, one can think of \( \tau \) as the response of consumers who have an incentive/enthusiasm for the remanufacturing of their used products as a result of the promotional/advertising activities of the agent in the reverse channel. To characterize the diminishing returns to investment, we use the cost structure \( \tau = \sqrt{I/C_I} \), where \( C_I \) is a scaling parameter. Similar forms of response functions have been widely used in the advertising response models of consumer retention and product awareness (Lilien et al. 1992, Fruchter and Kalish 1997, Zhao 2000), and in salesforce effort response models in the marketing literature (Coughlan 1993). In the operations literature, Porteus (1986) and Fine and Porteus (1989) use similar investment functions to investigate opportunities for process improvement and lot sizing by investing in setup cost reduction and quality improvement. This paper investigates tradeoffs that are similar to those in the above studies in a remanufacturing context.

From Assumptions 1 and 2, the average unit cost of manufacturing can be written as \( c = c_m(1 - \tau) + c_r \tau \). Note that when every consumer returns his or her used product (i.e., \( \tau = 1 \)), \( c = c_r \). If the return rate of used products is zero, then all demand will be satisfied from manufactured units and, therefore, \( c = c_m \). If we denote unit cost savings from reuse by \( \Delta \) (i.e., \( \Delta = c_m - c_r \)), the average unit cost is given by \( c_m - \tau \Delta \). Similar cost structures can also be found in the literature, which looks at process improvement decisions and cost reduction incentives in supply chains (Gupta and Loulou 1998, Gilbert and Cvsan 2000). The similarity in formulation is consistent because, in this paper, an increase in product remanufacturing would also mean a reduction in the average unit cost of manufacturing.

Assumption 3. There is a variable unit cost of collecting and handling a returned unit, which is denoted by \( A \). For remanufacturing to be economically viable, we assume that \( A < \Delta \), i.e., the fixed payment per unit is less than the savings generated per unit from remanufacturing.

One can think of \( A \) as the fixed payment given to the consumer who returns a used product. Consider the reverse vending machines that collect used products (e.g., soft drink cans). Each customer who returns a used product to these machines receives a fixed payment per unit, which is represented by the parameter \( A \) in our model. Note that \( A \) is exogenous to the model and does not affect the demand for the product. This would be the case for consumer products (i.e., cameras, cartridges) with low salvage value and no secondary markets (Swan 1972). In the case of durable goods, one should also consider how the secondary market/trade-in value of a used product affects the consumer’s decision to replace his or her used product. Assumption 3 also implies that \( A \) does not increase with the scale of operations. This is consistent with supply chain structures, where the reverse channel has sufficient capacity so that the product return rate is driven by the collection effort and by induced awareness and participation of consumers in product remanufacturing.

The total cost of collection \( C(\tau) \) can then be characterized as a function of the return rate of used products and is given by \( C(\tau) = I + ArD(p) = C_I + \Delta^2 + ArD(p) \), where \( \Delta D(p) \) is the total number of units returned from consumers and remanufactured into new products. Because we intend to compare the incentives of collecting agents across different reverse channel structures, we assume the same cost structure for all closed-loop supply chain models. In the conclusion section, we discuss other factors that may bring in collection cost advantages to some reverse channel structures.

Assumption 4. We consider a two-echelon supply chain and model a bilateral monopoly between a single manufacturer and a single retailer. This enables us to explore the implications of assigning a dual role to a forward channel member. Specifically, if the retailer undertakes the collection effort, he not only determines the quantity demanded in the market by setting the retail price of the product, but also by his collection effort level, he influences the average manufacturing cost of the product. Even though we consider a single manufacturer-retailer structure, the manufacturer can, in fact, sell to different retailers if the retailers are not in direct competition. The manufacturer can also manufacture competing brands, but she does not sell competing brands to the same retailer and the brands do not share the same manufacturing process. The retailer can also carry many brands, but for simplicity, we consider only one brand for which he takes decisions independent of the other existing brands.

Assumption 5. \( D(p) = \phi - \beta p \), with \( \phi \) and \( \beta \) being positive parameters and \( \phi > \beta c_m \).

We assume a downward sloping linear demand function. Lee and Staelin (1997) show that the vertical interaction between the channel members and the optimality of the channel strategies depend on
the convexity of the demand functions. Therefore, it should be pointed out that while the linear demand assumption is consistent with the literature (Bulow 1982, Weng 1995) and enables us to develop a first-cut analysis of the closed-loop supply chain decision of the manufacturer, the generalizability of the results to nonlinear demand functions is a question of future research. The last part of Assumption 5 is a condition for nonnegative demand.

To model the supply chain members’ incentives under different closed-loop supply chain structures, we use the principal-agent framework (Laffont and Tirole 1993) and make the following assumption concerning the power structure in the supply chain.

**Assumption 6.** In all our supply chain models with remanufacturing, the manufacturer has sufficient channel power over the retailer and the third party to act as a Stackelberg leader.

This assumption states that in determining the outcome of the game played between the manufacturer and the retailer, the manufacturer uses her foresight about the retailer’s and the third party’s reaction functions in her decision making. The Stackelberg structure for the solution of similar games has been widely used in the supply chain literature (Tayur et al. 1998).

**Assumption 7.** While optimizing their objective functions, all supply chain members have access to the same information.

This assumption enables us to control for inefficiencies and risk-sharing issues resulting from information asymmetry (Corbett and De Groote 2000). Savaskan et al. (1999) examine the closed-loop supply chain performance when the manufacturer faces an adverse selection problem.

**Assumption 8.** The closed-loop supply chain decisions are considered in a single-period setting.

We assume the previous existence of the product in the market. Those products sold in the previous periods can be returned to the manufacturer for reuse. Hence, the focus of analysis is on the average supply chain profits per period when similar products are introduced to the market repeatedly.

### 4. Supply Chain Models with Remanufacturing

This section presents three decentralized supply chain models with remanufacturing, viz., the closed-loop supply chain with the manufacturer collecting used products (Model M, Figure 1b), the closed-loop supply chain with the retailer collecting used products (Model R, Figure 1c), and the closed-loop supply chain with third party collecting used products (Model 3P, Figure 1d). As a benchmark case, the Centrally Coordinated System (Model C, Figure 1a) is analyzed to highlight inefficiencies resulting from decentralization of decision making, and is later used for deriving the channel coordinating pricing scheme.

We compare the models with respect to the wholesale price, the retail price, the product return rate, and the total supply chain profits. The profit functions of the supply chain members are shown to be concave in the decision variables, so the first-order conditions are used throughout to characterize the optimality of the decision variables. The single manufacturer-single retailer bilateral monopoly model without product remanufacturing is well known in the literature (Jeuland and Shugan 1983) and has been extensively analyzed. For comparison, we will refer to the results of this model in some parts of this paper.

#### 4.1. Model C—Centrally Coordinated System

The centrally coordinated system (Model C) provides a benchmark scenario to compare the decentralized models with respect to the supply chain profits and the reverse channel performance. Because there is a single decision maker, the wholesale price \( w \) and the transfer price \( b \) are irrelevant to the formulation of the objective function. Hence, the central planner optimizes

\[
\begin{align*}
\max_{\hat{p}_c, \hat{\tau}} & \quad \Pi_C = (\phi - \beta p) [p - \epsilon_m + \tau \Delta] - C_c \tau^2 - A \tau (\phi - \beta p). \\
\end{align*}
\]

\( (1) \)

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6 We conjecture that the results of the model would hold for all demand patterns with nonpositive elasticities with respect to price.

7 Kodak introduces new models of disposable cameras, which can incorporate components from previous generations.

8 We conjecture that the results of the model would hold for all demand patterns with nonpositive elasticities with respect to price.

9 For the clarity of the text, all proofs are provided in the appendix.

9 The results of the bilateral monopoly model without remanufacturing are given in Appendix A.
The simultaneous solution of the first-order conditions results in

\[ p^* = \frac{\frac{\phi + \beta c_m}{2\beta} - \frac{1}{2}(\Delta - A)^2}{\frac{\phi - \beta c_m}{4C_L - \beta(\Delta - A)^2}} \]

and

\[ \tau^* = \frac{(\phi - \beta c_m)(\Delta - A)}{4C_L - \beta(\Delta - A)^2}. \]

To ensure comparison of the interior point solutions to all four models, we impose the condition of \( \dot{a}r^c(\tau)/\dot{\tau} |_{\tau = 0} < 0 \) on \( \tau \) (see Lemma 1 in Appendix A). From this condition follows Assumption 9.

**Assumption 9.** The parameter \( C_L \) defined in the collection cost function is assumed to be sufficiently large, such that \( \tau^c < 1 \). More specifically, \( 4C_L > [(\phi - \beta c_m) + \beta(\Delta - A)](\Delta - A) \).

Assumption 9 ensures that remanufacturing is sufficiently costly so that it is not economically viable to manufacture all units from used products (similar assumptions have been made in Gupta and Loulou 1998 and Gilbert and Cvsa 2000). Note that the right-hand side is the maximum savings contribution of remanufacturing if all the products were remanufactured, i.e., \( \tau^c = 1 \).

From the best-response functions, it follows that the retail price charged is lower than the retail price in the centrally coordinated system without remanufacturing (see Table 1 in Appendix A). Part of the system profit gains from reduced unit variable costs are passed on to the customers as a lower retail price, which also enhances the demand of the product. The demand and total profits in the coordinated distribution and collection system can be found by evaluating \( D(p) \) and \( \Pi^{HC}(p, \tau) \) at \( p^* \) and \( \tau^* \). The results are shown in Table 1 in Appendix A.

### 4.2. Model M—Manufacturer Collecting

In this model, the manufacturer undertakes the used product collection effort, and decides on the wholesale price \( w \) and the product return rate, \( \tau \). An example of this form of closed-loop supply chain structure is Xerox’s network for printer and copy cartridges in Europe and in the United States. Specifically, Xerox provides prepaid mailboxes so that the used cartridges can be returned to Xerox at no expense to the customers.

Because the manufacturer is the Stackelberg leader, we begin by characterizing the best-response function of the retailer. For a given \( w \), the retailer’s problem is

\[ \max_{p, \tau} \Pi^{RM} = (p - w)(4 - \beta p) + \beta r(4 - \beta p) - \frac{\phi - \beta c_m}{2}[w - c_m + \tau \Delta] - C_L \tau^2 - A\tau \frac{\phi - \beta w}{2}. \]

Again, the objective function is jointly concave in \( w \) and \( \tau \), and the manufacturer’s first-order conditions characterize the unique best response,

\[ w^{*M} = \frac{\phi + \beta c_m - (\Delta - A)^2(\phi - \beta c_m)}{2\beta} \]

and

\[ \tau^{*M} = \frac{(\phi - \beta c_m)(\Delta - A)}{8C_L - \beta(\Delta - A)^2}. \]

Optimal retail price and equilibrium channel profits can easily be found by substitution of \( w^{*M}, \tau^{*M} \). The results are listed in Table 2 in Appendix A.

### 4.3. Model R—Retailer Collecting

In this model, the retailer also engages in the promotion and collection of used products in addition to distributing new products. One characteristic of this channel format is that the ownership of used products initially rests with the retailer after the collection. To take the products back, the manufacturer pays a transfer price \( b \) per product returned to her from the retailer. The transfer price, \( b \) is determined by the manufacturer. As an example of this closed-loop supply chain structure, Kodak currently engages retailers that sell their products to participate in the collection activity of the disposed cameras. Similarly, for electronics products, such as television sets, home appliances, personal computers, retailers also act as collection points when used products are returned to them at the moment of new sales. In this model, the retailer decides the retail price \( p \), and the product return rate \( \tau \), through his collection effort.

The retailer’s problem is given by

\[ \max_{p, \tau} \Pi^{RC} = (p - w)(4 - \beta p) + \beta r(4 - \beta p) - \frac{\phi - \beta c_m}{2}[w - c_m + \tau \Delta] - C_L \tau^2 - A\tau \frac{\phi - \beta w}{2}. \]

Because the objective function is jointly concave in \( p \) and \( \tau \), from the first-order conditions, the best responses are

\[ p^{*R} = \frac{\phi + \beta w}{2} \]

and

\[ \tau^{*R} = \frac{(b - A)/(2C_L)(\phi - \beta p^{*R})}. \]

Given \( p^{*R} \) and \( \tau^{*R} \), the manufacturer optimizes

\[ \max \Pi^{RM} = (\phi - \beta p^{*R})(w - c_m + \Delta \tau^{*R}) - b \tau^{*R}(\phi - \beta p^{*R}). \]

From the concavity of the objective function in \( w \), it follows that for a given \( b \),

\[ w^{*R} = \frac{\phi + \beta c_m}{2\beta} - \frac{(\Delta - b)(b - A)(\phi - \beta c_m)}{2[4C_L - \beta(\Delta - A)(b - A)]}. \]
The optimal value of the wholesale price can then be used to compute the demand and the profits for the manufacturer. The manufacturer’s profits are given by

\[ \frac{(\phi - \beta c_m)^2/(8\beta)}{1 - \beta(\Delta - A)(b - A)/(4C_1)}. \]

Note that because the manufacturer’s problem is concave in \( w \) for a given \( b \), this enables us to find the value of \( b \), which maximizes the optimal value of the manufacturer’s profit function.\(^{10}\) Thus, we make the following observation.\(^{11}\)

**Observation 1.** Because the manufacturer’s profits are increasing with respect to \( b \), the transfer price \( b \) is set equal to its upper bound of \( \Delta \), i.e., \( b = \Delta \).

Surprisingly, in the R Model, we find that the manufacturer does not extract any of the direct savings from remanufacturing, but prefers to pass them on to the retailer. There are two main driving forces to this seemingly counterintuitive result.

When the retailer undertakes the used-product collection activity, a unit increase in demand increases his profits in two distinct ways. First, an additional unit of demand contributes the margin (i.e., \( p - w \)) from the purchase of the new product. Second, it partially contributes the salvage value (i.e., \( b - A \)) of a used product. In other words, in the R Model, the retailer has a higher marginal profitability from a unit increase in demand than the cases when the manufacturer or the third party are the collecting agents. Hence, for a given level of \( \tau \), passing on all the savings to the retailer results in increased payoffs for the retailer (i.e., \( (A - A)\tau = (b - A)\tau \)), and this acts as an incentive to reduce the retail price of the product and increase demand, which results in increased profits. A second-degree effect of increasing demand also acts to set \( \tau \). A larger market size makes used-product collection more profitable for the retailer (i.e., scale effect) and, hence, the retailer has an incentive to increase his investment in used-product collection, and increase \( \tau \).

The manufacturer prefers to transfer cost savings directly to the retailer for the following reasons. Even though she does not internalize the unit cost savings from remanufacturing directly, her profits increase because of an increase in the product demand. Note that if the manufacturer had retained part of the unit cost savings (i.e., \( \tau(\Delta - b) > 0 \)), because of double marginalization in the channel, a part of these savings would be internalized in the retailer’s profit margin later when the retailer set the product price. As a consequence, the demand and the investment in used-product collection would be lower.

The analysis of the R Model shows that the way the cost savings from remanufacturing are shared between supply chain members has important implications for pricing decisions in the forward supply chain, both for the manufacturer and the retailer.

### 4.4. Model 3P—Third-Party Collecting

It is also not unusual to see the used-product collection activity contracted by the manufacturer to a third party, who is engaged only in the collection of the used products from the market. The Internet has been an essential enabler for such third-party managed product return channels (Hickey 2001). In these closed-loop supply chains, the third party acts as a broker between the customer and the manufacturer.

In the 3P Model, for a given transfer price \( b \) of a used product, the third party maximizes his profits to determine the investment in used-product collection and, hence, decides the value of \( \tau \). As in Model M, the retailer engages only in the distribution of the product. The retailer solves for \( p*3P \) as a function of the wholesale price set by the manufacturer. The third party assumes the collection activity and maximizes \( \Pi_3^P = b\tau(\phi - \beta p*3P) - C_t \tau^2 - A\tau(\phi - \beta p*3P) \). From the concavity of the objective function in \( \tau \), the optimal value of the product return rate \( \tau*3P \) is given by

\[ ((b - A)/(2C_t))(\phi - \beta p*3P). \]

Given \( p*3P \) and \( \tau*3P \), the manufacturer solves

\[ \max_w \Pi_M^P = (\phi - \beta p*3P)[w - c_m + (b - b)\tau*3P]. \]

From the first-order conditions, the manufacturer sets the wholesale price of the product to

\[ w*3P = \frac{\phi + \beta c_m}{2\beta} - \frac{\phi - \beta c_m}{2\beta} \left[ \frac{b(\Delta - b)/(4C_t)}{1 - \beta(b - A)(\Delta - b)/(4C_t)} \right], \]

from which the optimal retail price and profits for the three parties are calculated. These results are tabulated in Table 2 in Appendix A.

Substituting the value of \( w*3P \) back in \( \Pi_M^P \), the manufacturer’s profits are given by

\[ \frac{(\phi - \beta c_m)^2/(8\beta)}{1 - \beta(\Delta - b)(b - A)/(4C_t)}. \]

We make the following observation regarding the optimal \( b \) value in the 3P Model.

**Observation 2.** The manufacturer’s profit function is maximized when \( b = (\Delta + A)/2 \).

Note that in the 3P Model, the incentive of the third party to invest in the used-product collection effort is directly driven by \( b \), the transfer price. Hence, the manufacturer faces the following tradeoff. If she...
assigns a large value to b, she observes a high level of investment by the third party in used-product collection, but at the same time, her net savings from remanufacturing diminish, i.e., \( \Delta - b \) decreases and, therefore, her profits decrease as b approaches \( \Delta \). We find that the direct and indirect effects of b on the manufacturer’s profits balance when \( b = (\Delta + A)/2 \), i.e., when both parties equally share the gains from remanufacturing.

From Observations 1 and 2, it is important to note that the transfer price b has completely different implications for the supply chain profits in the R and the 3P Models. In the R Model, through b, the unit cost savings from remanufacturing are directly reflected in the final demand of the product (i.e., the retailer sets a lower price for the product), whereas in the 3P Model, b is a direct cost (i.e., a reduction in unit cost savings) for the manufacturer and the supply chain. Hence, the choice of the manufacturer concerning with whom and how she shares the cost savings from product remanufacturing has important consequences for forward channel pricing decisions.

In the following section, we compare the three supply chain models with respect to the retail price, the product return rate, and the profits of the channel members.

5. Comparison of the Three Closed-Loop Supply Chain Models

Based on the results summarized in Tables 1 and 2, some interesting observations can be made on the performance of decentralized closed-loop supply chain structures.

**Proposition 1.** The optimal product return rates are related as \( \tau^C > \tau^R > \tau^M > \tau^3P \).

Note that while the total savings from remanufacturing in the third-party model are given by \( (b - A)\tau D(\rho(w)) \), the total savings are given by \( (\Delta - A)\tau D(\rho(w)) \) in the manufacturer collecting model. From Observation 2, we see that the marginal benefit of investing in increasing \( \tau \) in the 3P Model is less than the marginal gains in the M Model (i.e., \( (b^3P - A) < (\Delta - A) \)). Hence, the third party invests less in used-product collection compared to the manufacturer in the manufacturer collecting system. In addition, the manufacturer can strategically set \( w \) in a way that would make used-product collection more profitable resulting in a second-degree effect on \( \tau \).

Comparing the closed-loop supply chain models where the manufacturer versus the retailer collects, from Observation 1, we see that while both the manufacturer and the retailer face the same marginal gains from investing in increasing \( \tau \) (i.e., \( b^R - A = \Delta - A \)), the retailer can impact the market size by choosing \( p \), whereas the manufacturer can influence the demand only by strategically choosing the wholesale price, w. The M Model has a lower product return rate than the R Model, because of double marginalization, the unit cost savings from remanufacturing is only partially reflected in the final price of the product.

The centrally coordinated system leads to the highest investment level in used-product collection, because the decisions are fully coordinated in the channel.

The implications of Proposition 1 form the basis of an interesting finding of the paper, vis-à-vis, the closer an agent is to the market, the more efficient is the collection of used products for all parties involved in the channel. The effective loss of efficiency in the decentralized system is mitigated, in part, by the ability to act more closely at influencing the underlying demand.

**Proposition 2.** The retail prices in the coordinated channel and the three cases in the decentralized channel are related as follows: \( p^C < p^R < p^M < p^3P \). Consequently, \( D^C > D^R > D^M > D^3P \).

The investment in collecting used products from the market benefits only the third party directly in the 3P system, and there is only a second-order effect on the retail price in the form of a lower wholesale price offered by the manufacturer to the retailer. The effect on the retail price is more direct in the M Model, as the manufacturer sets a lower wholesale price to increase demand, and thereby increases her savings through product remanufacturing. The reduction in retail price in the R Model is the largest among the decentralized channels, as the retailer can directly reflect the unit cost savings in the final demand through his pricing decision. The price in the coordinated channel is lower than all three decentralized channels, because the gains in efficiency from the coordination effort can be effectively shared with the market to increase both demand and profits.

**Proposition 3.** The manufacturer’s and retailer’s profits in the decentralized channel are related as follows: \( \Pi^C > \Pi^R > \Pi^M > \Pi^3P \). The total profits in the coordinated channel with recovery always dominate the total profits in the decentralized channel with recovery. Specifically, \( \Pi^C > \Pi^R > \Pi^M > \Pi^3P \).

The implication of Proposition 3 and this section is that the ranking of the different closed-loop supply chain structures (in terms of benefits to the manufacturer and retailer) mirrors the benefits to nontupply chain members as well. The benefits to society, in terms of an increased used-product return rate (greater reuse of products) as well as an increased
ability to buy the product (greater demand), complement the increased profits for the manufacturer and retailer in the coordinated system and R Model. McCartney (1999) and our own observations in the industry empirically corroborate the findings of the model. When the manufacturer owns the distribution channel, as in the coordinated case, the manufacturer undertakes the used-product collection effort herself, as in the case of Xerox (Xerox Corporation 2001).

In the next section, we show that the manufacturer can further improve the profits in the R Model by coordinating the forward and the reverse channel decisions via a single two-part tariff. To highlight incentive issues, we consider an environment where all cost parameters are common knowledge to the channel members (i.e., no adverse selection problem) and the used-product collection effort is fully observable (i.e., no moral hazard problem).

6. Improvements on the R Model

Under the assumption of complete information about cost and demand data and full observability of the retailer’s cost when he undertakes the used-product collection effort, we show that the manufacturer can offer a wholesale price to the retailer to induce him to choose an effort level that maximizes total channel profits. The details of the proof are presented in Appendix C. Proposition 4 states one form of the optimal contract that the manufacturer offers the retailer. The optimal coordinating wholesale price is linear in the unit cost of manufacturing, $c_m$ and $\tau$. $w(\tau)$ stands for a wholesale pricing scheme contingent on the return rate of used products from the market, and $F$ is a fixed payment made by the retailer to the manufacturer that distributes the efficiency gains. $\Pi^C$ is the total profits in the coordinated channel, and $\Pi^R$ is the retailer’s profits in the decentralized channel with the retailer collecting.

**Proposition 4.** The form of the optimal contract, $W'(\tau) = (w'(\tau), F')$, which ensures that the retailer undertakes the coordinated collection effort level and charges the optimal coordinated retail price is given by $w'(\tau) = c_m - (\Delta - b)\tau$, $0 < \tau < 1$, and $F' = \Pi^C - \Pi^R$.

As stated in the above proposition, to ensure that the retailer’s profit-maximizing product return rate is, indeed, equal to the coordinated channel product return rate, the manufacturer offers a wholesale pricing scheme contingent on the product return rate characterized by the term $(\Delta - b)\tau$. Specifically, the manufacturer puts the retailer in a position where he directly internalizes the cost consequence of his used-product collection effort. Besides ensuring the coordinated channel product return rate, the contract also provides a means for making the retail price of the product equal to the coordinated channel price level, thereby overcoming inefficiencies because of double marginalization. The optimal contract makes the transfer price $b$ unnecessary, as the manufacturer’s profits are the same for any $b \leq \Delta$.

Our findings are consistent with the extant literature in marketing and economics, which shows that if the manufacturer transfers the products at the realized unit cost of production, coordinated channel retail price and profit levels can be attained in the decentralized setting. The fixed payment, $F$ can be seen as a franchisee fee paid by the retailer to the manufacturer to have the rights not only to sell the product in the market, but also to collect the used units. It is interesting to note that by using a single contractual agreement (i.e., a simple two-part tariff, linear in $\tau$), the manufacturer can coordinate the decisions in the closed-loop supply chain. Simple forms of contractual agreements for channel coordination also make the R Model more attractive for the manufacturer compared to a third-party system, for which one would need to design separate contractual agreements for the retailer and for the third party (Savaskan et al. 1999).

7. Contributions, Limitations, and Future Research

Designing effective remanufacturing systems has important ramifications for firms, regulatory bodies, and the market. The first contribution of this paper has been to identify the appropriate closed-loop supply chain structure for OEMs. We show that firms can design closed-loop supply chains to enhance their profits and market demand, and in conjunction, can increase the used-product return rate. Coordination mechanisms are suggested to better achieve the above aims by providing suitable incentives in the form of simple two-part tariffs (a per unit wholesale price linear in product return rate coupled with a franchisee fee) to align the objectives of the members in the closed-loop supply chain.

In the early phase of this research, we have made a number of assumptions that must be relaxed in future research to develop a more comprehensive understanding of remanufacturing systems. We assume that, for the agent who implements the used-product collection effort, an infrastructure for the logistics of getting products back from customers and delivering them to the OEM already exists independently. Based on the model analysis and insights, we conjecture that a high cost of establishing such a network would make it even more appropriate for a forward distribution channel member (e.g., the retailer) to undertake the used-product collection effort, as such networks are already in place in typical systems for forward distribution activities. This paper considered only a single agent undertaking the collection
process. If there are multiple agents (e.g., retailers) undertaking the collection process in separate markets (such as geographically dispersed markets), the results of the model are not affected. Savaskan and Van Wassenhove (2001) study the impact of product collection on retail competition. The location of the agents vis-a-vis the consumers and proximity to the market was not considered in this research. If the used-product collection is done through a fixed-price delivery system (such as prepaid envelopes in the regular mail), then the results of the model are not affected. We also did not model the multiperiod game, an extension of this study could find the impact of possible nonstationary material flows on the channel choice decision.

We also assume that the firm has designed the product platform for reuse so that older products can be easily incorporated partially or wholly into the new product (Xerox Corporation 2001). An extension of this study should consider product platform design issues to make products easier to remanufacturing process. This research also assumed that all the products that are collected are usable for remanufacturing into new products. Future research should investigate the effect of the degree of product durability in the remanufacturing process.

We assume that the marginal cost of collecting a used unit is independent of the scale of the operations. This is consistent with supply chain structures where there is already an existing reverse logistics network in place with sufficient capacity. There are two interesting avenues of research concerning costs of the collection effort. First, one can examine the choice of closed-loop supply chain when the marginal cost of collection increases with the scale of operations.12 Second, one can also investigate capacity investment decisions in the closed-loop supply chain. Third, we assumed that the cost of collecting a used unit is the same for all the agents of collection. An extension of this research could consider the case of unequal costs of collection because of economies of scale (we conjecture that this would favor third-party collection), proximity to the consumer (this would favor retailer collection), and other possible reasons. The modeling framework in this paper can easily be extended to address such related questions.

In summary, this paper makes a contribution to the literature on distribution channel design by drawing attention to closed-loop supply chains, and developing a model of the tradeoffs underlying such systems. Our recommendation is that firms make a conscious choice of supply chain, as different closed-loop supply chains may be appropriate in different environments, depending on the cost structures of the agents of collection. Section 5, which summarizes the results of the model, provides some guidance in this regard. A combined design of the closed-loop supply chain can not only provide the firm with much-needed flexibility to reduce logistics costs for forward and reverse activities, but also enable it to signal continued concern and action on environmental issues.

Appendix A

**Lemma 1.** If the condition of $\frac{g_1}{p}|_{\tau = 0} < 0$ is satisfied, then $\tau^{C}, \tau^{M}, \tau^{S}, \tau^{C}$ are all between 0 and 1.

**Proof.** Consider Model C—Centrally Coordinated Model.

For $\Pi^{C}$ to be concave in $p$ and $\tau$, $\Pi^{C}$ should satisfy conditions (i) $\frac{\partial^2 \Pi^{C}}{\partial \tau^2} < 0$, (ii) $\frac{\partial^2 \Pi^{C}}{\partial p^2} < 0$, and (iii) $\frac{\partial^2 \Pi^{C}}{\partial \tau \partial p} > 0$. Note that (i) holds since $\frac{\partial^2 \Pi^{C}}{\partial \tau^2} = -2C_{c}$ and $C_{c} > 0$. Likewise, (ii) holds since $\frac{\partial^2 \Pi^{C}}{\partial p^2} = -2\beta < 0$ and $\beta > 0$. To have an interior point solution for $\tau^{C}$ (i.e., $\tau^{C} < 1$), we impose $\frac{\partial \Pi^{C}}{\partial \tau}|_{\tau = 0} < 0$. This condition can be written as $4C_{c} \beta > (\phi - \beta c_{n}) \cdot (\Delta - \Delta) + \beta (\Delta - \Delta)^2$. For condition (iii) to hold, we need $(-2C_{c})(-2\beta) > \beta^2(\Delta - \Delta)^2$, which follows from the previous condition. Note that from condition (iii) and $\Delta > \Delta$, $\tau^{C} > 0$ follows automatically (see Table 1).

Consider Model M—Manufacturer Collecting Model.

$\Pi^{M}_{f}$ is concave in $p$ since $\frac{\partial^2 \Pi^{M}_{f}}{\partial p^2} = -2\beta < 0$ and $\beta > 0$. $\Pi^{M}_{c}$ is concave in $w$ and $\tau$ since $\frac{\partial^2 \Pi^{M}_{c}}{\partial w^2} = -\beta < 0$, $\frac{\partial^2 \Pi^{M}_{c}}{\partial \tau^2} = -2C_{c} < 0$ and $\frac{\partial^2 \Pi^{M}_{c}}{\partial w \partial \tau} = 2C_{c} > (\beta^2/4)(\Delta - \Delta)^2$. Note that the last inequality holds from the condition $\frac{\partial \Pi^{C}}{\partial \tau}|_{\tau = 0} < 0$ of Model C, and it ensures $\tau^{M} > 0$ (see Table 2).

From Tables 1 and 2, we have $\tau^{M} = (\phi - \beta c_{n})(\Delta - \Delta) \cdot \frac{8C_{c} - \beta (\Delta - \Delta)^2}{(8C_{c} - \beta (\Delta - \Delta)^2)^{-1} < \tau^{C}}$, hence, $\tau^{M} < 1$.

<table>
<thead>
<tr>
<th>Appendix Table 1</th>
<th>Comparison of Coordinated Supply Chain Models with or Without Remanufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel decision and profits</td>
<td>Decentralized supply chain without remanufacturing</td>
</tr>
<tr>
<td>$\Pi_{f}$</td>
<td>$3(\phi - \beta c_{n})^2/16\beta$</td>
</tr>
<tr>
<td>$\rho^{*}$</td>
<td>$3\phi + \beta c_{n}/4\beta$</td>
</tr>
<tr>
<td>$\tau^{*}$</td>
<td>N/A</td>
</tr>
<tr>
<td>$w^{*}$</td>
<td>$\phi + \beta c_{n}/2\beta$</td>
</tr>
<tr>
<td>$\Pi^{M}_{f}$</td>
<td>$(\phi - \beta c_{n})^2/8\beta$</td>
</tr>
<tr>
<td>$\Pi^{M}_{c}$</td>
<td>$(\phi - \beta c_{n})^2/16\beta$</td>
</tr>
</tbody>
</table>

12 One way to model this aspect is to assume a total cost function, which is convex (e.g., quadratic) in the number of returned products.
Table 2  Comparison of Decentralized Supply Chain Models with Remanufacturing

<table>
<thead>
<tr>
<th>Channel decision and profits</th>
<th>R Model</th>
<th>M Model</th>
<th>3P Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_r$</td>
<td>$(3/4)(\phi - \beta c_m)^2/((4\beta) [3/4 - (\Delta - A)^2/(16C)] )$</td>
<td>$[1 - \beta/(\Delta - A)^2/(2C)]^2$</td>
<td>$[1 - \beta/(\Delta - A)^2/(16C)]^2$</td>
</tr>
<tr>
<td>$\rho^*$</td>
<td>$3\phi + \beta c_m$</td>
<td>$(\phi - \beta c_m)(\Delta - A)^2/(<a href="16C">4\beta - \beta/(\Delta - A)^2</a>)] )$</td>
<td>$[1 - \beta/(\Delta - A)^2/(16C)]^2$</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>$(\phi - \beta c_m)/(\Delta - A)^2/(8C) + (\phi - \beta c_m)/(\Delta - A)^2/(16C)] )$</td>
<td>$[1 - \beta/(\Delta - A)^2/(8C)]^2$</td>
<td>$[1 - \beta/(\Delta - A)^2/(16C)]^2$</td>
</tr>
<tr>
<td>$\psi^*$</td>
<td>$3\phi + \beta c_m$</td>
<td>$(\phi - \beta c_m)/(\Delta - A)^2/(16C)] )$</td>
<td>$[1 - \beta/(\Delta - A)^2/(16C)]^2$</td>
</tr>
</tbody>
</table>

To show that the manufacturer’s profits are concave in $w$ for a given $b$, we examine the sign of $\partial^2 \Pi_r / \partial w^2$. It follows that

$$\partial^2 \Pi_r / \partial w^2 = \frac{2C_l}{4C_l - \beta(b - A)(\Delta - b)} - 2 + \frac{4C_l \beta(b - A)(\Delta - b)}{4C_l - \beta(b - A)^2)^2}. $$

To show $\partial^2 \Pi_r / \partial w^2 < 0$ for a given $b$, one needs to show that $4C_l \beta(b - A)(\Delta - b) < 4C_l - \beta(b - A)^2$, which reduces to $\beta(b - A)(\Delta - A) < 4C_l$. Because $4C_l > \beta(b - A)^2$ holds from the interior point restriction on $\tau^*$ in Model C (Lemma 1) and $b < \Delta$, it follows that $\beta(b - A)(\Delta - A) < 4C_l$ holds. Hence, $\Pi_r$ is concave in $w$ for a given $b$.

In the second part of the proof, we examine the effect of $b$ on the profits of the manufacturer. Hence, the proof of Observation 1 for the R system follows from the fact that $\partial^2 \Pi_r / \partial b^2$ and $\partial^2 \Pi_r / \partial w^2 \geq 0$. To show these statements, note that

$$\partial^2 \Pi_r / \partial b^2 = \frac{(\phi - \beta c_m)^2}{32C_l} \frac{\Delta - A}{(1 - \beta/(\Delta - A)(b - A)/(4C)]^2}, $$

which is always positive. From the value of $\Pi_r$, after some simplification,

$$\partial^2 \Pi_r / \partial b^2 = \frac{(\phi - \beta c_m)^2}{32C_l} \frac{\Delta - b}{(1 - \beta/(\Delta - A)(b - A)/(4C)]^2}, $$

which is always nonnegative from the assumption that $\Delta > b$ and $\beta(b - A)(\Delta - A)/(4C) \leq \beta(\Delta - A)^2/(4C) < 1$, which follows from our Assumption 9.

Proof of Observation. We follow a similar procedure for the proof of the optimal $b$ value in the 3P system. First, we show that the manufacturer’s profits are concave in $w$ for a given $b$ and then, we optimize over $b$. In the 3P system,
the retailer and the third party solve the following problems, respectively, for \( p^*(w) \) and \( *^r(b) \): \( \max_p (p - w)(\phi - \beta p) \) and \( \max_b \beta(b - \beta p) - C_m^2 - \frac{\alpha}{\beta}(b - \beta p) \).

The manufacturer takes into account the reaction functions of the retailer and the third party and solves the following optimization problem for \( w^* \):

\[
\max_w \left( \phi - \beta w \right) \left[ w - c_m + \frac{(A - b)(b - A)(\phi - \beta w)}{4C_m} \right]
\]

To show that the manufacturer's profit is concave in \( w \) for a given \( b \), we examine the sign of \( \frac{\partial^2 \Pi^M}{\partial w^2} \). Note that \( \frac{\partial^2 \Pi^M}{\partial w^2} = -\beta[1 - \beta(b - A)(\Delta - b)/(4C_m)] \). By rearranging the terms, it follows that the concavity condition \( \frac{\partial^2 \Pi^M}{\partial w^2} < 0 \) reduces to \( 4C_m \beta > (\beta - \Delta - b) \). From the interior point restriction on \( *^r \) in Model C (Lemma 1), this holds true. Because \( \beta(b - A)(\Delta - b) \) is equivalent to \( (\Delta - b)(\beta - A)^2 \), which is less than \( (\beta - A)^2 \), the concavity condition \( 4C_m > (\beta - A)^2 \) holds also for a given \( b \).

Next, we solve for the optimal \( b^* \) value, which maximizes the manufacturer's profits. Note that the optimization of

\[
\Pi^M_p = \frac{(\phi - \beta c_m)^2}{1 - \beta(b - A)(\Delta - b)/(4C_m)}
\]

w.r.t. \( b \) is equivalent to the minimization of the expression \( 1 - \beta(b - A)(\Delta - b)/(4C_m) \), and this expression is minimized when \( \beta(b - A)(\Delta - b) \) is maximized. It can easily be shown that \( b^* = (\Delta + A)/2 \).

**Proof of Proposition 1.** The proof of Proposition 1 can be trivially observed from the values of \( r^* \) (the product return rate) for the \( C, R, M, \) and \( 3P \) channels in Table 2.

**Proof of Proposition 2.** We divide the proof into three parts:

(i) To prove \( p^c < p^8 \), we have to show that

\[
\phi + \beta c_m - \frac{(\phi - \beta c_m)(\Delta - A)^2}{2\beta} < \left( \frac{1}{2}(4C_m - (\beta - A)^2)\right) \beta \left[ 4C_m - (\beta - A)^2 \right]
\]

After simplification, this reduces to showing that

\[
\phi + \beta c_m < \left( \frac{1}{2}(4C_m - (\beta - A)^2)\right) \beta \left[ 4C_m - (\beta - A)^2 \right]
\]

or \( \phi > \beta c_m \) which is true by Assumption 4 (nonnegative demand).

(ii) To prove \( p^8 < p^m \), we have to show that

\[
\frac{3C_m - (\beta - A)^2}{\beta} \phi + \beta c_m
\]

or

\[
\frac{3\phi + \beta c_m}{4\beta} - \frac{(\phi - \beta c_m)(\Delta - A)^2}{16C_m}
\]

or

\[
\frac{(\phi - \beta c_m)(\Delta - A)^2}{4(3\phi + \beta c_m)(4C_m - (\beta - A)^2)}
\]

\[
\left\{ (3\phi + \beta c_m)(4C_m - (\beta - A)^2) \right\}
\]

\[
- 3C_m - (\beta - A)^2 \phi + 4BC c_m
\]

\[
\left( 4\beta [4C_m - (\beta - A)^2] \right)^{-1}
\]

On simplification, this reduces to showing that

\[
\frac{(\phi - \beta c_m)(\Delta - A)^2}{4[8C_m - (\beta - A)^2]} < \frac{(\phi - \beta c_m)(\Delta - A)^2}{4[4C_m - (\beta - A)^2]},
\]

which follows from simple algebra.

(iii) To show that \( p^m < p^{3P} \), we have to show

\[
\frac{3\phi + \beta c_m}{4\beta} - \frac{(\phi - \beta c_m)(\Delta - A)^2}{4[16C_m - (\beta - A)^2]},
\]

which follows from simple algebra. Because the ordering for the retail price \( p^* \) holds, the ordering for the demands of the channels follows trivially.

**Proof of Proposition 3.** The proof of Proposition 3 follows from simple algebra. In Table 2, \( \Pi^C_p > \Pi^M_p \) and \( \Pi^M_p > \Pi^{3P} \) are obvious, as are the results of \( \Pi^C_p > \Pi^R_p \) and \( \Pi^R_p > \Pi^{3P} \). Because \( \Pi^C_m > \Pi^M_p \) and \( \Pi^R > \Pi^{3P} \), it trivially follows that \( \Pi^C_m > \Pi^{3P} \). To show that \( \Pi^C > \Pi^{3P} \), we have to show

\[
\frac{(\phi - \beta c_m)^2}{16C_m} < \frac{(\phi - \beta c_m)^2}{16C_m} + \frac{3(\beta - A)^2}{4(8C_m - (\beta - A)^2)}
\]

which follows from simple algebra. The proof of \( \Pi^C > \Pi^{3P} \) is analogous.

**Appendix C**

Following the theory of incentive contracts (see Laflont and Tirole 1993), we take a principal-agent approach, with the manufacturer as the principal. The role of the manufacturer as the principal is consistent with the spirit of the earlier sections of this paper, which give the manufacturer the role of the leader in the Stackelberg game. The manufacturer can influence the retailer's choice of the collection effort level by specifying a contract of the type \( W = \{t, F\} \). Here, \( w(t) \) stands for a wholesale pricing scheme contingent on the return rate of used products from the market, and \( F \) is a fixed payment made by the retailer to the manufacturer that distributes the efficiency gains. Thus, the manufacturer's problem is formulated as

\[
\max_{W(t), r} \left( w(t) - (c_m - \Delta t)\right) + \frac{\alpha}{\beta}(p - \phi p) + F
\]

subject to

\[
\tau = \arg\max_{\tau} \Pi^C \left( W(t), \tau \right)
\]

\[
p = \arg\max_{p} \Pi^C \left( W(t), \tau \right)
\]

\[
\Pi^C \left( W(t), \tau \right) \geq \Pi^C
\]

where \( \Pi^C \) is the retailer's profit level realized in the decentralized channel structure, and \( \Pi^C \left( W(t), \tau \right) = \left( p - w(t)\right) + \frac{\alpha}{\beta}(p - \phi p) - C(t) - F \) is the profit function of the retailer under the contract \( W(t) \). The first two constraints are the incentive compatibility constraints for \( \tau \) and \( p \),

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respectively, which ensure that $p$ and $\tau$ are maximizers of the retailer's profit function, while the last constraint is the individual rationality constraint of the manufacturer, which ensures that the retailer accepts the contract.

When the retailer retains private information about the cost of his collection effort (i.e., the adverse selection problem), or when the collection effort level of the retailer cannot be fully observed, or there is noise in the return rate of used products (i.e., the moral hazard problem), the optimal contract is updated to incorporate the information constraints. In Savaskan et al. (1999), it is shown that the adverse selection problem in the closed-loop supply chain can be overcome by using a menu of contracts, which would induce the retailer to reveal the true cost of his collection effort to the manufacturer. However, to achieve this, the manufacturer leaves some rent with the retailer, which leads to an inefficient level of product return rate and lower supply chain profits. If the manufacturer faces moral hazard issues in the closed-loop supply chain, and if the retailer is risk neutral, one can easily show that the full information outcome can be obtained by using the linear contract outlined above (Holmstrom 1979, 1982). When the retailer is risk averse, the optimal contract is adjusted by taking into consideration the risk aversion of the retailer and the information on the product return rate.

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