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A Fuzzy TOPSIS Approach for Finding Shortest Path in Multimodal Transportation Networks

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Abstract

The paper must have abstract. In this paper, we present a fuzzy shortest path algorithm in Multimodal Transportation Networks (MTN). To extend the classical problem of shortest path, an innovative framework is presented, which integrates Fuzzy Logic and Multi Criteria Decision Making (MCDM) techniques. The aim is to deal with an efficient design for the multimodal shortest path computation taking into accounts not only the expected travel time, but also additional constraints such as: delays at mode and arc switching points, costs, viability of the sequence of the used modes and the number of modal transfers. To this scope, all previously mentioned constraints are evaluated following a new approach based on Fuzzy Logic and aggregated using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to find the shortest path in MTN. A numerical example is also presented to demonstrate the computational process of the proposed approach.

Keywords: Fuzzy shortest path, Multimodal Transportation

1 Introduction

The shortest path problem (SPP) is a classical and important network optimization problem, appearing in many applications including transportation,

routing, communications, supply chain management or models involving agents. Although the weights of the edges in an SPP are assumed to be precise real numbers in conventional graph theory, these parameters (i.e., costs, capacities, demands, time, etc.) are naturally imprecise for most practical applications. In such cases, an appropriate modeling approach may justifiably make use of fuzzy numbers, and so does the name fuzzy shortest path problem (FSPP) appear in the literature [1, 3, 2, 23] [12] [11] [27].
Since it involves the addition and the comparison between fuzzy numbers (particularly triangular fuzzy numbers), the FSPP is very different from the conventional SPP, which involves only real numbers. In the FSPP, the final costs are fuzzy numbers. So, it is difficult to find a path that can be shorter than all the others since the comparison among fuzzy numbers is an operation that can be defined several ways. As it is very well known, it is hard to find a cost that is strictly smaller than all the others in a set of fuzzy costs. In this work, the fuzzy shortest path problem on MTN is studied. The multimodal path is executed by using several mode of transportation. Hence, the viable path is a multimodal path that respects a set of constraints on its sequences of used modes. The multimodal shortest viable path is one of the most important current routing problems, and its solution is very interesting to travel in MTN. Each traveler assigns his personal preferences to $O-D$ paths in the multimodal networks. He/she can give his/her preferred modes of transport, the maximum of modal transfers, and the maximum of the generalized cost (for example: to reach his destination, a user can choose subway and tramway, maximum three modal transfers, and a cost under \$4). The particularity of this kind of problem is that the path from a given point to a specified point over MTN could have an illogical sequence of the used modes (non-viable path). A path of this type cannot really be travelled by the user. Many works have been treated the SPP in MTN. An overview of this literature can be found in [13, 19, 18][10, 17, 4, 5][15, 20] [22, 6] [16, 7]. In this type of problem, three constraints are considered. The main one account into the set of transit modes used on the path. According to [19][4] a path in MTN could have an illogical sequence of the used modes. A path of this type cannot really be travelled by the user. The path is called viable if its sequence of modes is feasible with respect to a set of constraints. The second constraint concerns the number of modal transfers accomplished on the path. The third is the time constraint that accounts for travel time and switching delays. In this context, the following works treated only the second or the third constraint, and the optimal path resulted from their algorithm could be an illogical one (non-viable path). Hence, Ref. [10] studied the shortest path on bimodal networks, using a node for each modal transfer. Ref. [22] considered the number of modal transfers in a path as an attribute in the multi-criteria shortest path. Ref. [20] proposed a utility measure that takes into account the propensity of

the decision maker with respect to the given transportation modalities. Ref. [15] modeled the transit mode change with a modal transfer arc. Ref. [7] studied the time-dependent least time paths on MTN. Few studies studied the constraint of the path viability, eliminating paths with illogical sequence of used modes [13, 19, 18] [4, 5]. Ref. [18] proposed a time-dependent multimodal optimum algorithm that eliminates the paths which do not respect a set of constraints on the sequence of used modes and exceed maximum of modal transfers given by the user. Ref. [21] presented a model and an algorithm for computing the shortest path in a network having various types of fuzzy arc lengths. A genetic algorithm is proposed for finding the shortest path in the network. The fuzzy shortest path is also developed in [26] to solve the problem complexity in terms of the multi-criteria of lead time and capacity with an efficient computational method. The results and analysis indicate that the proposed two-stage programming with fuzzy shortest path surpasses the performance of shortest path problem with time windows and capacity constraint (SPPTWCC) in terms of less computational time and CPU memory consumption. Ref [25] treated the shortest path in multi-constrained network using multi-criteria decision method based on vague similarity measure. Each arc length represents multiple metrics. The multi-constraints are equivalent to the concept of multi-criteria based on vague sets.

Unfortunately, all the previously mentioned constraints are modeled in a deterministic and known manner, but these hypotheses are seldom satisfied because related parameters (such as: expected travel time, delay, etc.) are often uncertain and variable. The scope of this work is that the proposed method for solving the SPP on MTN is based on fuzzy theory and Multi-criteria Decision Making (MCDM) techniques. In particular, the travel time of each path as well as the additional parameters is evaluated using Fuzzy Triangular Numbers (FTN), while TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is applied to aggregate parameters and to rank paths in order of optimality. The choice of these techniques is motivated by the need to assure a straightforward applicability to real-life situations. Indeed, FTNs represent the easiest way to reproduce in a soundly way the estimation given by experts and, similarly, TOPSIS aggregates the decision parameters following a simple procedure that can be easily extended to the fuzzy case.

2 Fuzzy multimodal network

In this section, some basic concepts as the concept of Fuzzy Triangular Numbers (FTNs) and fuzzy graph from Ref. [3, 2] are included to deal with the presentation of the fuzzy multimodal graph. The concept of FTN is used to describe the travel time and the other parameters of the graph paths. The use of FTNs appears adequate, because it offers good compromise between accuracy and computational time.

2.1 Concept of Fuzzy Triangular Numbers

Definition: A Fuzzy Triangular Number (FTN) is represented by an ordered triplet F=(a,b,c), with the membership function $\mu_F(x)$, defined by the following expression:

$$\mu_F(x) = \begin{cases} 0; & if \quad x \leq a \\ \frac{x-a}{b-a}; & if \quad a \leq x \leq b \\ \frac{c-x}{c-b}; & if \quad b \leq x \leq c \\ 0; & if \quad x > c \end{cases}$$

Where b is the center, a is the left spread, and c is the right spread. FTNs can be used to perform common arithmetic operations in a simple way. The fuzzy generalization (obtained through the principle extension discussed in Ref. [9]) of the basic arithmetic operations on FTNs are defined as follows: Fuzzy addition:

$$F_1 \oplus F_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \tag{1}$$

Fuzzy subtraction :

$$F_1 \ominus F_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2) \tag{2}$$

Scalar multiplication:

$$k \in IR, \quad k \otimes F_2 = (k \times a_2, k \times b_2, k \times c_2) \tag{3}$$

Fuzzy multiplication:

$$F_1 \otimes F_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2) \tag{4}$$

Fuzzy division:

$$F_1 \oslash F_2 = (a_1/a_2, b_1/b_2, c_1/c_2) \tag{5}$$

Fuzzy inverse

$$F^{-1} = (\frac{1}{c}, \frac{1}{b}, \frac{1}{a}) \tag{6}$$

Fuzzy mean:

$$\frac{1}{n}(F_1 \oplus F_2 \oplus ... \oplus F_n) = (\frac{\sum_{i=1}^n a_i}{n}, \frac{\sum_{i=1}^n b_i}{n}, \frac{\sum_{i=1}^n c_i}{n}) \tag{7}$$

The best known defuzzification ways to transform a FTN into crisp value are the following ones [24]: Smallest of maxima (SOM), Mean of maxima (MOM), and Largest of maxima (LOM)

$$SOM(F) = MOM(F) = LOM(F) = \{b\} \tag{8}$$

Centre of gravity:

$$GOG(F) = \frac{a+b+c}{3} \tag{9}$$

A defuzzification method that gives even more importance to the core of a FTN is the Graded Mean Interaction [**?**]:

$$GM(F) = \frac{a+4b+c}{3} \tag{10}$$

2.2 Fuzzy multimodal graph

Definition: By the notation $G = (N, E, M, \Omega)$ we mean a fuzzy multimodal graph G, where N is the set of nodes, E is the set of arcs, M is the transit modes associated with arcs and Ω is the set of Fuzzy Triangular Numbers assigned to arcs.

We assume that each network in MTN is modeled by one monomodal sub graph $SG_m = (N_m, E_m)$, where N_m is the set of nodes and E_m is the set of arcs, $m \in M$. The multimodal graph G is partitioned to a set of monomodal sub graphs: $SG_{m_1} = (N_{m_1}, E_{m_1}), SG_{m_2} = (N_{m_2}, E_{m_2}), ..., SG_{m_q} = (N_{m_q}, E_{m_q})$, such that :

- $N = N_{m_1} \cup N_{m_2} \cup, ..., \cup N_{m_q}$;
- $N_{m_i} \cap N_{m_j} = \emptyset$;
- $E_{m_1} \cup E_{m_2} \cup, ..., \cup E_{m_q} \subset E$; and
- $E_{m_i} \cap E_{m_j} = \emptyset$;
- where $i \neq j$ and $m_i \in M$, for $1 \leq i \leq q$.

Let T be the set of modal transfers between the different modes, then the elements of T are the modal transfer arcs. Hence, an adjacent arc of a $mode - m$ arc is either a$mode - m$ arc or a modal transfer arc, and we have the following: $E_{m_1} \cup E_{m_2} \cup, ..., \cup E_{m_q} \cup T$ The path from a given origin node x to the destination node y is denoted by a sequence of nodes and arcs as follows:

$\Pi_{x,y} = (z_1, z_2, ..., z_m)$ Where $z_1 = x$ and $z_m = y$. In a multimodal context we find two types of paths: the monomodal path and the multimodal path. A monomodal path $\Pi_{x,y}$ is a path composed of nodes which belong to the same $N_r(zi \in N_r, \forall i \in \{1, ..., m\})$.

A multimodal path $\Pi_{x,y}$ is a path accomplished by using several modes. The path $\Pi_{x,y} = (z_1, z_2, ..., z_m)$ is composed of nodes which belong to several N_r.

3 Decision criteria

To apply MCDM it is necessary to identify all feasible alternatives and the decision criteria to discriminate among them. Here the alternatives coincide with the paths in the fuzzy multimodal graph G, while possible decision criteria are introduced in below. The notation $\Pi^k_{O,D}$ indicates the k^{th} path from the origin node O to the destination node D.

3.1 Expected path cost

The expected cost is an important parameter to be considered. FTNs can be effectively used to represent the overall costs of each arc. Therefore, for each arc (v, u) is assigned with its corresponding fuzzy cost $C^m_{v,u} = (a^m_{v,u}, b^m_{v,u}, c^m_{v,u})$ on mode m. Next, the cost of the path $\Pi^k_{O,D}$ can be simply evaluated by aim of the following equation:

$$\begin{aligned} PC_{\Pi^k_{O,D}} &= \sum_{\substack{(v,u)\in\Pi^k_{O,D} \\ m\in M\cup T}} C^m_{v,u} \\ &= \Big(\sum_{\substack{(v,u)\in\Pi^k_{O,D} \\ m\in M\cup T}} a^m_{v,u} \,,\, \sum_{\substack{(v,u)\in\Pi^k_{O,D} \\ m\in M\cup T}} b^m_{v,u} \,,\, \sum_{\substack{(v,u)\in\Pi^k_{O,D} \\ m\in M\cup T}} c^m_{v,u} \Big) \end{aligned} \tag{11}$$

3.2 Expected path time

MTN is comprised of set of lines of a public transit mode. Let L be a set of lines and L_u a set of lines with a stop at node u. A model of the users behavior consists in assuming that each user has a priori to choose his attractive subset of lines to go from node u to destination$d$, such that, at node u, the user is willing to board the first carrier of subset Lu which arrives. We assume that each station u is associated with a set of scheduled departure list. Public rail modes are the modes such that at each stop, the user could take only one line with fixed schedule. Public surface modes are characterized by imprecise schedules, and at each stop the user is willing to board the first carrier of the attractive set of lines.

3.2.1 Delays

Let (v, u, w) a triplet nodes on the graph G. We denote by $\delta^{m,m'}_{v,u,w}$ the delay at node u, when entering node u from arc (v, u) on mode m and switching to arc (u, w) and mode m'. When m and m' belongs to the private mode, $\delta^{m,m'}_{v,u,w}$ represents the turning movement delay at intersection u, from arc (v, u) to arc

(u, w). When m is a transit mode and m' is a modal transfer we assume that leaving the transit arc (v, u) is accomplished without delays. When m and m' belongs to the public rail mode, $\delta_{v,u,w}^{m,m'}$ is the times of waiting the coming schedule departure. In the case where m and m' are surface modes, let φ_l be the frequency of line $l \in L_u$, then $\delta_{v,u,w}^{m,m'} = \frac{1}{\phi(u)}$, where $\phi(u) = \sum_{l \in L_u} \varphi_l$. Therefore, when delays are defined as FTNs.

$$\delta_{v,u,w}^{m,m'} = (a_{v,u,w}^{m,m'} \ , \ b_{v,u,w}^{m,m'} \ , \ c_{v,u,w}^{m,m'}) \tag{12}$$

3.2.2 Travel time

On graph G, the travel time is commonly used to determine the shortest path. Since a FTN already contains the information regarding travel time, their evaluation is straightforward and can be simply obtained summing (in the fuzzy sense) the travel time of the arcs belonging to a path. We denote by $T_{v,u}^m$ the fuzzy travel time from node$v$ to node u on mode m, their evaluation for the path $\Pi_{O,D}^k$ is given by aim of equation below:

$$T_{v,u}^m = (a_{v,u}^m \ , \ b_{v,u}^m \ , \ c_{v,u}^m)) \tag{13}$$

$$\begin{aligned} PT_{\Pi_{O,D}^k} &= \sum_{\substack{(v,u,w) \in \Pi_{O,D}^k \\ m,m' \in M \cup T}} T_{v,u}^m \oplus \delta_{v,u,w}^{m,m'} \\ &= \begin{pmatrix} \sum_{\substack{(v,u,w) \in \Pi_{O,D}^k \\ m,m' \in M \cup T}} a_{v,u}^m \oplus a_{v,u,w}^{m,m'} \\ \sum_{\substack{(v,u,w) \in \Pi_{O,D}^k \\ m,m' \in M \cup T}} b_{v,u}^m \oplus b_{v,u,w}^{m,m'} \\ \sum_{\substack{(v,u,w) \in \Pi_{O,D}^k \\ m,m' \in M \cup T}} c_{v,u}^m \oplus c_{v,u,w}^{m,m'} \end{pmatrix} \end{aligned} \tag{14}$$

3.3 Multimodal path viability

In MTN a path from a given point to a specified point could have an illogical sequence of the used modes. A path of this kind cannot be travelled by the user. The multimodal viable path is a path that respects a set of constraints on its sequences of used modes. A characteristic of multimodal path is the use of distinct modes of transportation, then consistent with the works [12], [8], we assume that no modal transfer allows to transfer from public modes to private mode since it is implicitly assumed that once path is not started with private modality, it is not possible anymore to take it for reaching the destination. For this reason a $O - D$ feasible path that use the private mode, it take it at the origin node O. Then, the viable path is a path that respects the following constraints:

- The number of modal transfer of the path must respect the maximum that the user is able to establish.
- If the path uses the rail mode, it must include only one consecutive sequence of rail mode,
- If the path uses the private mode, it must include only one consecutive sequence of private mode with initial node O.

The path viability is used as a key to identify a feasible sequence of the modes on the multimodal path. Hence, a simple procedure, similar to the one described in [18] can be used to check viability of the paths obtained from the concatenation. In this work we deal with fuzzy valuations to take into a proper account the different paths viabilities. On the graph G, we denote with $\wedge^m_{v,u}$ the fuzzy viability associated with arc (v,u) for mode m. To this scope, an index based on typical formula used in viability assessment is suggested:
$Viability = ConstrainedModes * Transfers$
In the present case : (i) the first term represents the possibility to use a constrained modes on the path $\Pi^k_{O,D}$ and (ii) the transfers are defined as the number of modal transfers expected on the path $\Pi^k_{O,D}$. In this case a set of Linguistic Variables (LV) should be defined to translate evaluations such as $'poor'$, $'fair'$, and $'good'$ to the corresponding FTNs.

$$PV_{\Pi^k_{O,D}} = \Big(\sum_{(v,u)\in\Pi^P} \wedge^P_{v,u} \oplus \sum_{(v,u)\in\Pi^R} \wedge^R_{v,u} \oplus \sum_{(v,u)\in\Pi^k_{O,D}} \wedge^S_{v,u} \Big) \otimes \sum_{(v,u)\in\Pi^k_{O,D}} \wedge^T_{v,u} \quad (15)$$

Where Π^P and Π^R are respectively, the unique private mode sub-path with the initial node O and the unique rail mode sub-path on the path $\Pi^k_{O,D}$.

4 The Fuzzy TOPSIS method

Yoon and Hwang introduced the TOPSIS method as a technique based on the order preference by similarity to ideal solution. The idea is that the positive ideal solution is a solution that maximizes the benefit criteria/attributes and minimizes the cost criteria/attributes, whereas the negative ideal solution maximizes the cost criteria/attributes and minimizes the benefit criteria/attributes. The best alternative should be closest to the ideal solution and farthest from the negative ideal solution. Suppose a MCDM problem has n alternatives, $A_1, A_2, ..., A_n$, and m decision criteria/attributes, $C_1, C_2, ..., C_m$. Each alternative is evaluated with respect to the m criteria/attributes. All the ratings are assigned to alternatives with respect to decision matrix denoted by $D = (d_ij)nm$, which contains the evaluations given to the i^{th} alternative with respect to the j^{th} decision parameter and a vector denoted by $W = (w_j)_m$, which contains the weights assigned to each one of the m decision parameters.

The fuzzy TOPSIS method consists of the following steps:

1) A set of LVs is used to convert the importance given to each decision criterion (i.e. the weight w_j) in a FTN. These values are defuzzified into a crisp value w_j by aim of the following relation, based on the graded mean integration:

$$w_j = \frac{\sum_{i=1}^{n} \frac{a_i+b_i+c_i}{6}}{n} \tag{16}$$

Where n is the number of decision makers and (a_i, b_i, c_i) is the fuzzy evaluation given by the ith decision maker.

2) All decision criteria are converted using the approach of Hsu and Chen [14]. Let $e_{ij} = (a_i, b_i, c_i)$ be the evaluation given to the i^{th} alternative with respect to the j^{th} criterion, conversion is made as follows :

$$\tilde{d}_{ij} = (\frac{a_{ij}}{c_j^*} \ , \ \frac{b_{ij}}{c_j^*} \ , \ \frac{c_{ij}}{c_j^*}) \tag{17}$$

when e_{ij} is a benefit criteria and $c_i^* = max_i \ c_{ij} \ , \ i = 1, ..., n$; and

$$\tilde{d}_{ij} = (\frac{a_j^*}{c_{ij}} \ , \ \frac{a_j^*}{b_{ij}} \ , \ \frac{a_j^*}{a_{ij}}) \tag{18}$$

when e_{ij} is a cost criteria and $a_i^* = min_i \ a_{ij} \ , \ i = 1, ..., n$

3) The evaluation values $\tilde{d}_{ii}$ are defuzzified using the graded mean integration and inserted in the decision matrix $D = (d_{ij})_{nm}$ as follows:

$$d_{ij} = GM(\tilde{d}_{ij}) \tag{19}$$

4) Calculate the weighted decision matrix $X = (x_{ij})_n m$:

$$x_{ij} = d_{ij} w_j \quad i = 1, ..., n \quad j = 1, ..., m \tag{20}$$

where w_j is the relative weight of the $j_t h$ criterion or attribute. 5) Determine the positive ideal and negative ideal solutions:

$$S^+ = \{s_1^+, s_2^+, ..., s_m^+\} \qquad s_j^+ = max_i \ x_{ij} \tag{21}$$

$$S^- = \{s_1^-, s_2^-, ..., s_m^-\} \qquad s_j^- = min_i \ x_{ij} \tag{22}$$

6) Calculate the distance of the existing alternatives from ideal and negative ideal alternatives: the two Euclidean distances for each alternative are, respectively:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (x_{ij} - s_j^+)^2} \ \ , \quad i = 1, 2, ..., n \tag{23}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} (x_{ij} - s_j^-)^2} \quad , \quad i = 1, 2, ..., n \tag{24}$$

7) Calculate the relative closeness of each alternative to the ideal solution. The relative closeness of the alternative Ai with respect to S+ is defined as:

$$RC_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad , \quad i = 1, 2, ..., n \tag{25}$$

Rank the alternatives according to the relative closeness to the ideal solution. The best alternative is the one with the greatest relative closeness to the ideal solution.

5 An algorithm for the fuzzy multimodal shortest path problem (FMSPP)

We aim at determining the fuzzy multimodal shortest path needed to traverse from source node O to destination node D. By combining the fuzzy shortest path method with TOPSIS method, the new algorithm is proposed as follows:

Step 1. *Form the possible paths from source node O to destination node D and compute the corresponding path parameters using equations (11)-(15) for possible m paths (* $\Pi_{O,D}^k$ *for* $k = 1, 2, ..., m$*).*
Step 2. *Employ TOPSIS method defined in section 4 to yield the similarity degree between ideal solution and* $\Pi_{O,D}^k$ *for k = 1,2,. . . ,m.*
Step 3. *Decide the multimodal shortest path with the highest relative closeness to the ideal solution.*

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6 Case study

To better explain the proposed algorithm, an example is presented. The Characteristics of the network used are shown in table 1 and table 2. This network is formed by 23 nodes and 34 arcs (29 travel arcs and 5 transfer arcs). In this case, the modes associated with the travel arcs are: Private modes, public rail modes (metro and train) and public surface modes (bus and collective taxi).

Table 1: The network characteristics.

Arc (v,u)	**mode(m)**	$C^m_{v,u}$	$T^m_{v,u}$
(O,1)	P	(6, 7, 10)	(0.68, 0.7, 0.73)
(O,2)	R	(6, 7.1, 8.8)	(0.3, 0.45, 0.7)
(O,3)	S	(3, 4, 6)	(0.8, 0.84, 0.87)
(1,3)	T	(0.88, 1, 1.15)	(0.03, 0.05, 0.08)
(1,4)	P	(7, 9.7, 11)	(0.77, 1, 1.3)
(1,5)	P	(11,13, 17)	(1.4, 1.7, 2)
(2,6)	R	(7.4, 7.7, 8)	(0.1, 0.4, 0.8)
(3,11)	S	(2, 3.7, 4)	(0.78, 0.8, 0.85)
(3,15)	S	(1, 3.9, 5)	(0.7, 0.75, 0.78)
(4,7)	P	(7, 8, 11)	(0.6, 0.81, 1.2)
(5,6)	T	(0.6, 0.9, 1.5)	(0.03, 0.05, 0.07)
(6,9)	R	(5, 7.9, 9)	(0.7, 1.1, 1.5)
(6,10)	R	(6.5, 8.7, 9.8)	(0.9, 0.95, 1.3)
(7,8)	P	(7.5, 9.7, 12)	(2, 2.5, 3)
(8,9)	T	(0.9, 1.2, 1.5)	(0.04, 0.07, 0.1)
(8,D)	P	(13, 17, 21)	(2.3, 2.5, 2.7)
(9, 21)	R	(8.4, 8.6, 8.9)	(0.95, 0.98, 1.2)
(10,20)	T	(0.8, 1, 1.3)	(0.03, 0.05, 0.08)
(11,12)	S	(2, 3.6, 6.5)	(0.6, 0.8, 1.1)
(12,13)	S	(4.3, 4.7, 5)	(1, 1.2, 1.5)
(13,14)	S	(4, 4.9, 6)	(1, 1.22, 1.5)
(14,19)	S	(3.5, 3.7, 4.2)	(0.3, 0.6, 1)
(14,21)	T	(0.7, 0.9, 1.2)	(0.05, 0.06, 0.08)
(15,16	S	(7, 9, 11)	(0.7, 0.75, 0.79)
(16,17)	S	(2, 3.8, 5.7)	(0.5, 0.81, 1)
(17,18)	S	(2, 3.9, 7)	(0.2, 0.58, 0.7)
(18,19)	S	(1, 3.5, 7)	(0.22, 0.5, 0.9)
(19,20)	S	(2, 4.1, 7)	(0.3, 0.7, 0.9)
(20, D)	S	(2.7, 4, 5)	(0.3, 0.7, 0.9)
(21, D)	R	(5.3, 7.7, 7.9)	(0.2, 0.6, 0.9)
(19,23)	S	(1, 4, 8)	(0.6, 0.69, 0.75)
(19,24)	S	(2, 3.8, 5.7)	(0.5, 0.81, 1)
(20,21)	R	(6, 7.1, 9)	(0.1, 0.4, 0.7)
(21,D)	R	(5.3, 7.7, 9)	(0.2, 0.6, 0.9)

Table 2: Delays of the triplet (v,u,w).

Triplet (v,u,w)	$\delta_{v,u,w}^{m,m'}$
(O, 1, 5)	(0.03, 0.05, 0.09)
(O, 1, 4)	(0.07, 0.1, 0.14)
(O, 2, 6)	(0.03, 0.08, 0.1)
(O, 3, 11)	(0.09, 0.13, 0.18)
(O, 3, 15)	(0.08, 0.1, 0.14)
(1, 3, 11)	(0.09, 0.15, 0.17)
(1, 3, 15)	(0.1, 0.12, 0.14)
(1, 4, 7)	(0.08, 0.1, 0.13)
(2, 6, 9)	(0.02, 0.04, 0.05)
(2, 6, 10)	(0.03, 0.07, 0.08)
(5, 6, 9)	(0.02, 0.05, 0.09)
(5, 6, 10)	(0.03, 0.05, 0.07)
(4, 7, 8)	(0.11, 0.13, 0.15)
(7, 8, D)	(0.09, 0.13, 0.17)
(6, 9, 21)	(0.04, 0.06, 0.09)
(8, 9, 21)	(0.07, 0.11, 0.19)
(3, 11, 12)	(0.12, 0.15, 0.17)
(11, 12, 13)	(0.1, 0.12, 0.14)
(12, 13, 14)	(0.08, 0.1, 0.12)
(13, 14, 19)	(0.11, 0.13, 0.15)
(3, 15, 16)	(0.07, 0.08, 0.1)
(15, 16, 17)	(0.08, 0.1, 0.13)
(16,17, 18)	(0.1, 0.15, 0.18)
(17, 18, 19)	(0.12, 0.14, 0.17)
(14, 19, 20)	(0.11, 0.13, 0.17)
(18, 19, 20)	(0.1, 0.13, 0.16)
(19, 20, D)	(0.09, 0.11, 0.13)
(10, 20, D)	(0.13, 0.15, 0.16)
(9, 21, D)	(0.07, 0.09, 0.11)
(14, 21, D)	(0.09, 0.12, 0.14)

To evaluate the objective parameters, data within tables 1 and 2 are processed with relations (11-15). The weights of the decision criteria w_j are assessed by three experts, using the following Linguistic Variables:
Very Low : VL =(0, 0.1, 0.3)
Low : L=(0.1, 0.3, 0.5)
Medium: M=(0.3, 0.5, 0.7)
High : H=(0.5, 0.7, 0.9)
Very High: VH=(0.7, 0.9, 1)
The subjective parameters are also assessed by three experts, using the follow-

ing Linguistic Variables:
Very Poor VP =(0, 0, 3)
Poor P=(0, 3, 5)
Fair F=(3, 5, 7)
Good G=(5, 7, 10)
Very Good VG=(7, 10, 10)

The O-D paths resulted from the example network are given in the following:
$\Pi^1_{O,D}$ = [O, 2, 6, 9, 21, D]
$\Pi^2_{O,D}$= [O, 2, 6, 10, 20, D]
$\Pi^3_{O,D}$= [O, 1, 4, 7, 8, D]
$\Pi^4_{O,D}$= [O, 1, 5, 6, 9, 21, D]
$\Pi^5_{O,D}$= [O, 1, 5, 6, 10, 20, D]
$\Pi^6_{O,D}$= [O, 1, 4, 7, 8, 9, 21, D]
$\Pi^7_{O,D}$= [O, 1, 3, 11, 12, 13, 14, 21, D]
$\Pi^8_{O,D}$= [O, 1, 3, 11, 12, 13, 14, 19, 20, D]
$\Pi^9_{O,D}$= [O, 1, 3, 15, 16, 17, 18, 19, 20, D]
$\Pi^{10}_{O,D}$= [O, 3, 11, 12, 13, 14, 21, D]
$\Pi^{11}_{O,D}$= [O, 3, 11, 12, 13, 14, 19, 20, D]
$\Pi^{12}_{O,D}$= [O, 3, 15, 16, 17, 18, 19, 20, D]

Table 3: List of the fuzzy evaluations of the parameters

$Pi^k_{O,D}$	**Cost**	*Time*	*Viability*
$\Pi^1_{O,D}$	[32.1, 39, 42.6]	[2.41, 3.8, 5.45]	[G,G,G]
$\Pi^2_{O,D}$	[23.4, 28.5, 32.9]	[1.82,2,85, 4.12]	[G, G ,F]
$\Pi^3_{O,D}$	[40.5, 51.4, 65]	[6.7, 7.97, 9.52]	[F,F,G]
$\Pi^4_{O,D}$	[22, 20, 36]	[4.12,5.38, 6.78]	[F,F,F]
$\Pi^5_{O,D}$	[27.6, 34.6, 44.6]	[3.53, 4.4, 5.4]	[P,P,F]
$\Pi^6_{O,D}$	[26.5, 23.7, 41.8]	[5.64,7.19, 9.15]	[P,F,F]
$\Pi^7_{O,D}$	[25.18,33.5, 43.1]	[4.82,6.07, 7.48]	[VP, P,F]
$\Pi^8_{O,D}$	[27.38,36.7, 48.85]	[5.69,7.66, 9.61]	[P,P,P]
$\Pi^9_{O,D}$	[24.58,40.2, 58.85]	[4.29,6.37, 7.79]	[F,P,P]
$\Pi^{10}_{O,D}$	[21.3, 29.5, 36.6]	[4.91,6.14, 7.55]	[P,F,F]
$\Pi^{11}_{O,D}$	[23.5, 32.7, 43.7]	[5.78,7.73, 9.68]	[F,F,G]
$\Pi^{12}_{O,D}$	[14.7, 31.2, 50.7]	[4,36,6.44, 7.85]	[F,P,F]
w_j	(H, H, H)	(VH,VH, VH)	(M, H, H)

To define the optimality ranking of the paths, the criteria are aggregated using Fuzzy TOPSIS method. By equation (15) (19) the weighted decision matrix $X = (x_{ij})_{nm}$ is constructed. By equations (22) and (23), the distances of alternative paths, $\Pi^k_{O,D}$ from S^+ and S^- are obtained. By equation (24), the closeness coefficient RC_k of each alternative path $\Pi^k_{O,D}$ to S^+ is obtained. According to Table 4, the best selection is $\Pi^k_{O,D}$.

Table 4: Relative closeness to ideal solution.

Alternative path	RC_k
$\Pi^1_{O,D}$	0,516688303
$\Pi^2_{O,D}$	0,615958641
$\Pi^3_{O,D}$	0,340796821
$\Pi^4_{O,D}$	0,430788218
$\Pi^5_{O,D}$	0,404666541
$\Pi^6_{O,D}$	0,394960768
$\Pi^7_{O,D}$	0,341712982
$\Pi^8_{O,D}$	0,305450859
$\Pi^9_{O,D}$	0,337897079
$\Pi^{10}_{O,D}$	0,396467616
$\Pi^{11}_{O,D}$	0,391899023
$\Pi^{12}_{O,D}$	0,400013628

7 Conclusion

A novel practical approach was proposed for computing a fuzzy shortest path in MTN. This approach integrates Fuzzy Logic and Multi Criteria Decision Making (MCDM) techniques. The aim was to deal with a novel design of the multimodal shortest path computation that accounts for the following parameters: the expected travel time, delays at mode and arc switching points, costs, viability of the sequence of the used modes and the number of modal transfers. In additionally, the previously mentioned parameters are evaluated using Fuzzy Logic and aggregated by means of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to find the shortest path in MTN. A numerical example was also presented to demonstrate the computational process of the proposed approach.

References

[1] A. Boulmakoul et al, Path-finding operators for fuzzy multimodal spatial networks and their integration in mobile -GIS, *IEEE International Symposium on Signal Processing and Information Technology,* (2002), 51-56.

[2] A. Boulmakoul, Fuzzy graphs modelling for HazMat telegeomonitoring, *European Journal of Operational Research,* **175, Issue 3** (2006), 1514-1525. http://dx.doi.org/10.1016/j.ejor.2005.02.025

[3] A. Bolumakoul, Generalized path-finding algorithms on semirings and fuzzy shortest path problem. *Journal of computational and applied mathematics* **162** (2004), 263-272. http://dx.doi.org/10.1016/j.cam.2003.08.027

[4] A. Lozano, G. Storchi, Shortest viable path algorithm in multimodal networks, *Transportation research part A,* **35** (2001), 225-241. http://dx.doi.org/10.1016/s0965-8564(99)00056-7

[5] A. Lozano, G. Storchi, Shortest viable hyperpath in multimodal networks, *Transportation research part B,* **36** (2002), 835-874. http://dx.doi.org/10.1016/s0191-2615(01)00038-8

[6] A. Tajdin, I. Mahdavi, N. Mahdavi-Amiri, B. Sadeghpour-Gildeh, Computing a fuzzy shortest path in a network with mixed fuzzy arc lengths using -cuts, *Computers & Mathematics with Applications,* **60, Issue 4** (2010), 989-1002. http://dx.doi.org/10.1016/j.camwa.2010.03.038

[7] A. Ziliaskopoulos, W. Wardell, An intermodal optimum path algorithm for multimodal networks with dynamic arc travel times and switching delays, *European journal of operational research,* **125**, (2000), 486-502. http://dx.doi.org/10.1016/s0377-2217(99)00388-4

[8] D. L. Olsen, Comparison of weights in TOPSIS models, *Mathematical and Computer Modelling* **40, Issues 7-8** (2004), 721-727. http://dx.doi.org/10.1016/j.mcm.2004.10.003

[9] D. Yong, Plant location selection based on fuzzy TOPSIS.*Int. J. Adv. Manufact. Technol,* **28, no. 7-8**, (2006) 839-844. http://dx.doi.org/10.1007/s00170-004-2436-5

[10] E. Fernandez et al., Network equilibrium models with combined modes, *Transportation Science,* **28** (1994), 182-192. http://dx.doi.org/10.1287/trsc.28.3.182

[11] E. Khorram, E. Keshavarz, A fuzzy shortest path with the highest reliability, *Journal of Computational and Applied Mathematics,* **230** (2009), 204-212. http://dx.doi.org/10.1016/j.cam.2008.11.007

[12] F. Hernandesa, M. Lamatab, J. Verdegayb, A. Yamakamic, The shortest path problem on networks with fuzzy parameters, *Fuzzy Sets and Systems* **158** (2007),1561-1570. http://dx.doi.org/10.1016/j.fss.2007.02.022

[13] G. Aifadopoulou, A. Ziliaskopoulos and C. Evangelia, Multiobjective Optimum Path Algorithm for Passenger Pretrip Planning in Multimodal Transportation Networks, *Transportation Research,* **2032** (2007), 26-34. http://dx.doi.org/10.3141/2032-04

[14] HM. Hsu, CT. Chen, Fuzzy credibility relation method for multiple criteria decision making, *Inf. Sci.* **96** (1997) 79-91. http://dx.doi.org/10.1016/s0020-0255(96)00153-3

[15] H. J. Miller, J. D. Storm, Geographic Information System design for networks equilibrium-based travel demand models, *Transportation Research. C* **4** (1996), 373-389. http://dx.doi.org/10.1016/s0968-090x(96)00020-4

[16] L. A. Zadeh, Fuzzy Sets, *Inform Control,* **8**, (1965), 338-353. http://dx.doi.org/10.1016/s0019-9958(65)90241-x

[17] Li. Q, Kurt. C., GIS-Based Itinerary Planning System for Multimodal and Fixed-Route Transit Network, *MID-Continent Transportation Symposium Proceedings,* (2000), 47-50.

[18] M. Bielli, A. Boulmakoul and H. Mouncif, Object Modeling and path computation for multimodal travel systems. *European Journal of Operational Research.* **175** (2006), 1705-1730. http://dx.doi.org/10.1016/j.ejor.2005.02.036

[19] M. G. Battista et al, *Path composition and multiple choice in a bimodal transportation network,* Proceedings of the seventh WCTR, Sydney.

[20] P. Modesti, A. Sciomachen, A utility measure for finding multiobjective shortest paths in urban multimodal transportation networks, *European Journal of Operational Research,* **111** (1998), 495-508. http://dx.doi.org/10.1016/s0377-2217(97)00376-7

[21] R. Hassanzadeh, I. Mahdavi, N. Mahdavi-Amiri, A. Tajdin, A genetic algorithm for solving fuzzy shortest path problems with mixed fuzzy arc lengths, *Mathematical and Computer Modelling,* **57, Issues 12** (2013), 84-99. http://dx.doi.org/10.1016/j.mcm.2011.03.040

[22] S. Pallottino, G. Gallo, Shortest path algorithms, *Annals of Operational Research,* **13** (1998), 3-79.

[23] T. Chuang and J. Kung, A new algorithm for the discrete fuzzy shortest path problem in a network, *App. Math. and Comp.,* **174** (2006), 660-668. http://dx.doi.org/10.1016/j.amc.2005.04.097

[24] X. Liu, Paramerized defuzzification with maximum entropy weighting function, *Math Comput Modell.,* **45**, (2007),177-188. http://dx.doi.org/10.1016/j.mcm.2006.04.014

[25] Y. Dou, L. Zhu, Ho S. Wang, Solving the fuzzy shortest path problem using multi-criteria decision method based on vague similarity, *Applied Soft Computing,* **12, Issue 6,** (2012), 1621-1631. http://dx.doi.org/10.1016/j.asoc.2012.03.013

[26] Y. Kristianto, A. Gunasekaran, P. Helo, Y. Hao, A model of resilient supply chain network design: A two-stage programming with fuzzy shortest path, *Expert Systems with Applications,* **41, Issue 1** (2014), 39-49. http://dx.doi.org/10.1016/j.eswa.2013.07.009

[27] Y. M. Wang et al., Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment, *Expert Syst. Appl.,* **31**, (2006), 309-319. http://dx.doi.org/10.1016/j.eswa.2005.09.040

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