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Constraints on a Model with Pure Right-Handed Third Generation Couplings

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Abstract

We examine constraints on a model with pure right-handed third generation charged couplings. The parameters of the right-handed mixing matrix and the right-handed coupling strength are constrained from semi-leptonic B decays, the mass difference of neutral mesons, the CP violating observables ϵ and ϵ'/ϵ, and the electric dipole moment of the neutron. We find the model to be tightly constrained by these parameters with several fine tuning conditions on the phases in the right-handed mixing matrix. There is also a necessarily non-zero value of the W_L-W_R mixing parameter, ζ_g. CP asymmetry phases in neutral B decays are discussed.

I. INTRODUCTION

The standard $SU(2)_L \times U(1)$ model of the weak interactions has achieved great success. Nevertheless, viable competing models with the $SU(2)_L \times SU(2)_R \times U(1)$ gauge group have been proposed [1,2]. In these models the left-handed Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix is that of the standard model and the parameters of the right-handed mixing matrix as well as the right handed coupling and the mass of the right-handed gauge bosons are constrained by experiment. In 1992, Gronau and Wakaizumi (GW) presented a model with this gauge group in which the flavor changing third generation decays occur only through right-handed currents [3]. This model owes its feasibility to the difficulty in differentiating between $(V-A)(V-A)$ quark-lepton couplings and $(V+A)(V+A)$ couplings. Experimental evidence has since ruled out the GW model as a possible alternative to the standard model. However, a more general choice than that chosen by GW for the right-handed mixing matrix, although tightly constrained by experiment, can not be entirely excluded on phenomenological grounds.

CP violation in the GW model and its more general extensions has been studied previously [3–6]. Previous authors have shown that the GW model parameters are constrained by the CP violating observables ϵ and ϵ' and the neutron dipole moment. They have also shown asymmetry values in nonleptonic neutral B decays differing from standard model predictions. It is, however, necessary to reexamine the constraints imposed by CP violation on these models in light of recent experiments [7,8].

In this paper we take the following approach. First, we briefly review non-symmetric left-right models and the GW model. Then, in section 3 we constrain the angles of the most general right-handed mixing matrix from observables not related to CP violation. We find that there is a tightly constrained region in which this model is viable, and we make a particular choice of angles. We then place constraints on the phases from CP violating observables in section 4. With constraints so imposed we examine various predictions in B decays in section 5. We summarize our results in section 6.

II. REVIEW OF $SU(2)_L \times SU(2)_R \times U(1)$ MODELS

Langacker and Sankar have reviewed $SU(2)_L \times SU(2)_R \times U(1)$ models [2]. In discussing these models below we follow much of their notation.

In $SU(2)_L \times SU(2)_R \times U(1)$ models, the left and right-handed quarks and leptons transform under doublets of separate $SU(2)$ gauge groups. This gives rise to a covariant derivative of the form

$$D^\mu = \partial^\mu + \frac{i}{2}(g_L \tau^a W_L^{\mu a} + g_R \tau^a W_R^{\mu a} + g' Y B^\mu), \tag{2.1}$$

where g' is the $U(1)$ gauge coupling, τ^a are the Pauli spin matrices, $W^a_{L,R}$ and B are the gauge boson fields and $g_{L,R}$ are the $SU(2)_L$ and $SU(2)_R$ gauge coupling constants. The gauge symmetry is spontaneously broken by introducing a Yukawa interaction with some Higgs sector and giving the Higgs a vacuum expectation value. This gives masses to the quarks, leptons and gauge bosons. We take a Higgs, Φ, that transforms as $\Phi \to L\Phi R^\dagger$ under

$SU(2)_L$ and $SU(2)_R$ and is neutral under hypercharge. A general choice for the vacuum expectation value gives

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \rightarrow \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}. \tag{2.2}$$

With this Higgs we have the relation $M_R = g_R/g_L M_L$, where M_L and M_R are the masses of the left and right-handed charged gauge bosons respectively. Taking the ratio g_R/g_L to be $\mathcal{O}(1)$, it is necessary to introduce additional Higgs to arrange for these masses to be much different. Minimally, one introduces two doublets or triplets under $SU(2)_L$ and $SU(2)_R$ which carry a hypercharge of 1. These obtain vacuum expectation values v_L and v_R.

The Yukawa couplings to the quarks are given as

$$-\mathcal{L}_Y = \sum_{i,j} \left(\bar{f}'_{iL} (r_{ij}\Phi + s_{ij}\tilde{\Phi}) f'_{jR} + \text{h.c.} \right), \tag{2.3}$$

where f' are the gauge eigenstate quark fields, r and s are general complex matrices and $\tilde{\Phi} = \tau^2 \Phi^* \tau^2$. This term gives rise to the mass matrices, $M^u = rk + sk'^*$ and $M^d = rk' + sk^*$. In the mass basis of quarks and leptons the charged current interaction is given by

$$-\mathcal{L}_{CC} = \frac{g_L}{\sqrt{2}} \bar{u}_{iL} \gamma_\mu V^L_{ij} d_{jL} W_L^{\mu +} + \frac{g_R}{\sqrt{2}} \bar{u}_{iL} \gamma_\mu V^R_{ij} d_{jL} W_R^{\mu +} + \text{h.c.}, \tag{2.4}$$

where V^L and V^R are the unitary mixing matrices for the quarks, the elements of which are

$$V^{L,R} = \begin{pmatrix} V_{ud}^{L,R} & V_{us}^{L,R} & V_{ub}^{L,R} \\ V_{cd}^{L,R} & V_{cs}^{L,R} & V_{cb}^{L,R} \\ V_{td}^{L,R} & V_{ts}^{L,R} & V_{tb}^{L,R} \end{pmatrix}. \tag{2.5}$$

The kinetic term for the Higgs gives a mass structure to the gauge bosons. There are two heavy neutral gauge bosons, the massless photon and charged gauged bosons from the left and right-handed sectors. The non-diagonal mass matrix of the charged left and right gauge bosons is

$$M_W^2 = \begin{pmatrix} \frac{1}{2} g_L^2 (|v_L|^2 + |k|^2 + |k'|^2) & -g_L g_R k' k^* \\ -g_L g_R k'^* k & \frac{1}{2} g_R^2 (|v_R|^2 + |k|^2 + |k'|^2) \end{pmatrix}. \tag{2.6}$$

where M_L and M_R are the upper and lower diagonal elements respectively. This matrix gives the mixing between the mass and the gauge eigenstates. In terms of the mixing angle this is

$$\begin{pmatrix} W_L^+ \\ W_R^+ \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ e^{i\omega}\sin\zeta & e^{i\omega}\cos\zeta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix}, \tag{2.7}$$

with

$$\tan 2\zeta = \frac{2 g_L g_R |k' k|}{M_R^2 - M_L^2}. \tag{2.8}$$

In this paper we will often use the following quantities:

$$\zeta_g \equiv \frac{g_R}{g_L}\zeta, \qquad \beta_g \equiv \frac{g_R^2}{g_L^2}\beta = \frac{g_R^2 M_1^2}{g_L^2 M_2^2}. \tag{2.9}$$

where M_1 and M_2 are the eigenvalues of the mass matrix (2.6). In the case where $M_R \gg M_L$ one has $M_1 \approx M_L$ and $M_2 \approx M_R$. ζ_g is the mixing parameter which determines the strength of the interactions due to mixing between left-handed and right-handed currents relative to pure left-handed current interactions. β_g determines the relative strength of right-handed to left-handed interactions.

Gronau and Wakaizumi proposed to modify the mixing matrices such that the third generation of quarks couples to the other generations only through the right-handed W bosons [3]. The specific parametrization is

$$V^L = \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.10}$$

for the left-handed mixing matrix, where θ_c is the Cabibbo angle. In the original GW model the right-handed mixing matrix was parametrized by a single angle and CP violation accommodated by a single phase. We choose to study the most general form of the right-handed mixing matrix, of which the GW model is a particular choice of angles and phases. This matrix involves three angles and four phases:

$$V^R = \begin{pmatrix} c_{12}c_{13}e^{i(\alpha+\beta)} & -c_{13}s_{12}e^{i(\gamma+\alpha)} & s_{13}e^{-i(\beta+\gamma-\alpha)} \\ (-c_{12}s_{23}s_{13}+s_{12}c_{23}e^{i\delta})e^{i(\beta-\alpha)} & (s_{12}s_{23}s_{13}+c_{12}c_{23}e^{i\delta})e^{i(\gamma-\alpha)} & s_{23}c_{13}e^{-i(\beta+\gamma+\alpha)} \\ (-c_{12}c_{23}s_{13}-s_{12}s_{23}e^{i\delta})e^{i\beta} & (s_{12}c_{23}s_{13}-c_{12}s_{23}e^{i\delta})e^{i\gamma} & c_{23}c_{13}e^{-i(\beta+\gamma)} \end{pmatrix}, \tag{2.11}$$

where c_{ij} denotes $\cos\theta_{ij}$ and s_{ij} denotes $\sin\theta_{ij}$. We will denote the model employing (2.10) and (2.11) as the generalized GW model. In the following section we will choose the angles in this matrix to satisfy experimental constraints.

III. GENERAL CONSTRAINTS ON THE MODEL

The b quark decays through pure right-handed couplings in this model. This allows us to use constraints from semi-leptonic B decays. In particular the decay $b \to cl\nu$ gives the important relation

$$|V_{cb}^R|(\beta_g^2+\zeta_g^2)^{1/2} = |V_{cb}^{SM}| = 0.036 - 0.042, \tag{3.1}$$

where SM denotes the standard model value. Results from CLEO measure the asymmetry in the decay $B \to D^* l\nu$ assuming a pure left-handed lepton current [8]. This puts an upper bound on the ratio

$$\left(\frac{\zeta_g}{\beta_g}\right)^2 < 0.30. \tag{3.2}$$

D0 performed direct searches for W^R [7]. For $g_R = g_L$ they obtain $M_{W_R} > 720$ GeV, which in turn implies $\beta_g < 0.012$. Because $|V^R_{cb}| < 1$ by unitarity, condition (3.1) gives $\beta_g > 0.03$. It is obvious that these two conditions can not simultaneously apply. It has been noted that the form of the right-handed mixing matrix and the size of the ratio g_R/g_L affect the lower bound on the mass of W_R [9]. In particular for $|V^R_{ud}| \ll 1$ and $g_R > g_L$ one lowers the bound on the W_R mass. In the region of parameter space of V^R in which $|V^R_{ud}| \ll 1$ we apply the constraints to be discussed below and find that the neutral B_s mass difference is too small to satisfy the experimental lower bound. We are left with the region where $g_R > g_L$, which may be unnatural in grand unified models [9]. D0 presents constraints on M_R for $g_R/g_L = \sqrt{2}$ but for no higher values of the ratio. It is not unreasonable to accept of β_g in the range $0.03 - 0.04$ for $g_R/g_L \sim 2$. In this paper we choose $\beta_g = 0.035$. Additionally, there is the ratio from semi-leptonic B decays

$$\left|\frac{V^R_{ub}}{V^R_{cb}}\right| = \left|\frac{V^{SM}_{ub}}{V^{SM}_{cb}}\right| = 0.09 \pm 0.03. \tag{3.3}$$

This gives the constraint $s_{13}/s_{23}c_{13} = 0.09 \pm 0.03$.

We now turn to the constraints imposed by the mass difference of the neutral mesons K, B_d and B_s. The neutral K mass difference is given as

$$\Delta m = -2\mathcal{R}e\langle K^0|H_{\Delta S=2}|\bar{K^0}\rangle. \tag{3.4}$$

The effective $\Delta S = 2$ Hamiltonian arises through the box diagram. In this model there are contributions due to the exchange of two W_R's and the exchange of a W_R and a W_L in addition to the two W_L exchange familiar from the standard model. We use the result of Mohapatra *et al.* for the box diagram without QCD corrections [10]. To calculate the matrix element of quark field operators we use the vacuum insertion approximation with bag factors equal to unity. The neutral B_d and B_s mass differences are determined by relations similar to (3.4) and we again use the results [10]. We use estimates of the B_d and B_s decay constants from the lattice [11]. For the experimental values

$$\Delta m_K = 3.49 \times 10^{-12}\,\text{MeV}, \tag{3.5}$$
$$\Delta m_{B_d} = 3.05 \times 10^{-10}\,\text{MeV}, \tag{3.6}$$
$$\Delta m_{B_s} > 8.16 \times 10^{-9}\,\text{MeV}, \tag{3.7}$$

we find the following satisfactory choice of angles:

$$\theta_{13} = 0.08, \qquad \theta_{12} = -0.04, \qquad \theta_{23} = 1.8, \tag{3.8}$$

where tuning of $\mathcal{O}(10^{-2})$ is necessary for θ_{13} and θ_{12}. Tuning of $\mathcal{O}(10^{-1})$ is necessary for θ_{23}. With these angles we satisfy the experimental conditions within theoretical uncertainties. We will use these values in the remainder of this paper. At this point there are no constraints on the four phases in this model. These will be adjusted by the CP violating observables discussed in the next section.

IV. CONSTRAINTS FROM CP VIOLATION

CP violation has been measured in the neutral K sector in the form of the observables ϵ and ϵ'. In addition, CP violation should give rise to a nonzero electric dipole moment of the neutron. We will now use the measurements of ϵ, ϵ'/ϵ and the upper bound on the neutron dipole moment to constrain the phases in our model.

The parameter ϵ is related to the $\Delta S = 2$ Hamiltonian.

$$\epsilon = \frac{e^{i\pi/4}}{\sqrt{2}} \frac{\mathcal{I}m\langle K^0|H_{\Delta S=2}|\bar{K^0}\rangle}{\Delta m_K}. \tag{4.1}$$

The effective Hamiltonian is that used to calculate the neutral K mass splitting. Using this and the choice of angles from the previous section we find several terms of $\mathcal{O}(10^{-1})$. The dominant contribution is from the box diagram involving the exchange of two W_R and two top quarks. This gives a strong contribution in this model because $V_{td}^R \sim 0.23$ and $V_{ts}^R \sim 1$, whereas in the standard model the two top exchange diagram is CKM suppressed. The leading terms in ϵ are

$$|\epsilon| = |0.14\cos 2(\beta-\gamma)\sin\delta - 0.14\sin 2(\beta-\gamma)\cos\delta + 0.23\sin(\beta-\gamma-\delta)|\,. \tag{4.2}$$

With the experimental value of $(\epsilon = 2.28 \pm 0.02) \times 10^{-3}$ and assuming no cancellation between terms, this suggests

$$\sin(\beta-\gamma) \lesssim \mathcal{O}(10^{-2}), \qquad \sin\delta \lesssim \mathcal{O}(10^{-2}). \tag{4.3}$$

There are other terms in the calculation of ϵ of order less than or equal to 10^{-2} which impose no further constraints on the phases.

The parameter ϵ' is given in terms of the K decay amplitude to two pions as

$$\epsilon' = \frac{e^{i(\frac{\pi}{2}+\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathcal{R}eA_2}{\mathcal{R}eA_0}\left(\frac{\mathcal{I}mA_2}{\mathcal{R}eA_2} - \frac{\mathcal{I}mA_0}{\mathcal{R}eA_0}\right), \tag{4.4}$$

with

$$A_i = \langle(\pi\pi)_{I=i}|H_{\Delta S=1}|K^0\rangle, \tag{4.5}$$

where i denotes the isospin channel and δ_i is the hadronic phase shift. The problem of calculating ϵ' is then to calculate the $\Delta S = 1$ Hamiltonian and with this, to estimate the decay amplitudes. Of course, this problem is plagued with hadronic uncertainties. The calculation of ϵ' in this model is interesting but lengthy. We relegate it to the appendix.

We find in the resulting expression for ϵ'/ϵ terms proportional to β_g and ζ_g. The terms proportional to β_g are too small to accommodate the measured value of ϵ'/ϵ of $(21.2 \pm 4.6) \times 10^{-4}$. Terms proportional to ζ_g must provide the dominant contribution. We will see the effects of a non-zero ζ_g in the following section. The dominant terms are

$$|\epsilon'/\epsilon| = \left|\zeta_g\left(1.8R_c^{LR}\sin(\alpha-\beta) - 4.0R_u^{LR}\sin(\alpha+\beta)\right)\right|, \tag{4.6}$$

where R_u^{LR} and R_c^{LR} are ratios of operators defined in the appendix. They are estimated to be $\mathcal{O}(1)$ and $\mathcal{O}(10^{-1})$ respectively. The constraint from ϵ'/ϵ then requires either small phases or a small ζ_g.

The electric dipole moment of the neutron arises in this model at the one loop level due to mixing of the W_L and W_R. In the standard model one loop diagrams do not contribute because they are proportional to the magnitude of CKM elements and so are real. W_L-W_R mixing permits imaginary coefficients in the loop diagram, allowing for a non-zero edm. Electric dipole moments of the u and d quark arise from diagrams involving the creation of a virtual W and the emission of a photon. These contributions have been calculated [12] and are given as

$$d_u = \frac{eG_F}{4\pi^2}\zeta_g \sum_{j=d,s,b} m_j \mathcal{I}m(V_{uj}^L V_{uj}^{R*}) f_1\left(\frac{m_j^2}{M_L^2}\right), \tag{4.7}$$

$$d_d = \frac{eG_F}{4\pi^2}\zeta_g \sum_{j=u,c,t} m_j \mathcal{I}m(V_{jd}^L V_{jd}^{R*}) f_2\left(\frac{m_j^2}{M_L^2}\right), \tag{4.8}$$

where f_1 and f_2 are dimensionless functions of the quark masses. In addition to the loop diagram there is also a contribution from the exchange of a mixed W_L-W_R from the u to the d with the emission of a photon. This contribution has large hadronic uncertainties and is estimated in the harmonic oscillator parton model of the neutron [12,13]. It is given by

$$d_{ex} = \frac{eG_F}{3\pi^{3/2}}\zeta_g\sqrt{2m_q\omega}(1-\beta_g)\mathcal{I}m(V_{ud}^L V_{ud}^{R*}), \tag{4.9}$$

where we use $\sqrt{m_q\omega} = 0.3$ GeV. The edm of the neutron is related to these contributions by

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u + d_{ex}, \tag{4.10}$$

Evaluated with our choice of angles we find

$$d_n = \zeta_g\left(-3.9\times10^{-21}\sin(\alpha+\beta) - 2.3\times10^{-22}\sin(\alpha+\gamma)\right)\text{ e-cm} \tag{4.11}$$

The experimental upper bound is $d_n < 2.6\times10^{-25}$ e-cm. This constraint could be accommodated by a small $\zeta_g \lesssim \mathcal{O}(10^{-5})$. However, this would make ϵ'/ϵ too small. Together the edm and ϵ'/ϵ require

$$\zeta_g \sim \mathcal{O}(10^{-2}), \qquad \sin(\alpha+\beta)\sim\mathcal{O}(10^{-3}). \tag{4.12}$$

We see that ζ_g must be close to the upper bound of (3.2).

V. DISCUSSION ON B DECAYS

We have now found a satisfactory albeit tightly constrained region in which the generalized GW model is valid. If we make the simplifying assumption

$$\beta = \gamma = -\alpha, \qquad \delta = \pi, \tag{5.1}$$

which is consistent with the constraints and necessarily correct to at least $\mathcal{O}(10^{-2})$, the right-handed mixing matrix becomes

$$V^R = \begin{pmatrix} 0.996 & 0.0399 & 0.0799e^{-i3\beta} \\ -0.0862e^{i2\beta} & -0.195e^{i2\beta} & 0.977e^{-i\beta} \\ -0.233e^{i\beta} & 0.979e^{i\beta} & -0.198e^{-i2\beta} \end{pmatrix}. \tag{5.2}$$

Although we found no others in our search, we do not suggest that this is the only possible choice of the seven free parameters in this matrix that satisfy the experimental constraints. We merely point out that this particular choice is phenomenologically acceptable and as such the GW ansatz of pure right-handed b decays is not completely ruled out. We now examine some consequences of this choice of parameters.

The ratio of branching ratios

$$R = \frac{\mathrm{Br}(B^- \to \psi\pi^-)}{\mathrm{Br}(B^- \to \psi K^-)} = 0.052 \pm 0.024. \tag{5.3}$$

has been measured. In the limit of dominant right-handed tree contributions to the decay we have

$$R \approx \left|\frac{V^R_{cd}}{V^R_{cs}}\right|^2 = 0.2, \tag{5.4}$$

where the ratio has been evaluated according to (5.2). To what degree should we trust the discrepancy here between experiment and our model? Certainly the ratios are the same within an order of magnitude. Penguin contributions will affect the theoretical prediction. There will also be a strong contribution due to W_L-W_R mixing because of the relatively large value of ζ_g. To estimate the mixing contribution in the decay $B^- \to \psi K^-$ we look at the ratio of mixed to unmixed tree level contributions

$$\frac{\zeta_g}{\beta_g} \times \frac{V^L_{cs}}{V^R_{cs}} \times \frac{\langle O^{LR}\rangle}{\langle O^{RR}\rangle} \lesssim 3.5, \tag{5.5}$$

where we have used the upper bound (3.2) and set the ratio of matrix elements

$$\frac{\langle O^{LR}\rangle}{\langle O^{RR}\rangle} = \frac{\langle K^-\psi|\bar{b}\gamma^\mu(1+\gamma^5)c\bar{c}\gamma_\mu(1-\gamma^5)s|B^-\rangle}{\langle K^-\psi|\bar{b}\gamma^\mu(1+\gamma^5)c\bar{c}\gamma_\mu(1+\gamma^5)s|B^-\rangle} = 1.4, \tag{5.6}$$

found in the vacuum insertion approximation. There is a substantial and possibly dominant contribution to the decay due to mixing. Our choice of phases and angles is then consistent with the ratio (5.3).

In the neutral B meson system one can write the physical mass eigenstates in terms of the gauge eigenstates as [14]

$$|B^0_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle. \tag{5.7}$$

Their decay amplitudes are

$$A = \langle f|H|B^0\rangle \qquad \text{and} \tag{5.8}$$
$$\bar{A} = \langle f|H|\bar{B}^0\rangle, \tag{5.9}$$

where f is a CP eigenstate. If there is a single dominant decay process (e.g. no strong penguin processes), then the decay asymmetry becomes [15]

$$a_f = \frac{\Gamma(B^0_{phys} \to f) - \Gamma(\bar{B}^0_{phys} \to f)}{\Gamma(B^0_{phys} \to f) + \Gamma(\bar{B}^0_{phys} \to f)} \propto \mathcal{I}m\left(\frac{q\bar{A}}{pA}\right), \tag{5.10}$$

where $B^0_{phys}(\bar{B}^0_{phys})$ denotes the time evolved $B^0(\bar{B}^0)$ meson. If the final state, f, is not a CP eigenstate but a neutral meson such as K_S, then it also contributes a mixing term to the asymmetry.

In the absence of mixing ($\zeta_g = 0$) all B decays occur through pure right-handed interactions. There are both tree and penguin diagrams contributing to the decay. However, due to the phase constraints imposed on this model, tree diagrams and penguin diagrams contribute with the same phase up to corrections to which (5.1) holds. This allows for clean asymmetry predictions which have been previously discussed [4,5].

However, in a situation in which mixing is large it is necessary to consider pollution from mixing. Previous work is no longer applicable to this situation. In the case of $B^0_d \to \psi K_S$, we see that the ratio of left-right to right-right tree amplitudes is given by

$$\left|\frac{T^{LR}}{T^{RR}}\right| = \left|\frac{V^R_{cb}V^{L*}_{cs}}{V^R_{cb}V^{R*}_{cs}}\right| \times \left|\frac{\zeta_g}{\beta_g}\right| \times \left|\frac{\langle O^{LR}\rangle}{\langle O^{RR}\rangle}\right|, \tag{5.11}$$

where

$$\langle O^{RR}\rangle = \langle K_S\psi|\bar{b}\gamma^\mu(1+\gamma^5)c\bar{c}\gamma_\mu(1+\gamma^5)s|B^0\rangle, \tag{5.12}$$
$$\langle O^{LR}\rangle = \langle K_S\psi|\bar{b}\gamma^\mu(1+\gamma^5)c\bar{c}\gamma_\mu(1-\gamma^5)s|B^0\rangle. \tag{5.13}$$

In the vacuum insertion approximation we find $\langle O^{LR}\rangle/\langle O^{RR}\rangle$ to be $\mathcal{O}(1)$. With the upper bound (3.2) this gives

$$\frac{T^{LR}}{T^{RR}} \lesssim 3.55. \tag{5.14}$$

Pollution is possibly over 100% and this decay ceases to be predictive. (It is clean in the standard model due to CKM suppression of the penguins.)

In the same way we examine the decays $B_d \to D^0_1\pi^0, D^+D^-, K_S\pi^0, \phi K_S, K_SK_S$ and $B_s \to D^+_S D^-_S, D^0_1 K_S, \psi K_S, \rho^0 K_S, K^+K^-, \eta'\eta', \phi K_S$. For pure penguin decay processes we determine the mixing contribution by assuming that the ratio of left-right to right-right matrix elements is $\mathcal{O}(1)$. For all of these decays we find pollution due to mixing on the order of 100%. There is little predictive power left from CP asymmetries in the neutral B sector. We stress that although disappointing this is a new result for models employing the GW ansatz. This model is not inconsistent with any values of the various CP asymmetries in the neutral B sector. In fact, this model is consistent with no correlations of any kind among the phases in these decays.

VI. CONCLUSIONS

In this $SU(2)_L \times SU(2)_R \times U(1)$ model where the third generation interacts weakly through pure right-handed couplings the parameters are highly constrained. Nevertheless, we have found a region in parameter space in which this model is consistent with measurements of the neutral meson mass differences Δm_K, Δm_{B_d} and Δm_{B_s} and semi-leptonic B decays. We find that it is necessary that the ratio of coupling constants, g_R/g_L, be on the order of two. Constraining the phases with CP violating observables leads to a second undesirable result. There are three fine tuning conditions on the four phases in the right-handed mixing matrix.

Constraints from CP violating observables also require that mixing between the left and right-handed W's is not small, but $\zeta_g \sim \mathcal{O}(10^{-2})$. This leads to pollution in CP asymmetries in B decays to CP eigenstates on the order of 100%. There are no definite predictions or clean phase measurements in these decays. If discrepancies between experiment and the standard model are found in the B decay asymmetries this model can not be ruled out. However, increasing the lower bound on the right-handed W from direct searches and a more stringent limit on ζ_g could decisively determine the fate of this model.

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APPENDIX A: CALCULATION OF ϵ'/ϵ

The expression (4.4) relates ϵ' to A_I, the neutral K decay amplitude to pions with isospin I, given in (4.5). To calculate ϵ'/ϵ we will use this expression employing an isoconjugate simplification due to Mohapatra and Pati [16,17]. In this procedure the $\Delta S = 1$ weak decay Hamiltonian is decomposed into scalar (S) and pseudoscalar terms (P) which are further decomposed into CP even and odd components (denoted by superscript + or −).

$$H^{\Delta S=1} = S^+ + S^- + P^+ + P^-. \tag{A1}$$

If a relationship can be found such that

$$[I_3, P^-] = i\alpha P^+, \tag{A2}$$

where α is a real constant, then it can be shown that the ratio $\mathcal{I}m A_I/\mathcal{R}e A_I$ is independent of I and $\epsilon' = 0$.

To see this notice that with $|K_{1,2}\rangle$ as CP eigenstates, $I_3|K_1\rangle = -1/2|K_2\rangle$ and $I_3|\pi^i\pi^j\rangle = 0$ where (i,j) denote $(+,-)$ or $(0,0)$. Now

$$\langle\pi^i\pi^j|P^-|K_2\rangle = i\alpha\langle\pi^i\pi^j|P^+|K_1\rangle \tag{A3}$$

holds independent of i and j. The amplitude, A_I, can then be written as a real factor containing matrix elements multiplied by a complex factor independent of I. The matrix elements cancel in $\mathcal{I}mA_I/\mathcal{R}eA_I$ and we have the desired result.

In the GW model the $\Delta S = 1$ Hamiltonian can be split into three terms pertaining to the tree and penguin amplitudes for pure left-handed couplings, pure right-handed couplings and mixed couplings. This can be written as

$$H = (T^{LL} + P^{LL}) + (T^{RR} + P^{RR}) + (T^{LR} + P^{LR}), \tag{A4}$$

where T and P denote tree and penguin contributions respectively. Because of the pure right-handed nature of the third generation couplings, of the penguin diagrams only P^{RR} has a contribution from the top quark. The elements have been calculated [18,19]. They are

$$T^{LL} = \frac{4G_F}{\sqrt{2}}(V^L_{ud}V^{L*}_{us})\bar{s}_L\gamma^\mu u_L\bar{u}_L\gamma_\mu d_L + \text{h.c.}, \tag{A5}$$

$$T^{RR} = \frac{4G_F}{\sqrt{2}}\beta_g(V^R_{ud}V^{R*}_{us})\bar{s}_R\gamma^\mu u_R\bar{u}_R\gamma_\mu d_R + \text{h.c.}, \tag{A6}$$

$$T^{LR} = \frac{4G_F}{\sqrt{2}}\zeta_g(V^L_{ud}V^{R*}_{us}\bar{s}_R\gamma^\mu u_R\bar{u}_L\gamma_\mu d_L + V^R_{ud}V^{L*}_{us}\bar{s}_L\gamma^\mu u_L\bar{u}_R\gamma_\mu d_R) + \text{h.c.}, \tag{A7}$$

$$P^{LL} = \frac{4G_F}{\sqrt{2}}\frac{\alpha_s(\mu)}{24\pi}\left(\sum_{q=u,c} V^L_{qd}V^{L*}_{qs} f\left(\frac{m_q^2}{M_L^2}\right)\right)(\bar{u}\gamma_\mu\tau^a u + \bar{d}\gamma_\mu\tau^a d)\bar{s}_L\gamma^\mu\tau^a d_L + \text{h.c.}, \tag{A8}$$

$$P^{RR} = \frac{4G_F}{\sqrt{2}}\beta_g\frac{\alpha_s(\mu)}{24\pi}\left(\sum_{q=u,c,t} V^R_{qd}V^{R*}_{qs} f\left(\frac{m_q^2}{M_R^2}\right)\right)(\bar{u}\gamma_\mu\tau^a u + \bar{d}\gamma_\mu\tau^a d)\bar{s}_R\gamma^\mu\tau^a d_R + \text{h.c.} \tag{A9}$$

$$P^{LR} = \frac{4G_F}{\sqrt{2}}\zeta_g\frac{\alpha_s(\mu)}{8\pi}(\bar{u}\gamma^\mu\tau^a u + \bar{d}\gamma^\mu\tau^a d)\frac{k^\nu}{k^2} \times\bar{s}i\sigma_{\mu\nu}\left(\sum_{q=u,c}(V^L_{qd}V^{R*}_{qs}\gamma_L + V^R_{qd}V^{L*}_{qs}\gamma_R)g\left(\frac{m_q^2}{M_L^2}\right)m_q\right)\tau^a d + \text{h.c.}, \tag{A10}$$

where f and g are dimensionless functions of quark masses and $\gamma_{R/L} = (1 \pm \gamma^5)/2$.

These terms are now separated into scalar and pseudoscalar components. In dealing with kaon decays to two pions only the pseudoscalar terms are relevant. We want to decompose the Hamiltonian into a part which satisfies an isoconjugate relation, H_0, and a part which accounts for the nonzero ϵ'. To do this we use the unitarity relations,

$$V^L_{ud}V^{L*}_{us} = -V^L_{cd}V^{L*}_{cs}, \tag{A11}$$

$$V^R_{cd}V^{R*}_{cs} = -V^R_{ud}V^{R*}_{us} - V^R_{td}V^{R*}_{ts}, \tag{A12}$$

to remove these two factors from the Hamiltonian. The pseudoscalar part of the Hamiltonian which satisfies an isoconjugate relation can be written as

$$H_0 = (-V^L_{ud}V^{L*}_{us} + \beta_g V^R_{ud}V^{R*}_{us})P + \text{h.c.}, \tag{A13}$$

where

$$P = \frac{G_F}{\sqrt{2}}\Bigg[\bar{s}\gamma^\mu\gamma^5 u\bar{u}\gamma_\mu d + \bar{s}\gamma^\mu u\bar{u}\gamma_\mu\gamma^5 d$$
$$+ \frac{\alpha_s(\mu)}{12\pi}(\bar{u}\gamma_\mu\tau^a u + \bar{d}\gamma^\mu\tau^a d)\bar{s}\gamma_\mu\gamma^5\tau^a d\left(f\left(\frac{m_u^2}{M_L^2}\right) - f\left(\frac{m_c^2}{M_L^2}\right)\right)\Bigg]. \tag{A14}$$

We have used the fact that the relation,

$$f\left(\frac{m_u^2}{M_L^2}\right) - f\left(\frac{m_c^2}{M_L^2}\right) = f\left(\frac{m_u^2}{M_R^2}\right) - f\left(\frac{m_c^2}{M_R^2}\right), \tag{A15}$$

holds up to negligible corrections of $\mathcal{O}(10^{-4})$. Splitting this term and its Hermitian conjugate into CP even and odd states, P^+ and P^- respectively, we arrive at the relationship

$$[I_3, P^-] = -\frac{i\mathcal{I}m(-V_{ud}^L V_{us}^{L*} + \beta_g V_{ud}^R V_{us}^{R*})}{2\mathcal{R}e(-V_{ud}^L V_{us}^{L*} + \beta_g V_{ud}^R V_{us}^{R*})}P^+. \tag{A16}$$

We now examine the mixing terms and the term proportional to $V_{td}^R V_{ts}^{R*}$. We define the operators

$$O^\pm = \bar{s}\gamma^\mu\gamma^5 u\bar{u}\gamma_\mu d \pm \bar{s}\gamma^\mu u\bar{u}\gamma_\mu\gamma^5 d, \tag{A17}$$
$$O^5 = \left(\bar{u}\gamma_\mu\tau^a u + \bar{d}\gamma_\mu\tau^a d\right)\bar{s}\gamma^\mu\gamma^5\tau^a d, \tag{A18}$$
$$O_{LR}^P = \frac{k^\nu}{k^2}\bar{s}i\sigma_{\mu\nu}\gamma^5\tau^a d(\bar{u}\gamma^\mu\tau^a u + \bar{d}\gamma^\mu\tau^a d). \tag{A19}$$

The dominant contributions to the ratios $\mathcal{I}mA_I/\mathcal{R}eA_I$ cancel in the difference of isospin channels. In terms of the above operators the following ratios are needed.

$$R^P = \frac{\frac{\alpha_s(\mu)}{12\pi}\left(f\left(\frac{m_t^2}{M_R^2}\right) - f\left(\frac{m_c^2}{M_R^2}\right)\right)\langle O_{1/2}^5\rangle}{\langle O_{1/2}^+\rangle + \frac{\alpha_s(\mu)}{12\pi}\left(f(\frac{m_u^2}{M_L^2}) - f\left(\frac{m_c^2}{M_L^2}\right)\right)\langle O_{1/2}^5\rangle}, \tag{A20}$$

$$R_u^{LR} = \frac{\langle O_{1/2}^-\rangle - \frac{\alpha_s(\mu)}{4\pi}m_q g(\frac{m_q^2}{M_L^2})\langle O_{LR1/2}^P\rangle}{\langle O_{1/2}^+\rangle + \frac{\alpha_s(\mu)}{12\pi}\left(f\left(\frac{m_u^2}{M_L^2}\right) - f\left(\frac{m_c^2}{M_L^2}\right)\right)\langle O_{1/2}^5\rangle} - \frac{\langle O_{3/2}^-\rangle}{\langle O_{3/2}^+\rangle}, \tag{A21}$$

$$R_c^{LR} = \frac{\frac{\alpha_s(\mu)}{4\pi}m_q g(\frac{m_q^2}{M_L^2})\langle O_{LR1/2}^P\rangle}{\langle O_{1/2}^+\rangle + \frac{\alpha_s(\mu)}{12\pi}\left(f\left(\frac{m_u^2}{M_L^2}\right) - f\left(\frac{m_c^2}{M_L^2}\right)\right)\langle O_{1/2}^5\rangle}, \tag{A22}$$

where we use $\langle O_{\Delta I}\rangle = \langle\pi\pi_I|O|K\rangle$. To first order in ζ_g and β_g, ϵ' is given by

$$\epsilon' = \frac{\omega}{\sqrt{2}}\frac{1}{V_{ud}^L V_{us}^L}\Big(\beta_g\mathcal{I}m(V_{td}^R V_{ts}^{R*})R^P$$
$$+ \zeta_g\mathcal{I}m(V_{ud}^L V_{us}^{R*} - V_{ud}^R V_{us}^{L*})R_u^{LR} + \zeta_g\mathcal{I}m(V_{cd}^L V_{cs}^{R*} - V_{ud}^R V_{us}^{L*})R_c^{LR}\Big), \tag{A23}$$

where $\omega = \mathcal{R}eA_2/\mathcal{R}eA_0 \approx 1/20$.

There are hadronic uncertainties in the ratios of matrix elements. To restrict the parameters in this model we need to obtain order of magnitude estimates of these ratios at the least. Assuming an $\mathcal{O}(1)$ estimate for $\langle O^5\rangle/\langle O^+\rangle$ and $\langle O^-\rangle/\langle O^+\rangle$ we obtain $R^P \sim \mathcal{O}(10^{-1})$. Assuming $k^\nu/k^2 \sim 1$ GeV^{-1} we estimate $R_c^{LR} \sim \mathcal{O}(10^{-1})$ and $R_u^{LR} \sim \mathcal{O}(1)$.

Evaluating the coefficients of the operators after imposing (4.3), the constraint from ϵ, we find the following expression.

$$\epsilon'/\epsilon = \zeta_g \left(1.8 R_c^{LR} \sin(\alpha - \beta) - 4.0 R_u^{LR} \sin(\alpha + \beta)\right) \tag{A24}$$

We use this result in section IV.

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