Lexicase Selection Beyond Genetic Programming

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Summary

Motivation

Why Lexicase?

- Proven to be helpful for several GP problems
- Not specific to GP
- Should be useful wherever there are many objectives (test cases), all of which we want to handle correctly

Why GAs? Why Boolean CSP?

- Lexicase selection is not necessarily unique to genetic programming
- We want to study lexicase selection in a less complex setting; GA provides this

- Boolean Constraint Satisfaction Problem (CSP) can easily be mapped to GAs
- Boolean CSP is more constrainted than most GP problems
- Lexicase does well with uncompromising problems
- Boolean CSP can serve as a proxy for problems with many interconnected constraints

Background: Lexicase Selection

Lexicase Selection

- Parent selection algorithm
- Employs repeated filtering steps of randomly chosen test cases

Result: Individual to be used as a parent

```
candidates := the entire population
cases := list of all test cases in a random order
while True do
    candidates := candidates who perform best on case[0]
    if only one candidate exists in candidates then
        return candidate
    end
    if cases is empty then
        return a randomly selected candidate from candidates
    end
    delete case[0]
end
```

Algorithm 1: Lexicase Selection

Lexicase Selection: An Example

- We want to evolve programs that do well over 4 objectives
- Our population size is 10
- Now we come to the point in our program that uses lexicase selection
- First we set our cases to be the number of objectives, and shuffle this list $[0, 1, 2, 3] \to [2, 0, 1, 3]$
- Then we set our candidates equal to the initial population.

Lexicase Selection: An Example

shuffled cases: 2, 0, 1, 3

case	0	1	2	3
e0	36	80	84	40
e1	47	2	84	30
e2	34	72	38	72
e3	32	96	84	72
e4	47	12	84	36
e5	17	37	84	80
e6	47	18	84	37
e7	47	23	84	84
e8	40	20	38	17
e9	87	25	6	84

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e0	36	80	84	40
e1	47	2	84	30
e2	34	72	38	72
e3	32	96	84	72
e4	47	12	84	36
e5	17	37	84	80
e6	47	18	84	37
e7	47	23	84	84
e8	40	20	38	17
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e7	47	23	84	84

Lexicase Selection: An Example

shuffled cases: ~~2~~, 0, 1, 3

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e0	36	80	40
e1	47	2	30
e3	32	96	72
e4	47	12	36
e5	17	37	80
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e7	47	23	84

Lexicase Selection: An Example

shuffled cases: ~~2~~, 0, 1, 3

case	0	1	3
e0	36	80	40
e1	$\boxed{47}$	2	30
e3	32	96	72
e4	$\boxed{47}$	12	36
e5	17	37	80
e6	$\boxed{47}$	18	37
e7	$\boxed{47}$	23	84

Lexicase Selection: An Example

shuffled cases: ~~2~~, 0, 1, 3

case	0	1		3
e1	47	2		30
e4	47	12		36
e6	47	18		37
e7	47	23		84

shuffled cases: ~~2~~, ~~0~~, 1, 3

case	1	3
e1	2	30
e4	12	36
e6	18	37
e7	23	84

Lexicase Selection: An Example

shuffled cases: ~~2~~, ~~0~~, 1, 3

case	1	3
e7	$\boxed{23}$	84

Lexicase Selection: A Short Analysis

- Let's assume we have a population of individuals, and that there exist only two objectives, or fitness cases
- Where do individuals selected by lexicase fall on the pareto front?
- What does this mean?
- Why is this important? In aggregation, these case **specialists are not often the most fit**.

- However, they may contain features good at solving niche portions of our problem.
- Contributes to diversity.

Background: Boolean CSP

Boolean Expressions

- Evaluate to either TRUE or FALSE (1 or 0)
- $(\neg x_1 \vee x_3 \vee x_0) \wedge (x_2 \vee x_0 \vee x_4 \vee \neg x_1)$
- This formula has 5 variables, all of which are either 1 or 0
- This formula is in CNF (conjunctive normal form)
- In 3CNF, all clauses must have 3 variables
- $(\neg x_1 \vee x_3 \vee x_0) \wedge (x_2 \vee x_0 \vee x_4) \wedge (x_0 \vee x_4 \vee \neg x_1)$

Boolean CSP

- Let's look at the following boolean constraints: $(x_0 \vee x_3 \vee \neg x_1)$ and $(x_2 \vee x_1 \vee \neg x_0)$
- Let's assign to each variable a value. $\alpha = [1, 1, 1, 1]$. In this case, both the constraints evaluate to TRUE. Hence, $\alpha = [1, 1, 1, 1]$ is a solution to the CSP.
- Correct assignment does not have to be unique. Another assignment for this problem is $\beta = [1, 1, 0, 0]$.

Experiments and Results

Mapping Boolean CSP to GA

- We experiment on GA with different selection algorithms: tournament selection (with replacement), roulette selection, and lexicase selection
- $(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_4 \vee x_5) \wedge (x_3 \vee x_5 \vee \neg x_3) \wedge (x_2 \vee x_4 \vee x_1)$
- How would we encode this?
- Candidate solutions are binary vectors of fixed length

Fitness Function

- Let's come back to our example expression
- $(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_4 \vee x_5) \wedge (x_3 \vee x_5 \vee \neg x_3) \wedge (x_2 \vee x_4 \vee x_1)$
- Split the formula into pieces
- piece 1: $(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_4 \vee x_5)$
- piece 2: $(x_3 \vee x_5 \vee \neg x_3) \wedge (x_2 \vee x_4 \vee x_1)$
- Essentially, each constraint is a subformula of our original expression
- We define our fitness function by the number of constraints our solution satisfied. We can interpret this as error. An assignment that solves a given problem then has a fitness value of 0.

Why Boolean Constraints?

- Remember that Boolean CSPs are a proxy for real world problems.
- Many real world problems have different components of error, and this is what our constraints represent.

Experimental Setup

- Tournament selection (various sizes)
- Lexicase selection
- Roulette (fitness proportionate) selection
- 15 different initializations
- 50 different runs for each initialization
- Hence, 750 runs for each parameter combination

Tournament Selection

- For integer-valued size t, we first form a tournament set of t individuals, each chosen with uniform probability (with replacement) from the entire population. We then return, as the selected parent, the individual in the set with the lowest total error.
- For a non-integer-valued size t between 1 and 2 we use tournament size 2 with probability t-1, and select a parent entirely randomly otherwise.

Roulette Selection

The probability of selection for an individual i that satisfies s_i constraints is s_i divided by sum of s_j for all individuals j across the population. In the degenerate case of no individuals satisfying any constraints, which would produce a denominator of zero, an individual is selected at random.

Parameters

Table 1: Problem parameters

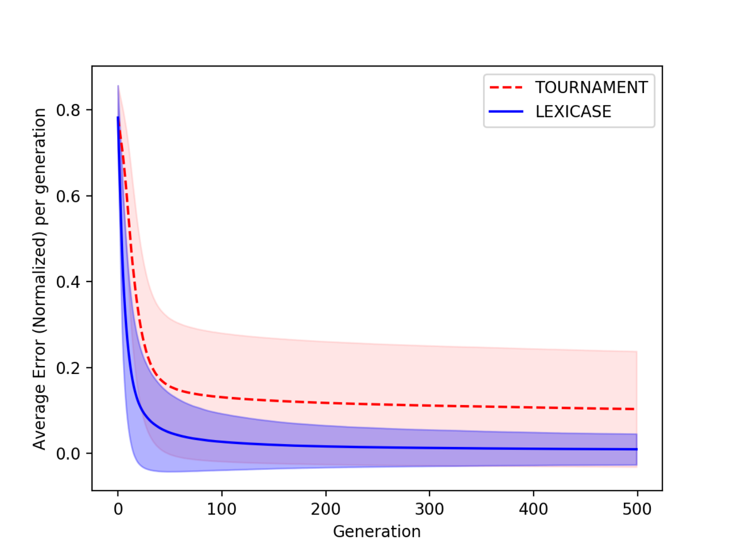
Parameter	Value
Number of variables (v)	20,30,40
Number of constraints (c)	8,12,16,32
Number of clauses per constraint (n)	20,25,30,35,40
Number of problems per combination of v, c, and n	15
Number of runs per method per problem	50
Total runs per method per combination of v, c, and n	750

Parameters

Table 2: Genetic algorithm parameters

Parameter	Value
Population size	200
Number of generations	500
Mutation operator	bit-flip
Probability of Mutation	0.1
Crossover operator	one-point
Probability of Crossover	0.9

Error Profile



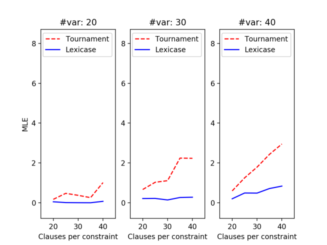
Quality Measures

- **Mean Least Error**:

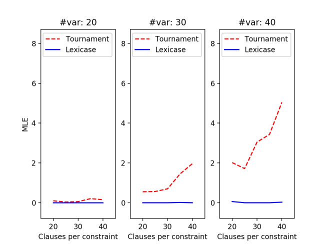
$$MLE = (1/N)\sum_i error(best\_prog_i)$$

- **Success Generation**: Number of generations the algorithm took to find a solution
- **Success Rate**: Fraction of the total runs that succeeded

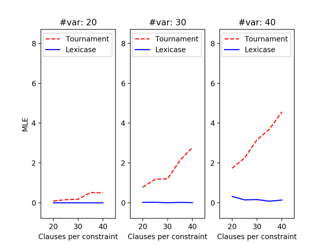
Mean Least Error



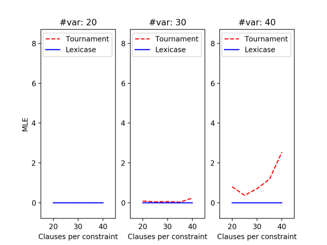
(a) $C = 8$



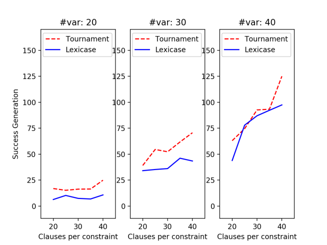
(a) $C = 16$



(b) $C = 12$



(b) $C = 32$



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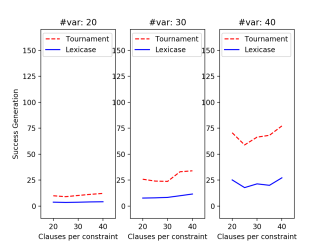

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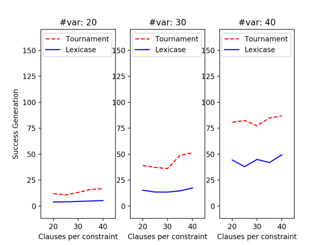
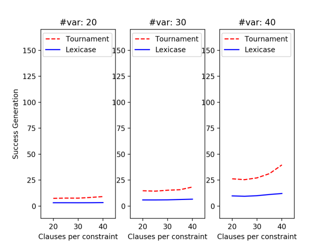


(b) $C = 12$



(b) $C = 32$

Success Rates: different selection algorithms

Table 3: Success rates. Underlines indicate statistically significant improvements, determined using a pairwise chi-square test with Holm correction and $p < 0.05$.

Number of Variables (v)	Number of Constraints (c)	Fitness Proportionate	Tournament (size 2)	Lexicase
20	8	0.835	0.867	0.992
20	12	0.940	0.954	1.000
20	16	0.980	0.987	1.000
20	32	0.999	1.000	1.000
30	8	0.415	0.475	0.889
30	12	0.614	0.697	0.995
30	16	0.815	0.869	1.000
30	32	0.983	0.995	1.000
40	8	0.205	0.257	0.689
40	12	0.224	0.310	0.927
40	16	0.433	0.576	0.993
40	32	0.861	0.944	1.000

Success Rates: different tournament sizes

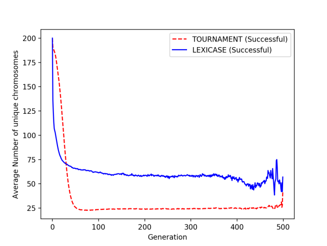
Table 4: Success rate for different tournament sizes. Boldfaced numbers indicate the highest success rate in a particular row.

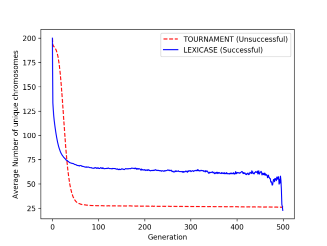
Number of Variables (v)	Number of Constraints (c)	Tournament Size 1.25	Tournament Size 1.5	Tournament Size 2	Tournament Size 4	Tournament size 8
20	8	0.850	**0.860**	0.856	0.818	0.777
20	12	0.948	0.955	**0.959**	0.952	0.934
20	16	0.982	0.987	0.988	**0.989**	0.979
20	32	**1.000**	**1.000**	0.999	**1.000**	0.999
30	8	0.443	**0.485**	0.471	0.428	0.367
30	12	0.644	0.702	**0.773**	0.712	0.618
30	16	0.850	**0.888**	0.879	0.846	0.766
30	32	0.993	**0.996**	**0.996**	0.990	0.974
40	8	0.226	**0.271**	0.137	0.120	0.105
40	12	0.254	**0.322**	0.293	0.245	0.213
40	16	0.510	**0.614**	0.503	0.423	0.335
40	32	0.938	**0.958**	0.901	0.794	0.680

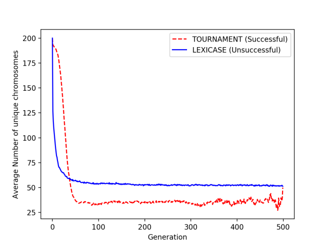
Analysis and Discussion

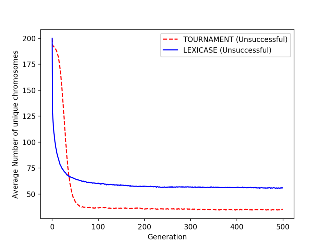
Diversity Analysis

Average number of unique chromosomes (individuals) in the population, over evolutionary time, under different conditions









Conclusions

Future Work

- Apply lexicase to problems that can be mapped to GA
- Lexicase is being used in GP and GA as a parent selection algorithm. However, it really is just a selection algorithm for optimization over many objectives
- Diversity analysis of error vectors. We only looked at the structure of bit strings. Considering error distributions might be interesting
- Study diversity of populations produced by other parent selection algorithms

Conclusions

- Lexicase is not necessarily unique to GP
- Lexicase outperforms tournament selection
- Lexicase maintains high genome diversity
- Studying where lexicase works and where it has difficulty in the Boolean CSP domain may help us improve it

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