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# Baryons as Solitons in Chiral Quark Models

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**Abstract.** We describe the formation of solitons in NJL-type models and discuss the influence of the regularization scheme on the stability of the solution. We concentrate on models with non-local regulators in which stable solutions exist without introducing additional constraints.

## 1 Introduction

The quark models of baryons can be divided into two main classes: (i) *constituent quark models* describing baryons as bound states of three massive quarks interacting via a phenomenological potential and (ii) *chiral models* with "bare" quarks surrounded by a cloud of either effective meson fields or $q\bar{q}$ pairs. The chiral model distinguishes itself from the constituent quark models in that the baryon masses, as well as the constituent quark masses are generated dynamically. The two classes of models describe hadrons at two different levels: at a higher level, the constituent quark model successfully predicts the spectrum of baryons and their excited states; at a lower level, the chiral models are able to describe the vacuum, meson and baryon sectors with the same effective Lagrangian and they model the origin of the masses of constituent quarks and in particular the role the chiral mesons play in the constituent quark model.

We present here the calculation of solitons in NJL-type models and discuss how different regularization schemes influence the stability of solutions. We focus on the version with non-local quark interactions as suggested from the instanton-liquid model which supports stable solutions without introducing artificial constraints as in previous calculations [1, 2].

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## 2 Basic Properties of the NJL

The NJL model assumes an attractive interaction between quarks constructed in terms of quark bilinears in a form that obeys the chiral symmetry. To describe low energy phenomena, it is enough to consider the non-strange quarks and keep only the scalar and pseudoscalar terms. (For a review of the model and its applications see ref. [3].) In the chiral limit, the Lagrangian becomes:

$$\mathcal{L}_{NJL} = \bar{q}\left(\imath\partial_\mu\gamma^\mu\right)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}\,\imath\gamma_5\tau_a q)^2\right] \ , \tag{1}$$

where $a = 1, \ldots 3$ is the isospin index. The interaction is point-like and the UV divergences have to be removed by a suitable regularization prescription.

To solve the model in the mean-field (Hartree) approximation it is practical to introduce auxiliary chiral fields

$$S = -G\,\bar{q}q \qquad \text{and} \qquad P_a = -G\,\bar{q}\,\imath\gamma_5\tau_a q \ , \tag{2}$$

such that the Lagrangian takes the *semi-bosonized* form:

$$\mathcal{L}_{NJL} = \bar{q}\beta\left(\imath\partial_t - h\right)q - \frac{1}{2G}\left(S^2 + P_a^2\right) \ , \tag{3}$$

$$h = -\imath\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta(S + \imath\gamma_5\tau_a P_a) \ . \tag{4}$$

The vacuum in the Hartree approximation is calculated by assuming a constant value, $M$, for $\langle S\rangle$ and evaluating $\langle\bar{q}q\rangle$ in (2) by summing up the bubble graphs or, equivalently, by performing functional integration over the quark fields. This yields the so called "gap equation":

$$\frac{1}{G} = 24\sum_{\Lambda}\frac{1}{k^2 + M^2} \ , \tag{5}$$

where $\sum_\Lambda$ denotes a regularized sum over quark states with a cut-off parameter $\Lambda$. Starting from a zero mass we have generated a finite quark mass. The model thus exhibits the *spontaneous breaking of the chiral symmetry*, yielding a nonzero value for the vacuum quark condensate $\langle\bar{q}q\rangle = -M/G$, and three Goldstone bosons (pions) corresponding to small oscillation of the pseudoscalar field around its vacuum value $\langle P_a\rangle = 0$. The model predicts the pion decay constant, $f_\pi$, and for a finite current quark mass, $m \neq 0$, also a finite pion mass satisfying the Gell-Mann–Oakes–Renner relation. Two of the three free parameters, $G$, $\Lambda$, or $m$ can be eliminated by fitting $f_\pi$ and $m_\pi$. The remaining free parameter is usually expressed in term of $M$, the so-called *constituent mass*.

In calculations of mesons and baryons several regularization schemes have been used including a *sharp cut-off* in 3- or 4-dimensions, the *proper-time* and *Pauli-Villars* regularizations and *non-local regulators* as suggested from the instanton-liquid model. All of them require a rather low cut-off parameter, $\Lambda \sim$ 600 MeV, which indicates that it may be derivable from the underlying theory.

Indeed, the instanton-liquid model [4] predicts a non-local interaction between quarks with $\Lambda$ related to the inter-instanton spacing $\rho^{-1} \sim 600$ MeV. The calculated vacuum properties (quark condensate, gluon condensate) suggest the value $M$ should lie in the range between 300 and 400 MeV.

The soliton, corresponding to a baryon, is constructed around three valence quarks that polarize the vacuum generating a spatially non-trivial configuration. The resulting mean-field potential may make the system stable against decaying into free quarks. This is not a mechanism of confinement but simply a mechanism to bind quarks. If this scenario is realized it means that the interactions of the quarks with the chiral field may play an equally important role in the formation of the baryon as the color confining forces.

The existence of a stable solution is closely related to the choice of regularization scheme. A simple sharp cut-off does not yield stable solutions. Using the proper-time or Pauli-Villars regularization a stable soliton can be formed only by restricting the chiral fields to lie on the chiral circle, i.e. $S(\boldsymbol{r})^2+P_a(\boldsymbol{r})^2 = M^2$, otherwise the scalar field acquires an arbitrarily low value at the origin producing a strongly localized state with a zero energy. Such a constraint does not have any justification in the model. It is only in the version with non-local regulators that solitons exist without introducing additional constraints.

## 3 Solitons in the NJL Model with Non-Local Regulators

In a non-local version of the NJL, the $q\bar{q}$ interaction is smeared by replacing the quark fields in the interaction part of (1) by *delocalized quark fields*:

$$\psi(x) = \langle x|\psi\rangle = \langle x|r|q\rangle = \sum_k r_k e^{\imath kx} \langle k|q\rangle \ . \tag{6}$$

Here $r_k$ is a regulator, diagonal in $k$-space, and $k$ is the *Euclidean* 4-momentum, $k^2 = \omega^2 + \boldsymbol{k}^2$. The instanton-liquid model predicts a form of the regulator. For model calculations it is simpler to take a Gaussian form, $r_k = e^{-k^2/2\Lambda^2}$. Using the Euclidean formulation the Dirac Hamiltonian takes the form

$$h = -\imath\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + r\beta S r + \imath\gamma_5\beta\tau_a r P_a r \ , \qquad r = r\left(-\partial_t^2 - \boldsymbol{\nabla}^2, \Lambda\right) \ . \tag{7}$$

The key point is the construction of the valence orbit. Let us first study the free quark propagator. In the chiral limit ($m = 0$) the inverse propagator takes the form $k_\mu\gamma_\mu + r(k^2)^2 M$. The pole (setting $\boldsymbol{k}^2 = 0$, $k_0^2 = -M_q^2$) occurs at the solution of

$$M_q{}^2 = M^2 \mathrm{e}^{2M_q{}^2/\Lambda^2} \ . \tag{8}$$

The most striking feature of (8) is that the solution exists only below a critical value of $M$. Above this value, free on-shell quarks do not exist in the vacuum.

The treatment of the quark propagator in a spatially non-homogeneous field configuration of the soliton is a non-trivial problem because of the presence of the time-dependent regulator in the Dirac Hamiltonian. For stationary chiral fields $S$ and $P_a$ the Dirac operator $h$ is diagonal in the energy representation:

$$h(\omega^2)\,|\,\lambda_\omega\,\rangle = \imath\omega\,|\,\lambda_\omega\,\rangle \ . \tag{9}$$

The lowest (valence) orbit is obtained as a solution of Eq. (9) for $\imath\omega \to \varepsilon_0$:

$$\left[\frac{\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}}{\imath} + \beta\,\mathrm{e}^{\frac{1}{2\Lambda^2}\,({\varepsilon_0}^2+\boldsymbol{\nabla}^2)}\left(S(\boldsymbol{r}) + \imath\gamma_5\tau_a P_a(\boldsymbol{r})\right)\mathrm{e}^{\frac{1}{2\Lambda^2}\,({\varepsilon_0}^2+\boldsymbol{\nabla}^2)}\right]|\lambda_0\rangle = \varepsilon_0|\lambda_0\rangle\ . \tag{10}$$

The solution of Eq. (10) exists for all $M$ beyond $M_{\mathrm{cr}}$ as shown in Fig. 1.

The soliton is sought iteratively by assuming a hedgehog shape for the chiral fields. Starting from an initial guess for $S(\boldsymbol{r})$ and $P_a(\boldsymbol{r})$ Eq. (9) is solved for the valence orbit, $|\lambda_0\rangle$, and the sea orbits, $|\lambda_\omega\rangle$. In the next iterations new values for the chiral fields are obtained using the Euler-Lagrange equations:

$$\begin{aligned}
\frac{S(\boldsymbol{r})}{G} &= N_c \mathrm{z}_0\langle\lambda_0|r|\boldsymbol{r}\rangle\beta\langle\boldsymbol{r}|r|\lambda_0\rangle + \int\frac{d\omega}{2\pi}\sum_{\lambda_\omega}\frac{\langle\lambda_\omega|r|\boldsymbol{r}\rangle\beta\langle\boldsymbol{r}|r|\lambda_\omega\rangle}{\imath\omega + e_\lambda\,(\omega^2)},\\
\frac{P_a(\boldsymbol{r})}{G} &= N_c \mathrm{z}_0\langle\lambda_0|r|\boldsymbol{r}\rangle\imath\beta\gamma_5\tau_a\langle\boldsymbol{r}|r|\lambda_0\rangle + \int\frac{d\omega}{2\pi}\sum_{\lambda_\omega}\frac{\langle\lambda_\omega|r|\boldsymbol{r}\rangle\imath\beta\gamma_5\tau_a\langle\boldsymbol{r}|r|\lambda_\omega\rangle}{\imath\omega + e_\lambda\,(\omega^2)}.
\end{aligned}$$

Here $z_0$ is the residue factor: $\mathrm{z}_0 = \left(1 - \imath\,\mathrm{d}\varepsilon_0(\omega)/\mathrm{d}\omega|_{\omega=\imath\varepsilon_0}\right)^{-1}$. The convergence is obtained for all values of $M$ beyond a critical value $M_{\mathrm{cr}}$.

In this approach the soliton acquires the correct baryon number [1].

## 4 Properties of the Soliton

Figure 1 (a) shows the soliton energy as a function of $M$ for three regularization schemes: (i) the non-local model described in Sect. 3 yields stable solutions above $M_{\mathrm{cr}} = 280$ MeV which are also energetically stable since the energy of three free on-shell quarks is always higher than the soliton energy, (ii) the model with a non-local regulator of the Gaussian shape that depends only on the 3-momentum $\boldsymbol{k}$ possesses stable solutions above $M \approx 325$ MeV; however, beyond $M \approx 600$ MeV the solution is again unstable since it becomes energetically favorable for two quarks in the soliton to acquire high momenta, lower their effective mass, and escape from the third quark (Such an awkward behavior is a consequence of breaking the Lorentz invariance and does not show up in the model with the 4-momentum regulator.) [6], (iii) the model using the proper time regularization and the chiral-circle constraint has stable solutions above $M = 340$ MeV that are energetically stable only beyond $M = 400$ MeV.

The model with the 4-momentum non-local regulator supports another type of solutions with one or two quarks polarizing the sea (Fig. 1). The solution with three valence quarks is stable against disintegrating into solitons with a lower number of valence quarks. This does not hold for a soliton with four or more valence quarks since in such a case the additional quarks would have to fill the grand spin 1 orbit which is energetically less favorable.

The calculation of observables in the model with non-local regulator is a non-trivial task since the Noether currents acquire additional terms originating from the momentum dependent regulator [2]. Fortunately, their contribution remains small for physically interesting values of $M$ below 400 MeV. The calculated properties reflect a relatively large size of the soliton compared to solutions using e.g. the proper time regularization as well as to the experimental values. In our opinion this is a consequence of the mean-field treatment rather than a serious deficiency of the model. Any improvement of the approximation such as elimination of spurious center-of-mass motion or projection onto subspace with good spin and isospin may considerably reduce the size [5].

## 5 Implications for Other Models

The solitons constructed in the chiral model with non-local regulators share several features found in more phenomenological models. From the model it is possible to derive a version of the linear $\sigma$-model that approximates well the full model [7].

The effective quark mass (i.e the square root of $S^2 + P^2$ shown in Fig. 2) drops almost to 0 in the center of the soliton for $M$ between 300 MeV and 350 MeV similarly as in models based on restoration of the chiral symmetry inside the baryon like different versions of the bag model.

The Goldstone-boson exchange constituent quark model also finds a qualitative support in the non-local model. The solitons consisting of only one valence quark (see Fig. 1) can be interpreted as constituent quarks. Above $M \approx 310$ MeV these solutions are dressed in a cloud of chiral mesons that generate attraction and a 3-quark soliton is formed. The interaction between

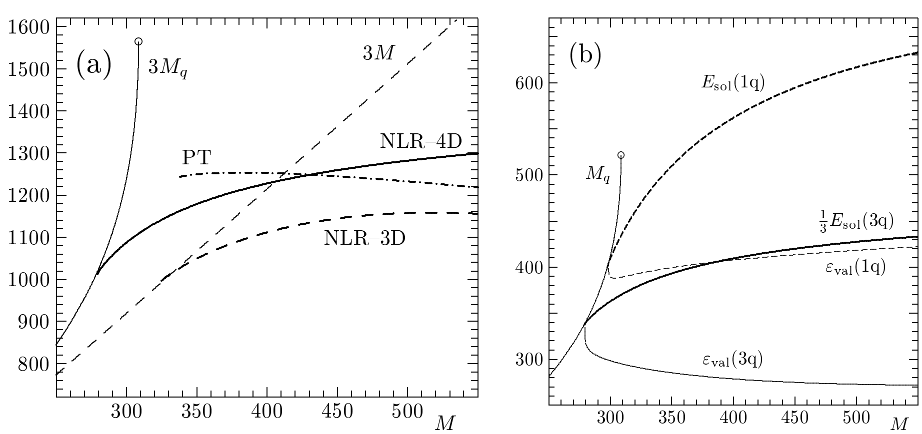


**Figure 1.** (a) The energy of the soliton for different regularization schemes plotted as functions of the parameter $M$ using the 4-dimensional non-local Gaussian regulator (solid line), the 3-dimensional Gaussian regulator (dashed line), and the proper-time regularization (dashed-dotted line). (b) The energy per quark of the soliton with three valence quarks (solid line) and with one valence quark (dashed line) plotted as functions of the parameter $M$. Also shown are the corresponding valence energies. $M_q$ is the free-space on-shell quark mass. All quantities in MeV. For each case the values of $f_\pi$ and $m_\pi$ have been fixed to their physical values.

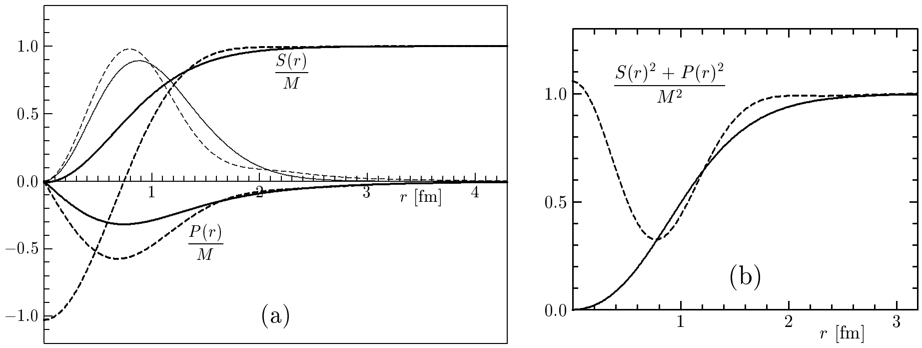


**Figure 2.** (a) Self consistently determined fields and baryon densities (multiplied by $4\pi r^2$); (b) effective squared quark mass for $M = 325$ MeV (solid line) and $M = 750$ MeV (dashed line), plotted as functions of the radial coordinate $r$.

such objects calculated in the linear $\sigma$-model [8] shows a typical behavior of the potential used in the Goldstone-meson exchange constituent quark model.

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