Robust Optimal Cross-Layer Designs for TDD-OFDMA Systems with Imperfect CSIT and Unknown Interference — State-Space Approach based on 1-bit ACK/NAK Feedbacks

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Abstract—Cross-layer designs for OFDMA systems have been shown to offer significant gains of spectral efficiency by exploiting the multiuser diversity over the temporal and frequency domains. In this paper, we shall propose a robust optimal cross-layer design for downlink TDD-OFDMA systems with imperfect channel state information at the base station (CSIT) and unknown interference in slow fading channels. Exploiting the ACK/NAK (1-bit) feedbacks from the mobiles, the proposed cross-layer design does not require knowledge of the CSIT error statistics or interference statistics. To take into account of the potential packet error due to the imperfect CSIT and unknown interference, we define average system goodput (which measures the average b/s/Hz successfully delivered to the mobile) as our optimization objective. We formulate the cross-layer design as a state-space control problem. The optimal power, optimal rate and optimal user allocations are determined as the output equations from the system states based on dynamic programming approach. Simulation results illustrate that the performance of the proposed closed-loop cross-layer design is very robust with respect to imperfect CSIT, unknown interference, model mismatch as well as channel variations due to Doppler.

Keywords: closed-loop, multiple antennas, imperfect CSIT, cross-layer

I. INTRODUCTION

Recently, cross-layer scheduling in OFDMA systems has received tremendous attentions. High spectral efficiency can be achieved by exploiting the multi-user selection diversity over the temporal and frequency domains[1], [2], [3], [4]. To exploit the multi-user selection diversity, knowledge of Channel State Information is required at the base station (CSIT). However, for TDD systems, obtaining perfect CSIT is very challenging, especially for large number of subcarriers or large number of users. When the base station has perfect knowledge of CSIT, the transmitted packet will be virtually error free (with powerful error correction coding) in slow fading channels, and hence, the system can achieve ergodic capacity. However, when the base station has imperfect CSIT or there is unknown interference at the mobile receivers, the scheduled data rate may be larger than the instantaneous channel capacity which is unknown to the base station. This results in packet transmission error even if powerful error correction code is applied. Moreover, the efficiency of the multi-user scheduling is reduced because the wrong set of users may be selected for transmission. Most of the existing cross-layer designs addressed the imperfect CSIT issue are based on heuristic approaches. For example, in [5], [6], the cross-layer schedulers are designed assuming CSIT is perfect and the effect of imperfect CSIT is evaluated by simulations. However, this approach does not offer any design insight on what should be the optimal design and performance with imperfect CSIT as the optimal design can be quite different from that with perfect CSIT. It is also found that the performance of the naive cross-layer scheduler (designed for perfect CSIT) is very sensitive to imperfect CSIT even at very small CSIT errors[7]. In [7], [8], the authors discuss the optimal cross-layer design with imperfect CSIT. However, knowledge of the CSIT error statistics (such as the error distribution or error variance) and interference statistics are required, which may not be available in practice.

In all the works mentioned above, the cross-layer design is open-loop. In open-loop scheduling, the set of admitted users, the power allocation and the rate allocation are determined based on the estimated CSIT (as well as estimated interference), and remain to be the same for the entire scheduling time slot. There are some existing works on the closed-loop adaptation with the ACK/NAK feedbacks [9], [10], [11], [12]. For example, in [9], the authors present a power and rate control policy for a point-to-point system with delay constrained traffic based on ACK/NAK feedback. However, the cross-layer scheduling (user selection) issue is not addressed. In [10], the authors present a heuristic adaptive rate control and randomized scheduling algorithm for flat-fading channels based on learning automata. In all these works, the solutions are heuristic and there is no insight on how good the heuristic solutions approach the optimal performance. Furthermore, knowledge of CSIT error statistics are needed and they did not address the potential issue of unknown interference.

In this paper, we shall propose a robust and optimal closed-loop cross-layer design for downlink TDD-OFDMA systems with imperfect CSIT and unknown interference for slow fading channels. We shall utilize the ACK/NAK (1-bit) feedbacks from the mobiles to adjust the power allocation, the rate allocation as well as user assignment per packet slot. No knowledge...
of the CSIT error statistics or interference statistics is required at the base station. To take into account of the potential packet error, we define average system goodput, which measures the average b/s/Hz successfully delivered to the mobiles, as the optimization objective. We formulate the cross-layer design as a state-space control problem, where the optimal power, optimal rate and optimal user allocations are determined as the output equations from the system states based on dynamic programming approach. Finally, simulation results illustrate that the performance of the proposed closed-loop cross-layer design is very robust with respect to imperfect CSIT, unknown interference, model mismatch as well as channel variation due to Doppler.

This paper is organized as follows. In section II, we outline the OFDMA system model as well as the imperfect CSIT and unknown interference model. In section III, we shall define the system goodput and formulate the closed-loop cross-layer design as a state-space control problem in the presence of imperfect CSIT and unknown interference. In section IV, we shall derive the optimal system outputs as well as the optimal state evolution in transient state based on dynamic programming approach. In section V, we shall discuss the convergence of our state-space approach. In section VI, numerical results are presented and discussed. Finally, we give a brief summary in section VII.

II. OFDMA SYSTEM MODEL

A. Slow Fading Channel Model

We consider a communication system with $K$ mobile users and one base station over a slow-varying frequency selective fading channel. Let $M$ be the number of subcarriers in the system. We consider a scheduling slot structure, which consists of $N$ packet bursts as illustrated in figure 1. We assume the channel is quasi-static within a scheduling slot in this paper.

Let $X_k,m,n$ be the transmit symbol on the $m$-th subcarrier in the $n$-th packet burst, the received signal $Y_k,m,n$ of the $k$-th user on the $m$-th subcarrier in the $n$-th packet burst can be expressed as:

$$Y_{k,m,n} = h_{k,m}X_{m,n} + Z_{k,m,n} + I_{k,m,n}$$

where $h_{k,m}$ is the channel coefficient of the $m$-th subcarrier and the $k$-th user, which is i.i.d. complex Gaussian distributed with zero mean and unit variance, $Z_{k,m,n}$ is the i.i.d. zero-mean complex Gaussian noise with variance $\sigma_z^2/M$ and $I_{k,m,n}$ denotes the zero-mean complex Gaussian interference (due to other cell interference) at the $k$-th mobile receiver with variance $\beta_k^2/M$.

B. Channel Estimation Model and Maximum Achievable Data Rate

In this paper, we consider the imperfect channel state information at the base station (imperfect CSIT), which can be modelled as:

$$h_{k,m}^b = h_{k,m} + \Delta_{k,m}$$

where $h_{k,m}$ is the actual CSI and $\Delta_{k,m}$ is the CSIT estimation error. We consider the case where there is interference to the mobile receivers, which may come from the surrounding cells. As it usually being in practical systems, we assume the base station has no idea about the mobile interference power $\beta_k$ of the $K$ users as well as the variance of the CSIT errors $\Delta_{k,m}$, denoted as $\sigma_z^2$.

For simplicity, we assume the CSIR as well as interference power measurement at the mobile station is perfect for the detection of downlink packets. Hence, based on the received signal model in (1), the maximum achievable data rate of the $k$-th user on the $m$-th subcarrier in the $n$-th packet burst is given by the maximum mutual information between $Y_{k,m,n}$ and $X_{m,n}$ conditional on CSIR $h_{k,m}$:

$$C_{k,m,n} = \max_{p_r(X_{m,n})} I(Y_{k,m,n}; X_{m,n} | h_{k,m})$$

$$= \log_2 \left( 1 + p_{m,n} \frac{|h_{k,m}|^2}{\sigma_z^2/M + \beta_k^2/M} \right)$$

where $p_{m,n}$ is the corresponding transmit power.

C. MAC Layer Model

The MAC layer is responsible for scheduling the radio resource at each scheduling slot based on the estimated CSIT as well as the ACK/NAK feedbacks. Figure 2 illustrates the structure of the cross-layer scheduler. The outputs of the MAC scheduler include the the power allocation \{p_{k,m,n}\}, the rate allocation \{r_{k,m,n}\} as well as user selection \{A_{m,n}\}. After the packets in the first packet slot are transmitted, the selected mobiles will send the ACK/NAK feedbacks to the base station before the next packet is delivered. For subsequent packet bursts in a scheduling slot, the cross-layer scheduler adapts the power allocation, rate allocation as well as user selection based on the CSIT $h_{k,m}^b = \{h_{k,m}^b\}$ and the ACK/NAK feedbacks from the mobiles $f_{m}^{n-1} = \{f_{m,i}^{n-1} \in \{1, n - 1\}, \forall n\}$ ($f_{k,m,n}^{n-1} = 1$ if an ACK is received from the $k$-th user after transmitting the $n$-th packet on the $m$-th subcarrier, and 0 otherwise). Hence, the MAC layer scheduler can be represented by the power allocation policy, rate allocation policy and user selection policy defined below.

Definition 1. Power Allocation Policy:

$$\mathcal{P} = \left\{ p_{k,m,n} (h_{k,m}^b, f_{1}^{n-1}) | \forall k,m,n and \sum_{k,m,n} p_{k,m,n} \leq P_t \right\}$$

Rate Allocation Policy:

$$\mathcal{R} = \left\{ r_{k,m,n} (h_{k,m}^b, f_{1}^{n-1}) | \forall k,m,n \right\}$$

User Selection Policy:

$$\mathcal{A} = \left\{ A_{m,n} (h_{k,m}^b, f_{1}^{n-1}) \subseteq \{1, K\} | \forall m,n and |A_{m,n}| \leq 1 \right\}$$

1For simplicity, we assume the delay of the ACK/NAK is small compared with the packet duration.
D. Packet Transmission Error and Average Goodput

Let \( r_{k,m,n} \) be the scheduled data rate for the user \( k \) on the \( m \)-th subcarrier in \( n \)-th packet. The instantaneous goodput of the \( k \)-th user on the \( m \)-th subcarrier in \( n \)-th packet, which measures the bits successfully delivered to the receiver, is given by:

\[
\rho_{k,m,n} = r_{k,m,n} \mathbb{1}[C_{k,m,n} \geq r_{k,m,n}]
\]  

(5)

where \( \mathbb{1}(I) \) is the indicator function which is equal to 1 if the event \( I \) is true and 0 otherwise. The average total goodput, which measures the average total b/s/Hz successfully delivered to the mobiles (averaged over ergodic realization of CSI), is defined as:

\[
U(P_0, A, \mathcal{R}, \mathcal{P}) = \mathbb{E} \left[ \sum_{n=1}^{N} \sum_{m=1}^{M} \rho_{A_{m,n,m,n}} \right]
\]

(6)

where \( \mathbb{E} \) denotes the expectation of the random variable \( X \), \( \rho_{A_{m,n,m,n}} \) denotes the actual channel coefficients; \( \mathbb{E}_h^b[X] \) denotes the expectation of the random variable \( X \) w.r.t. \( h^b \). \( G(\cdot) \) measures the conditional system goodput (conditioned on the estimated CSIT \( h^b \)). To account for the potential packet error \( (r_{k,m,n} > C_{k,m,n}) \), we shall design the cross-layer scheduler to optimize the total average system goodput \( U(\cdot) \).

III. CROSS-LAYER DESIGN FORMULATION WITH IMPERFECT CSIT AND UNKNOWN INTERFERENCE

A. Closed-Loop Structure of the Cross-Layer Scheduler

Figure 2 illustrates the structure of the closed-loop cross-layer scheduler. The scheduler is characterized by an internal state \( S_n \) and the state evolves based on the feedbacks of the users after each packet transmission. The scheduler outputs are uniquely determined by the system state. We first define the notations as follows:

- \( s_{k,m,n} \) denotes the state of user \( k \) on subcarrier \( m \) during the \( n \)-th packet burst, and \( S_n = \{s_{k,m,n}\}_{k,m} \).
- The causal state evolution policy \( S \) is defined as:

\[
s_{k,m,n} = S(S_{n-1}, f_{x}^{n-1})
\]

(7)

- The system outputs, including the admitted users, power allocation and rate allocation, are functions of the current system state \( S_n \).

From (3), the actual SINR (with unit power) \( \frac{|h_{k,m,n}|^2}{\sigma^2/M + \sigma^2/M} \) is a random variable with certain conditional pdf \( q_{k,m,n}(x|f_{x}^{n-1}, h^b) \) (The base station doesn’t know this distribution explicitly due to the lack of knowledge of the CSIT error statistics and interference statistics, however, the base station can make assumption on this distribution. We shall show the robustness of this assumption). We define the state \( s_{k,m,n} = [k,m,n,u_{k,m,n}] \) to be the lower bound and upper bound of the SINR given the knowledge of CSIT \( h^b \) and the ACK/NAK feedbacks \( f_{x}^{n-1} \):

\[
l_{k,m,n} = \inf_{x} \{x | q_{k,m,n}(x|f_{x}^{n-1}, h^b) > 0 \}
\]

(8)

\[
u_{k,m,n} = \sup_{x} \{x | q_{k,m,n}(x|f_{x}^{n-1}, h^b) > 0 \}
\]

(9)

B. Optimization Objective

To take into consideration of the potential packet errors, given any realization of the imperfect CSIT, we shall optimize the conditional average system goodput \( G(\cdot) \). Since the user selection, power allocation and rate allocation are functions of the system state \( S_n \), we rewrite (6) as:

\[
G(P_0, h^b, A, \mathcal{R}, \mathcal{P}, S) = \mathbb{E}_S \sum_{n=1}^{N} \tilde{g}_n(p_n, h^b, S_n)
\]

(10)

where \( S_n \) = \{S_1, \ldots, S_N \}, \( p_n \) is the total transmit power for the \( n \)-th packet burst, \( \tilde{g}_n \) denotes the conditional average goodput (conditioned on the CSIT \( h^b \) and current system state \( S_n \)) contributed by the \( n \)-th packet burst and is given by:

\[
\tilde{g}_n(\cdot) = \sum_{m=1}^{M} r_{A_{m,n,m,n}} \Pr[C_{A_{m,n,m,n}} > r_{A_{m,n,m,n}}| h^b, S_n]
\]

(11)

Thus, the closed-loop cross-layer scheduling problem with imperfect CSIT and unknown interference can be summarized as the following optimization problem:

Prob 1 (Cross-Layer Problem Formulation with Imperfect CSIT). Given any realization of the estimated CSIT for all mobile users at all subcarriers \( h^b = \{h_{k,m,n}^b\} \), determine the optimal state evolution policy \( S \), the optimal user selection policy \( A \), the optimal power allocation policy \( \mathcal{P} \) as well as the optimal rate allocation policy \( \mathcal{R} \) such that the conditional total goodput, \( G(\cdot) \) is maximized. That is,

\[
G^*(P_0, h^b) = \max_{A, \mathcal{R}, \mathcal{P}, S} \mathbb{E}_S \sum_{n=1}^{N} \tilde{g}_n(p_n, h^b, S_n)
\]

(12)

where the power allocation, rate allocation policies are subject to the following constraints:

- Total Transmit Power Constraint in (4)
- Quality of Service (QoS) Requirement: The conditional packet error probability of all the users is less than a target \( \epsilon \).

IV. OPTIMAL SOLUTION

A. Optimal State Evolution

At the base station, the actual SINR of user \( k \) on the \( m \)-th subcarrier in the \( n \)-th packet burst is a random variable with density \( q_{k,m,n}(x) = q_{k,m,1}(x|f_{1}^{n-1}) \). The event \( \{f_{1}^{n-1}\} \) is equivalent to the event \( \{LB_{k,m,n} \leq x \leq UB_{k,m,n}\} \), where

\[
LB_{k,m,n} = \max_{i} \left\{ \frac{2^{f_{k,m,i}} - 1}{p_{k,m,i}} \right\} 1 \leq i \leq n-1 \text{ and } f_{k,m,i} = 1
\]

\[
UB_{k,m,n} = \min_{i} \left\{ \frac{2^{f_{k,m,i}} - 1}{p_{k,m,i}} \right\} 1 \leq i \leq n-1 \text{ and } f_{k,m,i} = 0
\]

Hence, we have \( q_{k,m,n}(x) = q_{k,m,1}(x|LB_{k,m,n} \leq x \leq UB_{k,m,n}) \). According to the definition of the system state (8,9), we get

\[
l_{k,m,n} = LB_{k,m,n} \text{ and } u_{k,m,n} = UB_{k,m,n}
\]

(13)
And the optimal state evolution in (7) is given by (suppose $k \in A_{m,n}$):

$$l_{k,m,n+1} = \begin{cases} 
\max \{l_{k,m,n}, 2^{r_{k,m,n}-1} \} & \text{if } f_{k,m,n} = 1, \\
\min \{l_{k,m,n}, 2^{r_{k,m,n}-1} \} & \text{otherwise.}
\end{cases} \quad (14)$$

$$u_{k,m,n+1} = \begin{cases} 
\max \{u_{k,m,n}, 2^{r_{k,m,n}-1} \} & \text{if } f_{k,m,n} = 0, \\
\min \{u_{k,m,n}, 2^{r_{k,m,n}-1} \} & \text{otherwise.}
\end{cases} \quad (15)$$

B. Optimal System Output Equations

In fact, the optimization objective $G(.)$ can be divided and conquered into a set of recursive equations. This recursive relationship is summarized in the following lemma:

**Lemma 1 (Recursive Formulation of the Conditional Goodput).** Let $F_n(P, h^b, S_n)$ be the total average goodput from the $n$-th packet burst to the $N$-th packet burst conditional on the CSIT and the system state $S_n$ with total residual power $P$, i.e.,

$$F_n(P, h^b, S_n) = \tilde{g}_n(p_n, h^b, S_n) + \sum_{i=n}^{N} \Pr (S_i|S_n, h^b) \tilde{g}_i(p_i, h^b, S_i). \quad (16)$$

And let $F^*_n(P, h^b, S_n)$ be the optimized $F_n(.)$ subject to power constraint $\sum_{i=n}^{N} \sum_{m=1}^{M} p_{A_{m,n,m}} \leq P$. $F^*_n(.)$ can be expressed recursively as:

$$F^*_n(P, h^b, S_n) = \max \left\{ \tilde{g}_n(p_n) + \sum_{S_{n+1}} \Pr (S_{n+1}|S_n, h^b) F^*_n(P-p_n, h^b, S_{n+1}) \right\} \quad (17)$$

where $p_n = \sum_{m=1}^{M} p_{A_{m,n,m,n}}$ and we assume $F^*_{N+1} = 0$. Furthermore, the optimal conditional goodput in (12) is given by

$$G^*(P_0, h^b) = F^*_1(P_0, h^b) \quad (18)$$

**Proof 1.** The proof of this lemma is based on the recursive structure of $F_n(.)$. We omit it here due to the page limit.

As a result of Lemma 1, the optimization problem with respect to $\{A_{m,n}\}, \{p_{k,m,n}\}, \{r_{k,m,n}\}$ (given any CSIT realization $h^b$ and current system state $S_n$) can be divided and conquer into $N$ steps. The recursive equation in (17) is also called the Bellmen’s equation[13] and the optimization problem belongs to the Markov decision problem. The general solution of the Markov decision problem involves an offline recursion and an online strategy. We elaborate these two procedures as follows.

1) **Backward Recursion for User Selection Policy and Power/Rate Allocation Policies:** In the offline strategy, we shall partition the optimization for the average goodput $G^*(P, h^b)$ with respect to the user selection policy $\{A_{m,n}\}$, the power allocation policy $\{p_{k,m,n}\}$ and the rate allocation policy $\{r_{k,m,n}\}$ (for the $N$ packet bursts) into $N$ recursive optimizations using the recursive relationship of $F^*_n$ and $F^*_{n+1}$ in (17). These optimal policies will be used for the online algorithm when the actual ACK/NAK feedbacks are received. The offline recursive solution is elaborated in the following steps.

- **Step 1.** Consider the last packet burst $n = N$. Recall that the channel capacity is given by:

$$C_{k,m,N} = \log_2 \left( 1 + p_{k,m,N} \frac{|h_{k,m}|^2}{\sigma^2 + (\beta_k/\gamma)^2} \right) \quad (19)$$

where $\frac{|h_{k,m}|^2}{\sigma^2 + (\beta_k/\gamma)^2}$ is a random variable with density $q_{k,m,N}(x)$. Let $Q_{k,m,N}(x)$ be the corresponding cumulative distribution function. To satisfy the packet error requirement $\epsilon$, the scheduled data rate is given by:

$$r_{k,m,N} = \log_2 (1 + p_{k,m,N} \theta_{k,m,N}) \quad (20)$$

where $\theta_{k,m,N}$ is the SINR scaling factor given by:

$$\theta_{k,m,N} = Q^{-1}_{k,m,N}(\epsilon) \quad (21)$$

To determine the optimal power allocation policies, $\{p_{k,m,n}\}$, we form the Lagrangian as:

$$L = \sum_{m=1}^{M} (1 - \epsilon) \log_2 \left( 1 + p_{A_{m,n,m,n}} \theta_{A_{m,n,m,n}} \right) - \lambda_N \sum_{m=1}^{M} p_{A_{m,n,m,n}} \quad (22)$$

Using standard optimization technique, the optimal power allocation policy is given by:

$$p^*_{A_{m,n,m,n}} = \left( \frac{1}{\lambda_N} - \frac{1}{\theta_{A_{m,n,m,n}}} \right)^+ \quad (23)$$

where $(X)^+ = \max(0, X)$, $\lambda_N$ is the Lagrangian multiplier given by

$$\frac{1}{\lambda_N} = \frac{1}{M} (p_N - \sum_{m=1}^{M} \frac{1}{\theta_{A_{m,n,m,n}}}) \quad (23)$$

for sufficiently large $p_N$. Finally, substituting (22) and (23) into the objective function $F^*_N(.)$ (17), the optimal user selection is given by:

$$A_{m,n} = \arg \max_k \{ \theta_{k,m,N} \} \quad (24)$$

Hence, the closed form for $F^*_N(p_N)$ is given by:

$$F^*_N(p_N) = \tilde{g}_N(p_N) = (1 - \epsilon) \log_2 \left( p_N + \frac{1}{\sum_{m=1}^{M} \theta_{A_{m,n,m,n}}} \right)^M \quad (25)$$
Since \( \{\theta_{k,m,N}\} \) are functions of \( S_N \), the equations (24,22,20) give the optimal user selection, the optimal power allocation and the optimal rate allocation in terms of the system state.

- **Step 2.** Consider the packet burst \( n \), where \( n = \{N - 1, N - 2, \ldots, 1\} \). Given the target error probability \( \epsilon \), the state transition probability \( Pr(S_{n+1}|S_n, h^b) \) in (17) has the form of \((1-\epsilon)^n\epsilon^b\), where \( a \) is the total number of ACK feedbacks and \( b \) is the total number of NAK feedbacks after the transmission of the \( n \)-th packet. Since \( \epsilon \) is usually chosen to be very small, most of state transition probabilities are very small except the one when \( a = |A| \) and \( b = 0 \) (in this case, there is no transmission error). Hence, we have:

\[
F^*_n(P, h^b, S_n) 
\approx \max_{\{p_{A,m}, m, n\}} \{\bar{g}_n(p_n, h^b, S_n) + F^*_{n+1}(P - p_n, h^b, S_{n+1})\} \tag{26}
\]

where the state \( S_{n+1} \) is derived from its previous state \( S_n \) based on all ACK feedbacks. Similar to Step 1, the optimal power and rate allocation policies are given by:

\[
p_{A,m,n} = \left(1 - \frac{1}{\lambda_n} \right) \tag{27}
\]

\[
r_{A,m,n} = \log_2(1 + p_{A,m,n} \theta_{A,m,n}) \tag{28}
\]

where

\[
\frac{1}{\lambda_n} = \frac{1}{M} \left( p_n + \sum_{m=1}^{M} \frac{1}{\theta_{A,m,n}} \right) \tag{29}
\]

\[
p_n = \frac{P}{N-n+1} + \frac{1}{N-n+1} \sum_{i=n}^{N} \sum_{m=1}^{M} \frac{1}{\theta_{A,i,m,i}} 
- \sum_{m=1}^{M} \frac{1}{\theta_{A,m,n,n}} \tag{30}
\]

The optimal user selection is given by the following lemma:

**Lemma 2 (Optimal User Selection).** Let \( s_{k,m,n+1}, \ldots, s_{k,m,N} \) be the system states evolved from \( s_{k,m,n} \) with all ACK feedbacks and \( \theta_{k,m,n}, \ldots, \theta_{k,m,N} \) be the corresponding SINR scaling factors. The optimal admitted user in the \( m \)-th subcarrier and the \( n \)-th packet burst is given by:

\[
A_{m,n} = \arg \max_k \prod_{i=n}^{N} \theta_{k,m,i} \tag{31}
\]

**Proof 2.** Please refer to Appendix A

Hence, we have

\[
F^*_n(.,.) = (1 - \epsilon) \log_2 \left( \frac{\prod_{i=n}^{N} \sum_{m=1}^{M} \theta_{A,m,i}}{M(N-n+1)} \right) + (1 - \epsilon)
\]

\[
\times \log_2 \left( \frac{P}{N-n+1} + \sum_{i=n}^{N} \sum_{m=1}^{M} \frac{1}{\theta_{A,i,m,i}} \right)^{(N-n+1)} \tag{32}
\]

2) **Online Solution:** The online strategy is a realtime algorithm. For instance, upon receiving the specific ACK/NAK feedbacks \( f_n \), we update the system state \( S_n \) to \( S_{n+1} \) by (7), and then, select the optimal users, the optimal power and rate allocation by the optimal policies \( \{A_{m,n}\}, \{p_{k,m,n}\} \) and \( \{r_{k,m,n}\} \) (obtained in the offline backward recursion). The online processing is illustrate below:

- **Step 1.** At the first packet burst, the optimal users, the optimal power and rate allocation \( \{A_{m,1}\}, \{r_{k,m,1}\}, \{p_{k,m,1}\} \) based on the estimated CSIT \( h^b \) is obtained according to (31,27,28).

- **Step 2.** Before transmitting the \( n+1 \)-th packet burst (\( n = \{1, 2, \ldots, N-1\} \)), the base station has already obtained the specific ACK/NAK feedbacks of the previous packet \( f_n \) and updated the system state accordingly. The optimal user selection, the optimal power and rate allocation for the \( n+1 \)-th packet are obtained from (31,27,28) and (24,22,20) in the offline recursion.

V. **Steady State Analysis**

The convergence of the system state can be summarized in the following lemma:

**Lemma 3.** For sufficiently large \( n \) and quasi-static fading channel, we have:

\[
\lim_{n \to \infty} A_{m,n} = \arg \max_k \frac{|h_{k,m}|^2}{\sigma^2_2 + \beta_k^2 / M} \tag{33}
\]

Furthermore, if the user \( j \) has the largest SINR in the \( m \)-th subcarrier, we have

\[
\lim_{n \to \infty} S_{j,m,n} = \frac{|h_{j,m}|^2}{\sigma^2_2 + \beta_j^2 / M} \tag{34}
\]

In other words, for sufficiently large \( n \), the system state of the user with largest SINR will converge to the actual SINR and the user selection will converge to the best user selection (as if perfect CSIT were available).

**Proof 3.** Please refer to Appendix A

VI. **Numerical Result and Discussion**

In this section, we shall illustrate the performance of our closed-loop cross-layer scheduler design. In our simulation, the number of users \( K \) is 5, the number of multipaths \( L_p \) is 4 and the target packet error probability \( \epsilon \) is 0.01. For simplicity, we assume the unknown interference power \( \beta_k^2 \) of each user is the same. The unknown interference is quasi-static within a scheduling slot but random between scheduling slots according to \( U(0,1) \). In the simulation, the actual CSI is generated
according to complex Gaussian distribution $CN(0,1)$. We assume the base station does not have any knowledge on the actual interference power $\beta$, actual distribution of the SINR as well as the actual CSIT estimation error $\sigma_\Delta^2$. The base station has default values for these parameters ($\beta_{def} = 1$, $\sigma^2_{\Delta, def} = 0.5$) which is not the same as the actual parameters. We shall show by simulation that although the default parameters may not equal to the actual parameters, the system state in the proposed design can still converge to the actual SINR and the closed-loop system is very robust with respect to the mismatch even in high CSIT error and high interference power. Each point in the figures is obtained by averaging over 1000 independent fading realizations.

A. Performance of the closed-loop Cross-Layer Scheduler on Static Channel

We first consider the case of slow fading in which the channel fading is quasi-static within a scheduling slot. Figure 3 shows the average system goodput versus the transmit power of the proposed closed-loop scheduler at high CSIT errors $\sigma^2_\Delta = 0.1$, $I = 1$, $M = 4$ and the maximum unknown interference power $I = 0.1, 2$. For comparison, we also compare our proposed design with various baselines, namely the open-loop cross-layer scheduler, non-adaptive closed-loop cross-layer scheduler, naive scheduler (designed assuming perfect CSIT) and round robin scheduler. The open-loop scheduler, the round robin scheduler and the naive scheduler are considered as open-loop designs because they did not exploit the ACK/NAK feedbacks from the mobiles. The proposed closed-loop scheduler achieves a significant performance gain over these open-loop schedulers. This illustrates that with the ACK/NAK feedback, significant cross-layer gains can be achieved even at large CSIT errors and large unknown interference. Furthermore, the proposed closed-loop scheduler also achieves a significant performance gain over the non-adaptive closed-loop scheduler, where the CSIT is updated according to the feedbacks, however there is no power adaption among the packet bursts. This illustrates the importance of our proposed design of state-space based adaptions. The proposed design is also robust to the mismatch in the channel statistics and parameters.

Figure 4 illustrates the average goodput of each packet burst (averaged over multiple scheduling slots) at high CSIT errors $\sigma^2_\Delta = 0.1$, $I = 1$, $M = 4$ and $P_0 = 23dB$. The average goodput of the closed-loop scheduler increases with the packet burst index. There are two reasons for this. On one hand, because the scheduler can get better estimation of the actual SINR at later packet slots after receiving more ACK/NAK feedbacks, the decisions of user selection made in the later packet slots are more accurate. Since the scheduler can explore more multiuser diversity in the later packet bursts, the performance is better. On the other hand, since the CSIT is more accurate in the later packet slots, more power will be allocated to them to explore the performance gain of multiuser diversity. As a contrary, the two reference schedulers do not have such behavior because the knowledge of the actual SINR remains to be the same at all packet slots.

B. The Performance Sensitivity on Doppler Spread

In this part, we consider frequency selective fading channels with Doppler frequency $f_d$ from 20Hz to 100Hz, which corresponds to a speed of 9 and 45 km/hr at 2.4GHz. The duration of the packet slot is 0.2ms. Figure 5 illustrates the average system goodput versus the doppler frequency of the proposed closed-loop scheduler, round robin scheduler and naive scheduler at large CSIT errors $\sigma^2_\Delta = 0.1$, $I = 1, 2$, $M = 4$ and $P_0 = 23dB$ respectively. It can be observed that significant gain of the proposed closed-loop cross-layer design can be achieved at moderate to large Doppler.

C. The Convergence of the Close Loop Adaptation

Figure 6 and 7 illustrate the instantaneous scheduled data rate versus time in a scheduling slot at Doppler frequencies of $f_d = 0$ and $f_d = 20Hz$, high CSIT errors $\sigma^2_\Delta = 0.1$ and high interference $I = 1, 2$. In the simulation, $M = 4$, $K = 1$ and $P_0 = 29dB$. In both cases, the scheduled data rate of the proposed closed-loop cross-layer design converges to the instantaneous actual capacity quite well. This justifies the robustness of the proposed closed-loop scheduler with respect to the CSIT error, unknown interference, model mismatch and the channel variation due to Doppler.

VII. SUMMARY

In this paper, we propose a robust cross-layer design for the downlink OFDMA systems with imperfect CSIT and unknown interference for slow frequency selective fading channels. We formulate the cross-layer design as a state-space control problem. The optimal power, optimal rate and optimal user allocation are determined as the output equations from the system state. Based on dynamic programming approach, we work out the optimal state evolution using backward recursion and forward recursion algorithms. Simulations illustrate that the proposed closed-loop cross-layer scheduler has very robust goodput performance at moderate to high CSIT errors, interference power and moderate Doppler.

REFERENCES

APPENDIX A: PROOF OF LEMMA 2

\[ F^*_n(P, h^b, S_n) = \max_{\{A_{m,i}\}, \{p_{k,m,i}\}} (1 - \varepsilon) \sum_{m=1}^{M} \sum_{i=1}^{N} r_{A_{m,i},m,i} \]

\[ = \max_{\{A_{m,i}\}} (1 - \varepsilon) \sum_{m=1}^{M} \log_2 \left( \prod_{i=1}^{N} \theta_{A_{m,i},m,i} \right) \]

where the first equality comes from the target packet error rate constraint which is similar to (20); the second equality is obtained from standard water-filling approach over \( m = 1 \) to \( M \) and \( i = n \) to \( N \) with sufficient large power constraint \( P \); the last approximation is made for sufficiently large \( P \). We can observe from the above equation that the average goodput of a subcarrier is independent of the user selection of other subcarriers. In other words, the user selection of each subcarrier can be decoupled, i.e.:

\[ \{A_{m,n}, ..., A_{m,N}\} = \arg \max_{m=1}^{N} \prod_{i=1}^{N} \theta_{A_{m,i},m,i} \quad \forall m \in \{1, M\} \]  

(35)

Since we only consider ACK feedback, as the packet index grows from \( n \) to \( N \), the SINR scaling factor of the selected user \( \theta_{i,m,i} \) will increase. However, the SINR scaling factor of the un-selected user will remain the same (because they won’t be updated by feedbacks). Hence, the optimal user selection of any subcarrier must satisfy:

\[ A_{m,n} = A_{m,n+1} = ... = A_{m,N} \quad \forall m \in \{1, M\} \]  

(36)

combining (35) and (36), we complete the proof of lemma 2.

APPENDIX B: PROOF OF LEMMA 3

Let’s consider another lemma first.

Lemma 4. if user \( k \) is selected infinite times in the \( m \)-th subcarrier , the state of this user in this subcarrier will converge to the actual SINR.

Proof 4. This is because every selection will lead to update on the user state, which will make the lower bound of the state approach to the upper bound of the state. As a result, both bounds will converge to the actual SINR. We omit the detail of the proof here due to the page limit.

Assume the user \( j \) has the largest SINR \( B_{j,m} \) in the \( m \)-th subcarrier. We can argue that only this user will be selected infinite times in the \( m \)-th subcarrier when \( N \) tends to infinity. Otherwise, suppose another user \( i \) with SINR \( B_{i,m} \) is selected infinite times, we have the following inconsistent conclusions:

- Since the user \( i \) is selected infinite times, according to the above lemma, there should exists a packet burst indexed by \( n \) such that \( l_{i,m,n} \leq B_{i,m} \leq u_{i,m,n} < B_{j,m} \).
- According to the strategy of state evolution, we have \( u_{i,m,p} \leq B_{j,m} \).

Let’s assume \( \prod_{p=0}^{N} \theta_{j,m,l} \) and \( \prod_{p=0}^{N} \theta_{i,m,l} \) at \( p \)-th (\( p \geq n \)) packet burst. Since \( \theta_{j,m,l} \) and \( \theta_{i,m,l} \) (\( l = p + 1, ..., N \)) are derived by assuming all ACK feedbacks, we have \( \theta_{j,m,l} \rightarrow u_{j,m,p} \) and \( \theta_{i,m,l} \rightarrow u_{i,m,p} \) for sufficiently large \( l \). Due to \( u_{j,m,p} > u_{i,m,p} \), we can conclude that \( \theta_{j,m,l} > \theta_{i,m,l} \) for sufficiently large \( l \).

Hence, we have \( \prod_{p=0}^{N} \theta_{j,m,l} > \prod_{p=0}^{N} \theta_{i,m,l} \) when \( N \) tends to infinity. According to our user selection strategy, the user \( i \) will never been selected in the \( p \)-th packet burst. This conflicts with the statement that user \( i \) with SINR \( B_{i,m} \) is selected infinite times.

Hence, only the user \( j \) who has the largest SINR will be selected infinite times. Combining this result with Lemma 4, we can get the conclusion of Lemma 3.
Fig. 3. Average goodput performance versus transmit power at $\sigma^2_\Delta = 0.1$, $M = 4$, $I = \{0.1, 2\}$. *Open-loop* refers to the cross-layer design based on the imperfect CSIT knowledge obtained at the beginning of scheduling slot only. *Non-adaptive closed-loop* refers to the closed-loop cross-layer design where the CSIT can be updated according to the feedbacks and where there is no power adaption among the packet bursts. *Perfect CSIT* refers to the ideal system with perfect CSIT and this serves as performance upper bound for benchmarking. *Round robin scheduler* refers to the naive cross-layer design assuming the CSIT is perfect while selecting user randomly. *Naive scheduler* refers to the cross-layer scheduler assuming the CSIT is perfect.

Fig. 4. Average goodput performance of each packet burst at $\sigma^2_\Delta = 0.1$, $M = 4$, $P_0 = 23dB$, $I = 1$. *Round robin scheduler* refers to the naive cross-layer design assuming the CSIT is perfect while selecting user randomly. *Naive scheduler* refers to the cross-layer scheduler designed for perfect CSIT.

Fig. 5. Average goodput performance versus Doppler frequency at $\sigma^2_\Delta = 0.1$, $M = 4$, $I = \{1, 2\}$ and $P_0 = 23dB$. *Round robin scheduler* refers to the naive cross-layer design assuming the CSIT is perfect while selecting user randomly. *Naive scheduler* refers to the cross-layer scheduler designed for perfect CSIT.

Fig. 6. The transient of the instantaneous scheduled data rate and the actual instantaneous channel capacity versus time (packet slot) at $f_d = 0Hz$, $\sigma^2_\Delta = 0.1$, $M = 4$, $K = 1$ and $P_0 = 29dB$.

Fig. 7. The transient of the instantaneous scheduled data rate and the actual instantaneous channel capacity versus time (packet slot) at $f_d = 20Hz$, $\sigma^2_\Delta = 0.1$, $M = 4$, $K = 1$ and $P_0 = 29dB$. 