Local Contexts

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This (older) version contains technical material that was not included in the final version, in particular a comparison between the present approach and trivalent accounts of presupposition projection (see the Appendix). For all but technical purposes, I recommend reading the final version rather than this one.

Abstract: The dynamic approach posits that a presupposition must be satisfied in its *local context*. But how is a local context derived from the global one? Extant dynamic analyses must specify in the lexical entry of any operator what its 'context change potential' is, and for this very reason they fail to be explanatory. To circumvent the problem, we revise two assumptions of the dynamic approach: we take the update process to be derivative from a classical, non-dynamic semantics - which obviates the need for dynamic lexical entries; and we deny that a local context encodes what the speech act participants 'take for granted'. Instead, we take the local context of an expression *E* in a sentence *S* to be *the smallest domain that one may restrict attention to when assessing E* without jeopardizing the truth conditions of *S*. Local contexts may be computed *incrementally* or *symmetrically*: in the incremental case, only information about the expressions that precede *E* is taken into account; in the symmetric case, all of *S* (except *E*) is accessed. The resulting account of local satisfaction is shown to be equivalent to the 'Transparency theory' of presuppositions (Schlenker 2007a,b), whose incremental version is nearly equivalent to Heim's dynamic semantics. But unlike the Transparency theory, the present account makes it possible to compute in great generality the semantic contribution of an expression in its local context - and thus to offer a general theory of triviality, and possibly of presupposition generation. This account can thus be seen as a synthesis between the Transparency theory and dynamic semantics.

1 Introduction

1.1 The Dynamic Approach

A powerful intuition behind much recent research is that *a presupposition must be satisfied with respect to the context in which it is evaluated*. The relevant notion of context is, in Stalnaker's terminology (Stalnaker 1978), the 'context set', which encodes what the speech act participants take for granted (we will henceforth say 'context' for brevity[2]). But an unadorned version of this analysis faces immediate difficulties with complex sentences: *John is incompetent and he knows it* does not require that the speech act participants already take for granted that John is incompetent, since this proposition is asserted, not presupposed. The dynamic approach solves the problem by postulating that the second conjunct is evaluated with respect to a *local* context, which is obtained by updating the global one with the content of the first conjunct; this immediately explains why the presupposition of the second conjunct is automatically satisfied. This analysis is captured by the dynamic rule represented in (1): the update of a context C with a conjunction is the successive update of C with each conjunct:

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[2] In the literature on indexicals, the term 'context' refers to an object that determines the speaker, time and world of the utterance; the indexical notion should be clearly distinguished from the presuppositional one (a context set can sometimes be equated to a *set* of contexts in the indexical sense).

(1) C[F and G] = C[F][G]

Despite its considerable appeal, this analysis suffers from several well-known deficiencies. In its pragmatic incarnation (Stalnaker 1974), the dynamic approach takes the update to result from a rational process of information exchange. The theory works beautifully for unembedded conjunctions because the assertion of a conjunction can plausibly be equated with the successive assertion of each conjunct; since one may think that the context is updated after each act of assertion, the update rule in (1) has considerable intuitive force. But this analysis does not easily extend to environments in which an expression does not have assertive force. This is the case of embedded conjunctions, and of presupposition triggers that appear in the scope of other connectives or operators:

(2) a. None of my students is both incompetent and aware of it.
b. John never smoked or he has stopped.
c. None of these ten students knows that he is incompetent.

In (2)a, we do find a conjunction whose first element somehow satisfies the presupposition of the second element; but since the conjunction is embedded under a negative quantifier, neither conjunct is 'asserted' in any standard sense. In (2)b, *stop* is in the scope of a disjunction, whose point is precisely that the speaker can assert it without being committed to either disjunct. In (2)c, the presupposition trigger *know* is embedded under the quantifier *none of these ten students*, which yields an inference that *each of these ten students used to smoke*; but it is unclear how a non-propositional element (the Verb Phrase, which cannot be asserted on its own) can have a local context to begin with. In all of these cases, then, it is not obvious how an assertion-based analysis can be developed.

In fact, even in the most favorable cases (unembedded conjunctions, or sequences of sentences in discourse) there is little reason to assume that the addressee must necessarily *grant F* after he has heard the speaker assert it - after all, the speaker might well be wrong, and the addressee might have every reason *not* to believe him. One could argue that all that matters is that the addressee *pretends* to accept the speaker's claim; but even fictional acceptance leads to difficulties. Analyzed in terms of common belief, a context (whether real or fictional) is intrinsically symmetric between the beliefs of the speaker and those of the addressee. But this very symmetry makes it difficult to explain why (3)a is Moore-paradoxical while (3)b isn't (Schlenker 2007b):

(3) a. #It is raining but I (still) don't believe it.
b. It is raining but you (still) don't believe it.

If the context set is really updated with the first conjunct before the second one is processed, both sentences should be equally deviant: after the first update, the context set will entail that it raining; this means in particular that the speaker believes that it is raining, and that the addressee believes it too. When we come to the second conjunct, we should obtain exactly the same deviance in both cases. But in fact there is a clear difference between (3)a and (3)b: the former is Moore-paradoxical, the latter isn't; it seems that in (3)b the purported update process need not apply.

In its semantic incarnation (Heim 1983, following in part Karttunen 1974), the dynamic approach makes the update process part and parcel of the compositional semantics. Thus the rule for *and* in (1) is preserved, but it is interpreted in semantic rather than pragmatic terms - which avoids the technical problems raised by Stalnaker's analysis. As was noted almost from the start, however, the theory fails to explain why there doesn't exist a deviant conjunction *and** that has the same classical content as *and* but the opposite dynamic behavior, as in (4) (Rooth; Soames 1989).

(4) C[(F and* G)] = C[G][F]

(4) predicts that *John is incompetent and* he knows it* should come out as a presupposition failure, whereas *John knows that he is incompetent and* he is* should be entirely acceptable; this is of course the opposite of what we find in natural language. The problem is completely general: any classical operator can be 'dynamicized' in a variety of ways; its dynamic extensions make the same predictions with respect to non-presuppositional sentences, but they make conflicting predictions about presupposition projection - and the only way to choose between them is by invoking the presuppositional data that one sought to explain in the first place. This problem has been fully acknowledged by the pioneer of the approach (Heim 1990, 1992). The only way to avoid it within a lexicalist framework would be to impose constraints on possible dynamic connectives (see LaCasse 2007 for an ongoing attempt)[3].

1.2 A New Theory of Local Contexts

These difficulties have led some to throw the dynamic baby with its lexicalist bathwater (Schlenker 2007a-b). But this measure was premature: we will see that it is possible to reconstruct a notion of 'local context' which is extremely close to that of dynamic semantics, but is derived from a fully predictive algorithm. In order to do so, however, we depart from both sides of the dynamic tradition. Against the pragmatic line, we deny that local contexts result from an update of the beliefs of the speech act participants. Against the semantic line, we deny that they are the product of intrinsically dynamic meanings. Instead, we take the local context of an expression *E* in a sentence *S* to be *the smallest set-theoretic object* (of the right semantic type) *that one may restrict attention to when assessing E* without jeopardizing the truth conditions of *S* relative to the global context. We are in a context C, and we have heard the speaker say: *If John used to smoke, E.* Let us make the intuition clear with an example. We set out to assess the value of the consequent *E* of this conditional, which we analyze for simplicity as a material implication. One strategy would be to check the value of *E* in *all* possible worlds. But for the purposes of the conversation we are solely interested in those worlds that lie in C, because all other worlds are excluded by the shared assumptions of the conversation partners. For this reason, it won't hurt to replace *E* with $^{c'}E$, where *c'* denotes C and $^{c'}E$ is interpreted as the conjunction of *c'* and *E*; this makes it possible to only consider the value of *E* in the C-worlds, without paying attention the value it may have outside of C (since in any event those will make $^{c'}E$ false). However the interpreter can make his life even simpler by further restricting attention to those C-worlds in which John smoked, since all worlds in which John never smoked will make the conditional true no matter what the value of *E* turns out to be. We take the local context of *S* to be the *narrowest* restriction of this sort that we can make without jeopardizing the computation of the truth conditions of the entire sentence. In other words, we take the interpreter to be maximally lazy, in that he seeks to minimize the number of situations he must consider as he assesses the contribution of an expression to the conversation. The local context of an expression *E* is just the smallest domain the interpreter can get away with when he computes the semantic contribution of *E*. Following the processing metaphor of dynamic semantics, we assume that the computation of the local context is done 'on the fly', as a sentence is processed from left to right. This gives

[3] See also Moltmann 1997, 2003 for a different critique of dynamic semantics.

rise to an asymmetry: information that comes before *E* is known when the local context is computed, but information that comes after *E* isn't, and the interpreter must therefore ensure that *no matter how the sentence ends* the local context will indeed be innocuous (the theory is in this sense 'incremental'; we will explore below a variant which is 'symmetric' and does not have this left-right bias).

To sharpen our intuitions, let us first make sure that for a sentence *S* uttered in a context C, the local context of *S* is the global one, i.e. C itself. Certainly the worlds that don't lie in C are known to be irrelevant to the conversation, so the interpreter may without truth-conditional risk replace *S* with $^{c'}S$, where *c'* denotes C; since we interpret $^{c'}S$ as the conjunction of *c'* and *S*, this makes it possible for the interpreter to disregard the value of *S* in the worlds that are outside of C (these will automatically make $^{c'}S$ false). On the other hand, no further restriction can be made without risk. For suppose *c'* denotes a set that excludes some world w of C. Since the interpreter doesn't yet know the value of *S*, it might turn out that *S* is true in w, and false in all other worlds of C; if so, *S* will be true in w, but by computing $^{c'}S$ instead the interpreter will reach the erroneous conclusion that the sentence is systematically false, since *c'* excludes w. So we have obtained two results: restricting attention to C is truth-conditionally innocuous, and any innocuous restriction must include *all* of C. In other words, C is the *strongest* restriction one can make before one assesses *S*, and it is thus the local context of *S*. If *S* is, say, the sentence *John stopped smoking*, we correctly predict that C should guarantee that John used to smoke (since the local context of *S* must entail its presupposition).

To see a more interesting case, let us compute the local context of *S* in the complex sentence *John used to smoke and S*. We ask once again what is the smallest domain of worlds that the interpreter may restrict attention to when he starts interpreting *S*. As before, he may exclude from consideration all worlds that are not compatible with C. But he can do more: any world w in which John never smoked will make the first conjunct false, and thus the value in w of the second conjunct will be immaterial to the conversation. Thus it won't hurt to replace *S* with $^{c'}S$, where *c'* denotes those C-worlds that satisfy the first conjunct. On the other hand, *all* of these worlds must be considered: if *c'* excluded from consideration one world w of C in which John used to smoke, it could turn out that *S* is true in w but false in all other worlds; by computing *John used to smoke and* $^{c'}S$ rather than *John used to smoke and S*, the interpreter would wrongly conclude that the sentence has to be false. The set of C-worlds that satisfy the first conjunct is thus the strongest restriction that the interpreter can make without risk; it is thus the local context of the second conjunct. This correctly predicts that *John used to smoke and he has stopped smoking* does not presuppose anything: by construction, the local context of the second conjunct already entails its presupposition, and so no special demands are made on the global context C. In these simple cases, then, we derive from a classical semantics and a general definition of local contexts the results that dynamic semantics had to stipulate.

The rest of this paper is organized as follows. In Section 2, we define the incremental version of the theory, which most closely resembles traditional dynamic semantics. A symmetric version is motivated and developed in Section 3. In Section 4, we show that our reconstruction of dynamic semantics is equivalent to the Transparency theory, an *anti*-dynamic theory whose incremental version was shown in earlier work to be nearly equivalent to standard dynamic semantics. In Section 5 we discuss two further applications of local contexts: they make it possible to develop a general theory of 'local triviality', and they might be helpful to understand how some presuppositions are generated. Section 6 discusses two extensions of the theory: it offers an alternative definition of our incremental algorithm, suggested by Fox and Stabler; and it analyzes cases in which local contexts as defined do not exist. Finally, in Section 7 we situate our account within the new debate about presupposition projection which has emerged in the last year or so. A systematic formal comparison between five theories of presupposition projection is offered in the Appendix.

2 Incremental Contexts and Incremental Satisfaction

2.1 Preliminaries

We begin by defining precisely the notion of an 'incremental local context', and we apply it to the analysis of presuppositions. For simplicity, we work within a bivalent semantics, and we assume that the presupposition *d* of a an expression $\underline{d}d'$ in a syntactic context *a_b* must be entailed by its local context given the global context set. Following the spirit of Stalnaker's approach, which take this requirement to be pragmatic in nature, and thus presupposition failure need not be encoded in the semantics itself. In fact, for our purposes it suffices to treat the semantics as bivalent, and to interpret $\underline{d}d'$ as the conjunction of *d* and *d'*; the fact that $\underline{d}$ is underlined is crucial to indicate to the pragmatics that it must be entailed by its local context (thus $\underline{p_i}p_k$ will in the end represent a proposition with a presupposition p_i and an assertive content p_k; $\underline{P_i}P_k$ has the same interpretation, except that each element is predicative rather than propositional). In the rest of this discussion, we provide formal details as is needed to offer a self-contained presentation of the theory and of a few examples; systematic definitions and general results are found in the Appendix. We assume throughout a highly simplified formal syntax, summarized in (5), in which constituency is encoded by parentheses: conjunctions and disjunctions have the form *(F and G)* and *(F or G)*, negations have the form *(not F)*, and generalized quantifiers and conditionals appear as *(Q F . G)* and *(if F. G)* respectively.

(5) Syntax
-Generalized Quantifiers: Q ::= Q_i
-Predicates: P ::= P_i | $\underline{P_i}P_k$
-Propositions: p ::= p_i | $\underline{p_i}p_k$
-Formulas F ::= p | (not F) | (F and F) | (F or F) | (if F. F) | (Q_i P . P)

The 'official' object language is supplemented with a notation for local contexts, which we already introduced informally above: we write ${}^{c'}E$ for an expression *E* restricted to *c'*, and we interpret ${}^{c'}E$ as the generalized conjunction of *c'* and *E* (in general, we treat *c'* as a variable, whose value is provided by an assignment function).

(6) Local context notation
a. Syntax: ${}^{c'}F$ is *F* is a formula, ${}^{c'}P$ if *P* is a predicate.
b. Semantics: in all cases, for any expression *E*, ${}^{c'}E$ is interpreted as the (generalized) conjunction of *c'* and *E*.

(We will sometimes extend the object language with predicate conjunctions, which are written as *(P and P')* and receive the natural interpretation).

As mentioned, we view the local context of an expression *E* in a sentence *S* relative to a context set C as the smallest set-theoretic object (of the type determined by *E*) that one can restrict attention to when assessing the contribution of *E* to the truth conditions of *S* relative to C. To implement this idea, we must decide what 'small' and 'restrict attention to' mean. Both notions can easily be defined if *E* is of a type τ that 'ends in t', for instance <s, t>, <s, <e, t>>, <s, <et, t>>, etc. In this case, 'smaller' will mean 'entails', with a generalized notion of entailment; and one may 'restrict attention to' x when evaluating *E* if conjoining an expression that denotes x with *E* does not run the risk of affecting the truth conditions of *S* relative to C (here too we must use a generalized notion of conjunction). We will use the symbol $\leq$ to denote generalized entailment both in the object language and in the meta-language; for conjunction, we use $\wedge$ in the meta-language, and of course ${}^{c'}F$ to indicate that *F* is interpreted with the conjunctive restriction *c'*. We remind the reader in (7) and (8) of the definitions of generalized entailment and generalized conjunction, but these are entirely standard.

(7) Generalized Entailment
a. If x and x' are two objects of a type τ that 'ends in t', and can take at most n arguments, x $\leq$ x' just in case whenever $y_1, ..., y_n$ are objects of the appropriate type, if $x(y_1) ... (y_n) = 1$, then $x'(y_1) ... (y_n) = 1$
b. If E and E' are two expressions of a type τ that 'ends in t',
$w \models^s (E \leq E')$ iff $[\![E]\!]^{w,s} \leq [\![E']\!]^{w,s}$

(8) Generalized Conjunction
a. If x and x' are two objects of a type τ that 'ends in t', and can take at most n arguments of types $\tau_1, ..., \tau_n$ respectively, then
$x \wedge x' = \lambda y_{1\tau_1} \quad \lambda y_{n\tau_n} \quad x(y_1) ... (y_n) = x'(y_1) ... (y_n) = 1$
b. If E and E' are two expressions of a type τ that 'ends in t',
$[\![{}^{E'}E]\!]^{w,s} = [\![(E' \text{ and } E)]\!]^{w,s} = [\![E']\!]^{w,s} \wedge [\![E']\!]^{w,s}$

With these notions in place, we can define two notions of local context: the incremental and the symmetric local context of an expression. The rest of this section is devoted to a theory of incremental contexts and incremental satisfaction, which is the closest counterpart in our system of the local contexts of dynamic semantics. A symmetric version of these notions is developed in Section 3.

2.2 Incremental Local Contexts

Let us start with an example. As we saw above, when we evaluate relative to a context set C a sentence that starts with *((p and q ...* , we can be certain that we need not be concerned with the value that *q* has in those worlds that fail to satisfy *p* - since in these worlds the conjunction *p and q* will be false anyway. In other words, we can be certain that no matter what the end of the sentence - call it *b'* - turns out to be, the restriction to these worlds will be harmless. Calling 'good final' a string that turns the beginning of a sentence into a complete sentence[4], we can thus assert:

(9) For every constituent *d'*, for every good final *b'*, $C \models^{c' \rightarrow \mathbf{p}}$ ((p and ${}^{c'}$d' b' $\Leftrightarrow$ ((p and d' b'

Here we employ standard notations from modal logic: $C \models^{c' \rightarrow \mathbf{p}} F$ means that under an assignment function in which *c'* denotes **p**, every world w in C makes *F* true (i.e. every world w in C guarantees that $w \models^{c' \rightarrow \mathbf{p}} F$). We adopt the further convention of writing in bold the semantic value of an expression, so that for instance **F** is the proposition denoted by the formula *F*. With these conventions, (9) means that if *c'* denotes **p**, for any good final *b'* the formula *((p and ${}^{c'}$d' b'* is equivalent (relative to C) to the formula *((p and d' b'*: the

[4] This is the same thing as what was called in Schlenker 2007a a 'sentence completion'. Thanks to E. Stabler for pointing out that the term 'good final' belongs to established terminology.

restriction to *c'* is innocuous. But if we are *really* lazy when we evaluate the second conjunct, we can do better. Since the worlds that are outside C are excluded from consideration to begin with, we can restrict attention to those worlds *in C* that satisfy **p**. In other words, we may without risk restrict attention to **p** ∧ C:

(10) For every constituent *d'*, for every good final *b'*, $C \models^{c' \rightarrow \mathbf{p} \wedge C}$ ((p and $^{c'}$q b' ⇔ ((p and q b'

Can we be lazier still? No: if *c'* denotes a proper subset S of **p** ∧ C, one that excludes a p-world w of the context set, $^{c'}$*d'* will have to be false at w, while *d'* alone might well be true - this is exactly the reasoning we informally developed in the introduction. If the sentence turns out to be *((p and d') and t)*, where *t* is a tautology, we will have the unfortunate result that w makes *((p and d') and t)* true, but that it makes *((p and $^{c'}$d') and t)* false. Thus by restricting attention to *c'*, we will be led to make a mistake about the truth value of the sentence at w; in this case the restriction to S is not innocuous:

(11) $C \not\models^{c' \rightarrow S}$ ((p and d') and t) ⇔ ((p and $^{c'}$d') and t)

The moral is that if we want to be maximally lazy *without* taking any truth-conditional risk, we may restrict attention to **p** ∧ C, but *all* the worlds in that set must be inspected. In other words, this is the strongest restriction we can give ourselves without taking any risk - which means that **p** ∧ C is the incremental local context of *q*.

In the general case, local contexts are best defined in two steps. First, we find the set of denotations that make *c'* truth-conditionally harmless; we say in such cases (following the terminology of Schlenker 2007a) that (the value of) *c'* is 'transparent', or that it is a 'transparent restriction'. We then ask whether this set has a bottom element, i.e. one that entails all others; if so, it is the incremental local context of the expression (it is shown below that under broad conditions local contexts do exist). A notion of presupposition satisfaction is then easily defined. In this part of our discussion, we stick to the intuition that restrictions must be innocuous *no matter what the end of the sentence* turns out to be. Our theory is thus incremental, and to remind the reader of this fact all the relevant notions will carry the superscript *i* (analogous notions will be defined later for a symmetric version of theory, and at that point the superscript *i* will be replaced with *s*).

The first step, then, is to define the set of denotations that make *c'* (incrementally) transparent:

(12) tr^i(C, d, a_b) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, for every good final *b'*, $C \models^{c' \rightarrow x}$ a $^{c'}$d' b' ⇔ a d' b'}

We can then define the (incremental) local context of *d* as the bottom element of tr^i(C, d, a_b), if it has one:

(13) lc^i(C, d, a_b) = the bottom element[5] of tr^i(C, d, a_b), if it exists; # otherwise.

Using these notations, the informal reasoning we just developed shows that lc^i(C, p, ((_ and q) and r)) = C ∧ **p**.

[5] As mentioned, by 'bottom element' of tr^i(C, d, a_b), we mean an element e such that for all e' ∈ tr^i(C, d, a_b), $e \leq e'$. It is immediate that if a bottom element exists, it is unique: if e_1 and e_2 are both bottom elements, $e_1 \leq e_2$ and $e_2 \leq e_1$, so $e_1 = e_2$ (this is the case because e_1 and e_2 are set-theoretical objects rather than formulas).

Finally, we can say that in a context C, the presupposition *d* of an expression *dd'* that appears in a syntactic environment *a_b* is (incrementally) satisfied in its local context just in case it is entailed by it:

(14) If lc^i(C, dd', a_b) ≠ #, Sat^i(C, dd', a_b) just in case lc^i(C, dd', a_b) ≤ **d**

We can now apply this definition to some traditional examples; in each case, we derive on the basis of a classical semantics the results that Karttunen 1974, Heim 1983, and Beaver 2001 had to stipulate in the update rules of their operators. At this point the details of our formal syntax become rather important. If we seek to determine, say, the incremental context of *F* in the formula *(F and G)*, we must put ourselves in the position of someone who has seen a left bracket, and asks himself what is the narrowest restriction that he can get away with when assessing the meaning of *F,* which is not yet known. Now any formula that starts with *(d'* can be turned into a full sentence in a variety of ways, for instance by adding *and H)*, or for that matter by adding *or H)*, to obtain the formulas *(d' and H)* and *(d' or H)*. And *all* these possibilities must be taken into account to determine whether a restriction *c'* on *d'* does or does not carry any truth-conditional risk.

(i) To start with a particularly simple example, let us show again - slightly more formally than was done in the introduction - that the incremental context of *F* in any formula that starts with *(F ...* is just the context set C. It is clear that this restriction won't carry any truth-conditional risk. Furthermore, if *c'* denotes a set S that excludes a world w from C, it will (among others) fail to guarantee that C |= ($^{c'}$d' and t) ⇔ (d' and t) in case *d'* is true in w and *t* is a tautology. Thus any value for *c'* which excludes some world of C will fail to be transparent; in other words, any transparent restriction must include every world in C. This means that C entails every transparent value for *c'*, and thus that C is the incremental context of *F*.

(15) lc^i(C, F, (_ and G)) = lc^i(C, F, (_ or G)) = C

(ii) The incremental context of *F* in the formula *(not F)* is also the context set C: it certainly doesn't carry any truth-conditional risk; and by the same reasoning as in (i), we can see that if *c'* denotes a set S that excludes a world w of C, we will fail to guarantee that C $|=^{c' \to S}$ (not $^{c'}$d') ⇔ (not d') in case *d'* is true in w (since in this case the left-hand side is true at w because $^{c'}$*d'* is false, while the right-hand side is false at w). This shows again that the incremental context of *F* is C.

(16) lc^i(C, F, (not _)) = C

(iii) The incremental context of *G* in the formula *(F and G)* is C ∧ **F**, as was shown informally in the introduction and more formally in (10)-(11). In Heim's notation, this derives the result that C[F and G] = C[F][G]: the incremental context of *G* is the original context C, updated with *F*.

(17) lc^i(C, q, (p and _)) = C ∧ **F**

(iv) The incremental context of *F* in the formula *(if F. G)* is simply C. For simplicity, we follow Heim 1983 in treating conditionals as material implications. It is immediate that this restriction doesn't jeopardize the truth conditions of the formula relative to C. Furthermore, in case *c'* denotes a set S that excludes some world w of C, if the sentence turns out to be *(if d' . b')* where *d'* is true in w while *b'* is false in w, we will have that w $|=^{c' \to S}$ (if $^{c'}$d' . b') (because *c'* is false in w) but w $\not\models$ (if d' . b') (because *d'* is true and *b'* is false in w). Therefore every transparent value for *c'* must include all of C. C is thus the incremental context of *p*.

(18) lc^i(C, F, (if _ . G)) = C

(v) The incremental context of *G* in the formula *(if F. G)* is also C ∧ **p**. It is immediate that this restriction carries no risk. And if *c'* denotes a set S that excludes a p-world w of C, in case *d'* is true in w we will have both that w $\models^{c' \to S}$ (if p . d') and that w $\not\models^{c' \to S}$ (if p . $^{c'}$d'), and hence C $\not\models^{c' \to S}$ (if p . $^{c'}$d') ⇔ (if p . d'). So the narrowest restriction that one can get away with is C ∧ **F**.

(19) lc^i(C, G, (if F. _)) = C ∧ **F**

We note for future reference that the same reasoning extends to *(if F. (not G)):* C ∧ **F** is clearly a transparent restriction for *G*, and furthermore it entails all transparent restrictions for *G* (the argument is the same as for the preceding case, reversing the value that we consider for *d'*):

(20) lc^i(C, G, (if F. (not _))) = C ∧ **F**

(vi) More interestingly, the incremental context of *F* in *(F or G)* is C ∧ **(not F)** - which derives a result that Beaver 2001 argued for on the basis of presuppositional data. Within the present framework, the argument is quite direct: by propositional logic, *(F or d')* is always equivalent to *(F or ((not F) and d'))*, hence C $\models^{c' \to C \wedge \textbf{(not F)}}$ (F or $^{c'}$d') ⇔ (F or d'); this establishes that C ∧ **(not F)** is a transparent value for *c'*. On the other hand, if *c'* denotes a set S that excludes a (not F)-world w of C, in case *d'* is true in w we will have that w $\models^{c' \to S}$ (F or d') but w $\not\models^{c' \to S}$ (F or $^{c'}$d'), and thus C $\not\models^{c' \to S}$ (F or d') ⇔ (F or $^{c'}$d'); this shows that any transparent for *c'* must include *all* of C ∧ **(not F)**, which is thus the local context we were looking for.

(21) lc^i(C, q, (p or _)) = C ∧ **(not F)**

(vii) Importantly, the present approach also yields a fully explicit notion of 'local context' for expressions that are embedded under quantifiers. Let us first compute the incremental context of the nuclear scope *Q* in the quantified statement *(Every P . Q)*. In earlier examples, the value of the context variable *c'* was a proposition, i.e. an object of type <s, t>. Things are different in this case: for *c'* to be conjoinable with *Q* in the formula *(Every P . $^{c'}$Q)*, it must have the type of a predicate, i.e. <s, <e, t>>. It turns out that the narrowest possible value of *c'* is just $\lambda w_s\ \lambda x_e$. C(w) = 1 and **P**(w)(x) = 1 (we call this function ^{C}**P**); in other words, the local context of *Q* is just **P** restricted to the context set. To see that this is so, we argue in two steps. First, it is clear that such a restriction does not carry any truth-conditional risk: because natural language quantifiers are conservative, within C *(Every P. $^{c'}$D')* is equivalent to *(Every P. D')* whenever *c'* denotes ^{C}**P**. Second, whenever *c'* is transparent in *(Every P. $^{c'}$D')*, the denotation S of *c'* is entailed by ^{C}**P** (formally: ^{C}**P** ≤ S). Suppose, for contradiction, that this is not the case. Then there is some world w of C and some individual d in the domain of w for which ^{C}**P**(w)(d) = 1 but S(w)(d) = 0. At this point we assume that the language is extremely expressive, and that the nuclear scope *D'* could be true of everything except d (i.e. **D'**(w)(x) = 1 iff x ≠ d). If so, we will have the result that w $\models^{c' \to S}$ (Every P. $^{c'}$D') (because d does not belong to S(w)); on the other hand, w $\not\models^{c' \to S}$ (Every P. D'), because d belongs to ^{C}**P**(w) but not to **D'**(w). Thus $C \not\models^{c' \to S}$ (Every P. $^{c'}$D') ⇔ (Every P. D') - which shows that *c'* is not transparent after all.

(22) $lc^i(C, Q, (\text{Every } P\ .\ \_\ )) = {}^{C}\mathbf{P}$

(viii) The result is exactly the same if we are interested in the incremental context of *Q* in the formula *(No P . $^{c'}$Q):* the local context is just $^{C}\mathbf{P}$. This is an important observation because it guarantees that the projective behavior of *(No P . $\underline{Q}Q'$)* is identical to that of *(Every P.$\underline{Q}Q'$)* (as we will see shortly, both sentences presuppose that every *P*-individual is a *Q*-individual). Let us see how the result is derived. By Conservativity, the value $^{C}\mathbf{P}$ will not carry any truth-conditional risk. Now suppose, for contradiction, that some value S for *c'* is not entailed by $^{C}\mathbf{P}$, and thus that for some world w and individual d, $^{C}\mathbf{P}$(w)(d) = 1 but S(w)(d) = 0. Take the nuclear scope *D'* to be true of d and nothing else (i.e. **D'**(w)(x) = 1 iff x = d). In such a case, $w \models^{c' \to S}$ (No P. $^{c'}$D') (because the only member of **D'**(w), namely d, does not belong to S(w)); on the other hand, $w \not\models^{c' \to S}$ (No P. D'), because d belongs both to $^{C}\mathbf{P}$(w) and to **D'**(w). Thus $C \not\models^{c' \to S}$ (No P. $^{c'}$D') ⇔ (No P. D') - which shows that *c'* is not transparent after all.

(23) $lc^i(C, Q, (\text{No } P\ .\ \_\ )) = {}^{C}\mathbf{P}$

2.3 Incremental Satisfaction

Following the logic of dynamic semantics, we can now specify that the presupposition *d* of an expression *$\underline{d}d'$* must be entailed by its (incremental) local context; in fact, in (14) (copied for convenience in (24)a) we already introduced the notation *$Sat^i(C, \underline{d}d', a\_b)$* to indicate that the presupposition of *$\underline{d}d'$* is (incrementally) satisfied relative to C in the syntactic environment *a_b*. We can now add that a formula *F* is presuppositionally acceptable just in case for *all* expressions *$\underline{e}e'$* for which *F* is of the form *a' $\underline{e}e'$ b'* for some strings *a', b'*, *$Sat^i(C, \underline{e}e', a'\_b')$*. We write this stronger condition as $Sat^i(C, F)$, as is summarized in (24)b.

(24) a. $Sat^i(C, \underline{d}d', a\_b)$ just in case $lc^i(C, \underline{d}d', a\_b) \leq \mathbf{d}$
b. $Sat^i(C, F)$ just in case for all expressions *$\underline{e}e'$*, for all strings *a'*, *b'*, if F = a' $\underline{e}$e' b', then $Sat^i(C, \underline{e}e', a'\_b')$

We can immediately apply these definitions to derive the main results of the dynamic analysis of presuppositions. We assume throughout that the sentences are uttered in a context set C, and we complete the analysis of the examples discussed in the previous section.

(i) *($\underline{p}p'$ and q)* and *($\underline{p}p'$ or q)* both require that C |= p
Both (25)a and (25)b are understood to presuppose that *John is incompetent*, which motivates the claim that formulas of the form *($\underline{p}p'$ and q)* and *($\underline{p}p'$ or q)* presuppose *p*.

(25) a. John knows that he is incompetent and he is depressed.
b. John knows that he is incompetent or he is depressed.

We showed in (15) that the incremental context of *$\underline{p}p'$* in either formula is C itself, so we immediately obtain the desired result: C must entail p.

(26) $Sat^i(C, (\underline{p}p' \text{ and } q))$ iff $Sat^i(C, (\underline{p}p' \text{ or } q))$ iff C |= p

(ii) *(not $\underline{p}p'$)* requires that C |= p

(27) strongly suggests that *John is incompetent*, hence the conclusion that negations are 'holes' for presuppositions.

(27) John doesn't know that he is incompetent.

This result is immediate once it is established, as was done in (16), that here too the incremental context of *pp'* is C.

(28) Sat^i(C, (not pp')) iff C |= p

(iii) *(p and qq')* requires that C |= (if p . q)
Dynamic semantics posits that *(p and qq')* presupposes that *p entails q*. This is not entirely uncontroversial - van der Sandt 1992 and Geurts 1999 have argued that in many cases this prediction is too weak (this has been dubbed the 'proviso problem'). For the moment, let us observe with the literature that *some* examples do seem to argue for a conditional presupposition. Assertive examples don't show much because even if they don't *presuppose q*, they certainly entail it. But in questions the inference obtained is sometimes conditional in nature (see van Rooij 2007 for a recent discussion)[6]:

(29) Is it true that John is a diver and that he will be bring his swimming suite?
=> If John is a diver, he has a swimming suite.

We showed in (17) that the incremental context of *qq'* is C ∧ **p**. Incremental satisfaction thus demands that C ∧ **p** entail **q**. But this is just to say that C must guarantee that *p* entails *q*.

(30) Sat^i(C, (p and qq')) iff C |= (if p . q)

(iv) *(if pp' . q)* requires that C |= p
The projection of presuppositions out of the antecedent of indicative conditionals is a staple of presuppositional studies - so much so that it is sometimes taken as a characteristic feature of presuppositions (and certainly *if John stopped smoking, he made a wise decision* does presuppose that John used to smoke). This result is immediately derived since we showed in (18) that the incremental context of *pp'* is C.

(v) *(if p . qq')* requires that C |= (if p . q)
The case of conditionals is entirely parallel to that of conjunctions - and for a reason: in both cases, the incremental context of *qq'* is C ∧ **p**. The reasoning applied in (iii) carries over, and we get the result that C must guarantee that *(if p . q)*.

(31) Sat^i(C, (if p . qq')) iff C |= (if p . q)

(vi) *(p or qq')* requires that C |= (if (not p) . q)
On the basis of examples such as (32)a-b, one can argue (again, controversially) that *(p or qq')* presupposes that *(if (not p) . q)*: these examples certainly trigger the inference that *if John has lung cancer, he used to smoke* or that *If John is a diver, he has a swimming suite*,

[6] It is uncontroversial that in sentences of the form *(q and qq')* no presupposition is projected, but every theory can account for this fact. For van der Sandt and Geurts, this is because the presupposition of the second conjunct can find an anaphoric antecedent in the first conjunct. By contrast, when *p* only entails *q* relative to the context set (rather than in all possible worlds), predictions differ (dynamic semantics predicts a conditional presupposition, van der Sandt and Geurts predict an unconditional one).

but not necessarily anything stronger. This is also the prediction made by Beaver 2001 (Heim 1983 did not discuss disjunctions).

(32) John is not a diver, or (else) he will bring his swimming suite.

This result follows from the present theory once one has established, as was done in (21), that the incremental context of *$\underline{q}q'$* is C ∧ **(not p)**. Incremental Satisfaction requires that C ∧ **(not p)** entails **q**; in other words, C must guarantee that *(if (not p) . q)*:

(33) Sat^i(C, (p . $\underline{q}$q')) iff C |= (if (not p) . q)

(vii)-(viii) *(Every P . $\underline{Q}$Q')* and *(No P . $\underline{Q}$Q')* both require that C |= (Every P . Q)
The result about universal quantifiers is standard. The sentences in (34)a-b trigger universal presuppositions - a conclusion which is confirmed by Chemal 2007 with experimental means: almost 90% of his subjects derive a universal inference that *every P-individual is a Q-individual* in both cases (the demonstrative *these ten students* is designed to make it pragmatically unlikely that the speaker has an additional implicit restriction in mind).

(34) a. None of these ten students takes good of his computer.
=> Each of these ten students has a computer.
b. None of these ten students has stopped smoking.
=> Each of these ten students used to smoke.

This result follows from the present theory once it has been established, as was done in (22)-(23), that the local context of *Q* is ^{C}**P** (i.e. the function $\lambda w_s\, \lambda x_e$. C(w) = 1 and **P**(w)(x) = 1). Incremental satisfaction requires that ^{C}**P** entail **Q**. But this is just to say that for every world w in C, for every individual x in the domain of w, if **P**(w)(x) = 1, **Q**(w)(x) = 1:

(35) Sat^i(C, (Every P . Q)) iff Sat^i(C, (No P . Q)) iff C |= (Every P . Q)

2.4 Dynamic Implementation

Although our analysis did reconstruct a notion of local context, it also departed from dynamic semantics in... not being truly dynamic at all! Specifically, in our system local contexts are derivative from a classical semantics, together with a specification of the syntax of the language under consideration. Still, one could use our framework to constrain a more conservative version of dynamic semantics, one in which all expressions are intrinsically dynamic (but see LaCasse 2007 for an entirely different solution). We can thus require that for any unary or binary connective *, lexical rules specify that the presupposition of *$\underline{F}$F'* in C[*$\underline{F}$F'] really be checked with respect to the incremental context of *$\underline{F}$F'*; and in case no presupposition failure occurs, the update of C with *(*$\underline{F}$F')* is simply the subset of worlds of C that satisfy *(*F')* (given that in such a case *F* is entailed by the incremental context, this is the same thing as satisfying *(*$\underline{F}$F'))*.

(36) C [(* $\underline{F}$F')] = # iff lc^i(C, $\underline{F}$F', *_) ≤ **F**
If ≠ #, C[(* $\underline{F}$F')] = {w∈C: w |= (* F')}

The same reasoning can be applied to binary connectives:

(37) $C[(\underline{F}F' * \underline{G}G')] = \#$ iff (it is not the case that $lc^i(C, \underline{F}F', (\_ * \underline{G}G')) \leq \mathbf{F}$) or ($lc^i(C, \underline{F}F', (\_ * \underline{G}G')) \leq \mathbf{F}$ and (it is not the case that $lc^i(C, \underline{G}G', (\underline{F}F' * \_)) \leq \mathbf{G}$)). If $\neq \#$, $C[(\underline{F}F' * \underline{G}G')] = \{w \in C: w \models (F' * G')\}$.

It can be checked that these templates derive the rules posited for connectives by Heim 1983 (augmented with the asymmetric dynamic disjunction of Beaver 2001). The template in (37) can easily be extended to binary connectives that have a different syntax, such as *(if F . G)* or *(Q F . G)*; it is noteworthy that the same template applies to both cases because, in our highly simplified fragment, they share the same syntax:

(38) $C[(* \ \underline{F}F' . \underline{G}G')] = \#$ iff (it is not the case that $lc^i(C, \underline{F}F', (* \_ . \underline{G}G')) \leq \mathbf{F}$) or ($lc^i(C, \underline{F}F', (* \_ . \underline{G}G')) \leq \mathbf{F}$ and it is not the case that $lc^i(C, \underline{G}G', (\underline{F}F' * \_)) \leq \mathbf{G}$). If $\neq \#$, $C[(* \ \underline{F}F' . \underline{G}G')] = \{w \in C: w \models (F' * G')\}$.

(For reasons that we discuss below, the template for quantifiers derives something close, but not identical, to Heim's treatment of quantifiers; see in particular the Appendix, [22]-[23]).

3 Symmetric Contexts and Symmetric Satisfaction

Up to this point, we have attempted to derive in a principled fashion the results that dynamic semantics had to stipulate by way of lexical entries. Our analysis was incremental because local contexts were computed on the basis of information available *at a given point* during the interpretation of a sentence. But we can also develop a symmetric notion of local context; on this view, the context of an expression *E* in a sentence *S* is computed on the basis of *all* the information available in *S* - except for *E*, of course, whose interpretation the local context is intended to facilitate. Following Schlenker 2007b, we provide some motivations for a symmetric analysis; we then develop a theory of symmetric satisfaction, and explain how the two version of the theory (incremental vs. symmetric) can be integrated.

3.1 Motivations for a Symmetric Analysis

The incremental version of our analysis predicts (like Beaver 2001) that disjunction should display an asymmetric behavior: *($\underline{q}$q' or p)* presupposes *p*, but *(p or $\underline{q}$q')* only presupposes that *if not p, q*. However there are numerous cases in which a symmetric analysis would appear to be more adequate (see also Geurts 1999 for discussion):

(39) a. There is no bathroom or the bathroom is well-hidden (after Partee).
b. The bathroom is well-hidden, or there is no bathroom.

Although (39)b might be a bit less natural or somewhat 'harder' than (39)a, both sentences are understood not to imply that there is a bathroom. On the face of it, these data are problematic for an incremental account, which predicts that (39)b should presuppose that the house has a bathroom. By contrast, a symmetric account that posits that the negation of *either* disjunct can be used to satisfy the presupposition of the other disjunct would seem to fare better.

The discussion is complicated, however, by the issue of local accommodation: in non-presuppositional cases, *(F or G)* gives rise to a very strong implicature that the speaker is uncertain about the truth of *F* and of *G*. Now if the presupposition were computed in the 'standard' way in (39)b, it would contradict this implicature. Several researchers (in particular Gazdar 1979 and Heim 1983) have argued that to avoid such inconsistencies presuppositions can be 'locally accommodated', a process that can be assimilated to the nongeneration of the presupposition (which thus becomes - or in the present framework remains - part of the bivalent content of a clause). To avoid this confound, we must consider other examples[7]:

(40) a. Mary doesn't have cancer, or (else) her doctor will realize that she is sick.
b. Mary's doctor will realize that she is sick, or (else) she doesn't have cancer.

The presupposition predicted by an incremental account for (40)b is that Mary is sick. The speaker may well take this for granted without thereby being certain that Mary has cancer - and thus local accommodation should not be applied in this case (this is because local accommodation is taken to be a 'last resort' mechanism, which may only be applied in the face of some very bad pragmatic outcome - in particular of a possible inconsistency). The data are presumably graded: although (40)b might be more conducive to a presupposition than (40)a, it doesn't quite seem to *force* one. Pending more rigorous investigation of the data, one should presumably explain why a non-presuppositional reading is possible in (40)b, and why it is somewhat harder to obtain than in (40)a.

Importantly, the availability of 'symmetric' readings is not limited to disjunctions; the same data can be replicated with conditionals:

(41) a. If this house has a bathroom, the bathroom is well hidden.
b. If the bathroom is not hidden, this house has no bathroom.

(42) a. If Mary has cancer, her doctor will know / knows that she is sick.
b. If Mary's doctor doesn't know that she is sick, she doesn't have cancer.

An incremental analysis predicts that (41)a and (42)a should presuppose nothing; on the other hand, (41)b and (42)b should respectively presuppose that the house has a bathroom, and that Mary is sick. The theoretical issues and the data are similar to the case of disjunction: (41)b might conceivably be explained by local accommodation (because the potential presupposition contradicts the implicature, triggered by *if F, G*, that the speaker is uncertain about *G*); but (42)b can probably not be explained in this way. These facts appear less surprising if we adopt a symmetric perspective. Trading on the near-equivalence between *if F, G* and its contraposition *if not G, not F*, it is expected that the b-examples should behave like their a-counterparts *if* the computation of presuppositions is allowed to access *all* the semantic information accessible in a sentence (rather than just the information available to the left of the presupposition trigger). In this case, the difference between the two types of examples should be obliterated, as is to some extent the case (the canonical order is still preferred to the non-canonical one).

Potential arguments for a symmetric analysis might even be found in the behavior of conjunctions. Traditionally, the facts are taken to argue rather strongly for an asymmetric analysis: *John knows that he is incompetent, and he is* sounds odd. But upon further inspection there is an independent reason for this deviance - quite generally, it is infelicitous to utter a conjunction whose first element entails the second, as is suggested by the following non-presuppositional contrast:

(43) a. John lives in France and he resides in Paris.
b. #John lives in Paris and he resides in France.

[7] Thanks to B. Spector for discussion of this point.

This contrast will immediately follow from the theory of incremental triviality to be developed in Section 5.1. But in any event, once this factor is controlled for, the asymmetry of conjunction becomes somewhat less clear[8]:

(44) a. John stopped smoking and he used to smoke five packs a day!
b. Is it true that John stopped smoking and (that he) used to smoke five packs a day?
c. I doubt that John stopped smoking and that he used to smoke five packs a day.

Judgments on these sentences are somewhat split, but it doesn't seem impossible to understand them without a presupposition. Even in the case of conjunctions, then, symmetric readings appear to be marginally available.

3.2 Symmetric Local Contexts and Symmetric Satisfaction

The analysis is straightforward: the data we just discussed suggest that local contexts may be computed on the basis of information contained in the entire sentence. The key notion is that of a symmetrically transparent value for a restriction *c'*; it is identical to its incremental counterpart, except that the universal quantification over good finals *b'* is eliminated, and the end of the sentence *b* is taken as given:

(45) $tr^s(C, d, a\_b)$ = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, $C \models^{c' \rightarrow x} a\ {}^{c'}d'\ b \Leftrightarrow a\ d'\ b$}

Just as in the incremental case, the notions of 'local context' and 'local satisfaction' are derivative from that of a transparent restriction:

(46) Symmetric Local Context
$lc^s(C, d, a\_b)$ = the bottom element of $tr^s(C, d, a\_b)$, if it exists; # otherwise.

(47) Symmetric Satisfaction
a. $Sat^s(C, \underline{d}d', a\_b)$ just in case $lc^s(C, \underline{d}d', a\_b) \leq \mathbf{d}$
b. $Sat^s(C, F)$ just in case for all expressions $\underline{e}e'$ for which $F = a'\ \underline{e}e'\ b'$ for some strings *a', b'*, $Sat^s(C, \underline{e}e', a'\_b')$.

It is worth noting that the set of symmetrically transparent restrictions for an expression *E* is always a superset of the set of incrementally transparent restrictions for *E*. This immediately follows from the notions involved, since an incrementally transparent restriction must be transparent *no matter how the sentence ends*, and thus it is in particular symmetrically transparent (i.e. it is transparent when one takes the end of the sentence as given). As a result, a symmetric context always entails the corresponding incremental context, because the former is the bottom element of a 'larger' set than the latter. It is thus easier for the global context set C to guarantee that a presupposition is symmetrically satisfied than to guarantee that it is incrementally satisfied; in other words, incremental satisfaction predicts presuppositions that are at least as strong as symmetric satisfaction. These results are summarized in (48).

(48) For any context set C, for all expressions $\underline{d}d'$ and for all strings *a, b*,
a. $tr^i(C, \underline{d}d', a\_b) \subseteq tr^s(C, \underline{d}d', a\_b)$.

[8] Thanks to B. George, V. Homer, N. LaCasse, and A. Lima for discussion of this point.

Furthermore, if $lc^s(C, d, a\_b) \neq \#$ and $lc^i(C, d, a\_b) \neq \#$,
b. $lc^s(C, d, a\_b) \leq lc^i(C, d, a\_b)$
c. if $Sat^i(C, d, a\_b)$, then $Sat^s(C, d, a\_b)$

In some of the examples we considered earlier, there is no difference between the symmetric and the incremental analysis. Consider for instance negation. We saw earlier that a context-denotation S is incrementally transparent in the environment *(not _)* (for instance for a sentence *(not pp'))* just in case the condition in (49) is satisfied:

(49) For every propositional expression *d'*, $C \models^{c' \to S}$ (not $^{c'}$d') ⇔ (not d')

The key observation was that once the initial string *(not d'* is known, it can be determined that the end of the sentence must consist of a right parenthesis. As a result, the incremental and the symmetric versions of the analysis make the same prediction in this case: a value for *c'* is symmetrically transparent in *(not pp')* under the very condition stated in (49). The same conclusion applies to the local context of *qq'* in the sentence *(p and qq')*. Thus the symmetric version of the theory predicts that *(not pp')* presupposes *p*, and that *(p and qq')* presupposes *(if p. q)*.

In other cases, symmetric satisfaction predicts weaker presuppositions than its incremental counterpart. Consider sentences of the form *(qq' and p)* and *(qq' or p)*: both are predicted by the incremental analysis to presuppose *q*; by contrast, the symmetric analysis predicts that they presuppose *(if p, q)* and *(if (not p), q)* respectively. To see this, we observe that the reasoning is the same as for the *incremental* analysis of *(p and qq')* and *(p or qq')*. This is because in the latter case syntactic considerations guarantee that a formula of the form *(p and d' b'*, where *p* and *d'* are constituents, must end with a right parenthesis, so that *b' =)*. As a result, *c'* is symmetrically transparent in *($^{c'}$qq' and p)* just in case it is incrementally transparent in *(p and $^{c'}$qq')*. Likewise, *c'* is symmetrically transparent in *($^{c'}$qq' or p)* just in case it is incrementally transparent in *(p or $^{c'}$qq')*. Since the notion of 'local context' is defined on the basis of the corresponding set of transparent restrictions, we can immediately determine the value of the symmetric local context of *qq'* in both cases:

(50) a. lc^s(C, qq', (_ and p)) = lc^i(C, qq', (p and _)) = C ∧ **p**
b. lc^s(C, qq', (_ or p)) = lc^i(C, qq', (p or _)) = C ∧ **(not p)**

Thus the theory of symmetric satisfaction predicts that *(qq' and p)* presupposes that *if p, q*, while *(qq' or p)* presupposes that *if not p, q* - a desirable result in view of the symmetric patterns of presupposition projection we observed earlier.

By the same reasoning, the symmetric analysis predicts that *(if qq'. p)* has the presupposition that the incremental analysis predicts for *(if (not p). (not qq'))*. Here the argument is a bit less direct because conditionals are not semantically symmetric. We reason as follows:
-S is *symmetrically* transparent for *c'* in *(if $^{c'}$qq' . p)* just in case:

(51) for every propositional constituent *d'*, $C \models^{c' \to S}$ (if $^{c'}$d' . p) ⇔ (if d' . p)

-S is *incrementally* transparent for *c'* in *(if (not p) . (not $^{c'}$qq'))* just in case for every propositional constituent *d'*, for every good final *b'*, $C \models^{c' \to S}$ (if (not p) . (not $^{c'}$d' b' ⇔ (if (not p). (not d' b'. With a bit of syntactic reasoning (based on the formal fragment in (5)), it can be argued successively that *d'* must be immediately followed by *)*, and that *(not $^{c'}$d')* must itself be followed by *)*, so that the condition is in effect that in (52):

(52) for every propositional constituent *d'*, C $\models^{c' \to S}$ (if (not p) . (not $^{c'}$d')) ⇔ (if (not p). (not d'))

But since we have treated the conditional as a material implication, the condition in (52) is equivalent to that in (51) (just take the contraposition of each side of the biconditional). It follows that the transparent values of *c'* are the same in both cases, and that the corresponding local contexts are also identical:

(53) lc^s(C, $\underline{q}$q', (if _ . p)) = lc^i(C, $\underline{q}$q', (if (not p) . (not _)))

We showed in (20) that lc^i(C, G, (if F . (not *))) = C ∧* ***F***. *Taking F = (not p)* and *G = $\underline{q}$q'*, we obtain lc^i(C, $\underline{q}$q', (if (not p) . (not *))) = C ∧* ***(not p)***. Thus symmetric satisfaction predicts that *(if $\underline{q}$q' . p)* just presupposes *(if (not p) . q)*.

As is well-known, contraposition does not hold in full generality of natural language conditionals, but in the cases at hand the rule is arguably close enough to being true to account for the symmetric reading we observed in (41) and (42)[9].

3.3 Biases: Incremental vs. Symmetric Satisfaction

Following Schlenker 2007b, we propose that the acceptability judgments we obtain for presuppositional sentences are in fact gradient:

-A sentence with a single presupposition trigger is most acceptable if the presupposition is satisfied in its incremental context.

-If not, the sentence may still be 'saved' in case the presupposition is satisfied in its symmetric context.

In other words, we assume that there exists an incremental bias in the computation of the value of the local context: it is easiest to compute the local context of an expression *E* on the basis of the information that has been heard before *E*; but if necessary, information that comes after *E* may be taken into account as well (though at some cost). This bias accounts for the slight deviance of, say, *John has stopped smoking and he used to smoke five packs a day!*. The present account forces us to make the same prediction for disjunctions - *The bathroom is well hidden or there is no bathroom* should be somewhat less acceptable than *There is no bathroom or the bathroom is well hidden*. A systematic empirical investigation would be needed to establish these data.

Further biases could be explored. In general, when more information is obtained about the final form of the sentence, the local context becomes *stronger* (because as one learns more about the final shape of the sentence, it becomes possible to add stronger restrictions without jeopardizing the computation of truth conditions); and as a result the presuppositions that get projected become *weaker*. One case of interest is that in which one only has access to a very small portion of the sentence. To give an example, suppose that we

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[9] The Symmetric Satisfaction might well encounter serious problems when a sentence contains several presupposition triggers. Thus in the example in (i), it is predicted that no presupposition failure obtains, despite the fact that both *$\underline{p}$p'* and *$\underline{q}$q'* trigger a failure on their own.

(i) a. ($\underline{p}$p' and $\underline{q}$q')

b. C = {w_1, w_2}, $w_1 \not\models$ p, $w_1 \not\models$ q, $w_2 \models$ p and $w_2 \models$ q

The Appendix discusses this example in the context of the Transparency theory (see 39). But due to the equivalence between Symmetric Satisfaction and the symmetric version of the Transparency theory (discussed in Section 4.2 and in the Appendix), the reasoning applied in the Appendix carries over to Symmetric Satisfaction. It should be pointed out that the symmetric version of alternatives based on Strong Kleene semantics do not suffer from the same problem, as is discussed in the Appendix.

compute the value of the local context *c'* in *(p and* $^{c'}\underline{q}q'$*) without* regard to the syntactic context of $^{c'}\underline{q}q'$. We will end up with the same result as was obtained for $^{c'}\underline{q}q'$ alone: the local context is just C in this case. Local satisfaction applied to a local context computed in this fashion will yield an unconditional presupposition that *q* should hold in the context. Interestingly, van der Sandt 1992 and Geurts 1999 have argued against dynamic semantics on the grounds that it often predicts conditional presuppositions that are too weak, and they have offered an algorithm that often predicts unconditional presuppositions. In principle, we can achieve similar results *if* we are willing to restrict the information on the basis of which local contexts are computed. Future research will have to determine whether such an analysis can be constrained in an interesting way.

4 Local Satisfaction, Transparency and Dynamic Semantics

In this section, we show that our reconstruction of dynamic semantics is equivalent to the Transparency theory, an analysis that was initially presented as *anti*-dynamic. The incremental version of the Transparency theory was itself shown in earlier work (Schlenker 2007a) to be equivalent to Heim's dynamic semantics under relatively broad conditions; when these are satisfied, we thus have an indirect proof that the incremental version of the present proposal is equivalent to standard dynamic semantics. Throughout this section, we assume that local contexts do exist, which is not always the case; we revisit this question in Section 6.1, where we show that a natural extension of our proposal yields full equivalence with the Transparency theory even when local contexts fail to exist.

4.1 The Transparency Theory

The Transparency theory purports to do without any notion of local context, and to explicate presupposition projection in purely pragmatic terms, on the basis of two Gricean principles of manner. Starting from a sentence *S* and a specification of its classical semantics (with distinguished presupposition triggers), the reasoning is as follows.
-A presupposition is viewed as a distinguished entailment, one that 'wants' to be articulated as a separate conjunct. All things being equal, then, one should say *It is raining and John knows it* rather than *John knows that it is raining*. The constraint that demands that presuppositions be articulated separately is called *Be Articulate*; it can be seen as a Gricean principle of manner, since it imposes a condition on the way in which certain meanings should be expressed.

(54) Be Articulate
Say *a (d and* $\underline{d}$*d') b* rather than *a* $\underline{d}$*d' b*.

-A second principle of manner, *Be Brief*, limits the effects of *Be Articulate*. The intuition is that in any syntactic environment *a _ b*, one should *not* say *a (d and blah) b* in case the words *d and* are certain to be eliminable without truth-conditional loss. *Be Brief* was taken to come in an incremental and in a symmetric version.
• In the incremental version, *d and* is considered idle in case *no matter what follows,* these words are certain to be eliminable given what is already assumed in the conversation. For instance, if it is already assumed that John lives in Paris, it will be idle to start any sentence with ***John lives in France and*** ... . Similarly, no matter what is assumed, a sentence that starts with *If John lives in Paris,* ***he resides in France and*** ... will contain a redundancy, because the words in bold are certain to be eliminable without truth-conditional loss.

• In the symmetric version of *Be Brief*, the entire syntactic environment of a conjunction ... *F and G* ... is taken into account when deciding whether the words *F and* are redundant. All the cases excluded by the incremental version are excluded by the symmetric version, but additional cases are ruled out by the symmetric version. For instance, ***John resides in France and*** *he is happy, if he lives in Paris* is prohibited by the symmetric but not by the incremental version; for no matter what the second conjunct *blah* turns out to be, one can be certain that *John resides in France and blah, if John resides in France* is equivalent to *blah, if John resides in France*.

(55) Be Brief (slightly generalized from Schlenker 2007b)
Let C be a context set, and let *d* be an occurrence of an expression whose type 'ends in t' in a sentence *a (d and d') b*.
a. Incremental Version
d is 'incrementally transparent' - and violates the incremental version of *Be Brief* - just in case for any expression *g* of the same type as *d*, for any good final b',
C |= a (d and g) b' ⇔ a g b'.
b. Symmetric Version
d is 'symmetrically transparent' - and violates the symmetric version of *Be Brief* - just in case for any expression *g* of the same type as *d*,
C |= a (d and g) b ⇔ a g b.

With these principles in place, a theory of presupposition projection was developed by simply postulating that *Be Brief* cannot be violated, while *Be Articulate* can be. This may be encoded by postulating (for instance in an optimality-theoretic framework) that *Be Brief* is more highly ranked than *Be Articulate*:

(56) Be Brief >> Be Articulate

Together, these principles predict that in any syntactic environment a presupposition trigger *d̲d'* must be expressed as *(d and d̲d')*, unless *d* is (incrementally or symmetrically) transparent. To give an example, *p̲p'* presupposes that *p*, because it is precisely in case C |= p that we can be sure that for any *g* C |= (p and g) ⇔ g. Similarly, *(if p . p̲p')* does not presuppose anything, because no matter what C and *g* are, C |= (if p . (p and g)) ⇔ (if p. g). The same reasoning could in principle apply to the sentence *p̲p' if p,* but only if one applies the symmetric rather than the incremental version of *Be Brief*. Furthermore, we can posit that *both* versions of *Be Brief* are in fact at work, but that an articulated sentence is most deviant - and hence its unarticulated counterpart *most acceptable* - if it is ruled out by the incremental version of *Be Brief*. This immediately derives the preference, say, for sentence of the form *(p and q̲q')* over *(q̲q' and p)* in case *p* entails *q*.

Taken together, *Be Brief* and *Be Articulate* imply that a presupposition trigger *d̲d'* in a syntactic environment *a_b* satisfies the incremental or the symmetric version of the Transparency theory just in case its competitor *a (d and d̲d') b* is ruled out by the relevant version of *Be Brief*. To indicate that *d̲d'* is acceptable according to the incremental or symmetric version of the Transparency theory, we write $Transp^i$(C, d̲d', a _ b) or $Transp^s$(C, d̲d', a _ b).

(57) $Transp^i$(C, d̲d', a _ b) iff for any expression *g* of the same type as *d*, for any good final b', C |= a (d and g) b' ⇔ a g b'

(58) $Transp^s$(C, <u>d</u>d', a _ b) iff for any expression *g* of the same type as *d*, C |= a (d and g) b' ⇔ a g b'

We can then say that formula *F* is acceptable according to the Transparency theory just in case every occurrence of any presupposition trigger *<u>d</u>d'* is acceptable; and here too the notion comes in two versions, though both are defined in the same way relative to the relevant version of the *Transp*.

(59) For any v ∈ {i, s}, $Transp^v$(C, F) iff for every expression <u>d</u>d', for all strings a, b, if F = a <u>d</u>d' b, then $Transp^v$(C, <u>d</u>d', a _ b)

Three important remarks must be made at this point.
1. The two versions of *Be Brief* assumed here are far too special to count as primitive: they only explain under what conditions the first conjunct of an expression *F and G* is redundant. But they are hopelessly silent about innumerable cases of redundancy; for instance, they are powerless to explain why *If F, F* or *F or F* are felt to be redundant, since these examples do not even include a conjunction[10]. We will see shortly that the present theory offers a general account of redundancy which is directly applicable to these cases.
2. The incremental version can to some extent be motivated on the basis of a processing metaphor: the beginning of a conjunction, *F and*, is incrementally transparent just in case one can determine *as soon as one has heard it* that it is certain to be eliminable without truth-conditional loss. But as stated the symmetric version is much less natural: one must somehow pretend that one has heard the beginning of the sentence *a*, the end of the sentence *b*, and the beginning of the conjunction *d*, but crucially not the end of the conjunction *[and] d'*! Unfortunately this odd wrinkle is arguably essential to make the right predictions. Consider for instance the sentence *It is John who won*. The negation and the question tests suggest that its presupposition is that *exactly one person won*; and the assertive component has to be that *John won*. But in most cases, if John won, nobody else did, so the assertive component (quasi-)entails the presupposition. If one *did* take into account the second conjunct when determining whether the first one is redundant, one would have to predict that *Exactly one person won and it is John who did* must be ruled out by the symmetric version of *Be Brief*. Since the articulated competitor is ruled out, the sentence *It is John who won* should in general be acceptable without a presupposition! This appears to be incorrect.
3. The analysis (in particular in its incremental version) is most easily implemented if it is assumed that the object language contains quite a few brackets to disambiguate structure. In this respect, the Transparency theory is in exactly the same situation as our reconstruction of dynamic semantics. We come back to this point in Section 6.2.

In Schlenker 2007a, it was shown that for the very fragment we have assumed throughout the present discussion, with expressions of the form *(not F), (F and G), (F or G), (if F. G), (Q F . G)*), the incremental version of the Transparency theory derives almost all the results of Heim 1983. We will now extend these results to our reconstruction of dynamic semantics by showing that the latter is itself equivalent to the Transparency theory; near-equivalence with Heim's dynamic semantics will immediately follow.

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[10] Redundancies that arise because of the second conjunct of an expression *F and G* are also left out of the analysis.

4.2 Equivalence with the Transparency theory

Our reconstruction of dynamic semantics does things in two steps:
-it starts by defining the local context of an expression $\underline{d}d'$ in an environment *a_ b* as the strongest **c'** for which **c'** is (incrementally or symmetrically) transparent in $a\ ^{c'}\underline{g}\ b$ relative to the context set C.
-it then requires that this **c'** should entail **d**.
The Transparency theory does essentially the same thing, but in a single step: given a sentence *a* $\underline{d}d'$ *b*, it simply asks whether $\underline{d}$ is (incrementally or symmetrically) transparent no matter what the assertive component *d'* turns out to be. Because the theory is based on a competition between *a* $\underline{d}d'$ *b* and its 'articulated' competitor *a (d and* $\underline{d}d'$*) b,* the relevant notion of 'transparency' involves a full conjunction (i.e. we ask whether *d and* could be eliminated without truth-conditional loss), but the end result is still that the presupposition must be transparent.

It can be shown that whenever the local context of $\underline{d}d'$ exists, $\underline{d}d'$ satisfies Transparency (in its incremental or symmetric version) just in case *d* is entailed by its (incremental or symmetric) local context:

(60) **Equivalence with Transparency - Special Case**
For any v ∈ {i, s}, for every formula that has the form a $\underline{d}$d' b, if lc^v(C, $\underline{d}$d', a _ b) ≠ #, then $Transp^v$(C, $\underline{d}$d', a _ b) iff Sat^v(C, $\underline{d}$d', a _ b).

The argument is straightforward; we only sketch it for the incremental version (the argument is analogous for the symmetric version, taking *b' = b*).
-First, suppose that $Transp^i$(C, $\underline{d}$d', a_b). Then for every *g* of the same type as *d* and for every good final *b'*, C |= a (d and g) b' ⇔ a g b'. Using our superscript notation, this also means that C |= a ^{d}g b' ⇔ a g b', and thus that **d** is a transparent restriction for *g*. Since lc^i(C, $\underline{d}$d', a_b) is the bottom element of the set of transparent restrictions, it immediately follows that lc^i(C, $\underline{d}$d', a _ b) ≤ **d**.
-Second, suppose that lc^i(C, $\underline{d}$d', a _ b) ≤ **d**. Then for every *g* of the same type as *d*, for every good final *b'*:

(61) a. C $|=^{c' \rightarrow\ lc^{i}(C,\ \underline{d}d',\ a\ \_\ b)}$ a $^{c'}$g b' ⇔ a g b'
b. C $|=^{c' \rightarrow\ lc^{i}(C,\ \underline{d}d',\ a\ \_\ b)}$ a $^{c'}$(d and g) b' ⇔ a (d and g) b'

Since lc^i(C, $\underline{d}$d', a _ b) ≤ **d**, replacing *g* with *(d and g)* in *a* $^{c'}$*g b'* won't affect the truth conditions:

(62) C $|=^{c' \rightarrow\ lc^{i}(C,\ \underline{d}d',\ a\ \_\ b)}$ a $^{c'}$g b' ⇔ a $^{c'}$(d and g) b'

Putting (61)a-b and (62) together, we conclude that C $|=^{c' \rightarrow\ lc^{i}(C,\ \underline{d}d',\ a\ \_\ b)}$ a (d and g) b' ⇔ a g b'. Since *c'* does not occur in this formula, the value assigned to *c'* is irrelevant and we obtain the result that C |= a (d and g) b' ⇔ a g b', which shows that $\underline{d}d'$ satisfies incremental Transparency.

It follows, of course, that an entire formula *F* satisfies the incremental version of Transparency just in case *each* presupposition is entailed by its local context:

(63) Consequence
For any a any v ∈ {i, s}, for any formula *F*, for every expression $\underline{d}d'$ and for all strings *a,*

b, if F = a $\underline{d}$d' b and if lc^v(C, $\underline{d}$d', a _ b) ≠ #, then:
$Transp^v$(C, F) iff Sat^v(C, F).

Arguably, the present theory makes more sense than the Transparency theory when it comes to symmetric readings - as mentioned, the symmetric version of Transparency was based on a somewhat phony metaphor, and things might be a tad easier to conceptualize within the present framework. When an interpreter decides on the narrowest restriction he can get away with when interpreting an expression *E*, he may well take into account all of the sentence except *E* - though of course this requires that he wait until the end of the sentence to do so. This might well be costly, but there is no requirement that he somehow take into account the end of the sentence while ignoring part of a conjunction, since conjunctions *per se* do not play any role for the present theory (thought they do for the Transparency theory).

4.3 Equivalence with Standard Dynamic Semantics

It was shown in Schlenker 2007a that in the propositional case the incremental version of the Transparency theory is fully equivalent to Heim's dynamic semantics (augmented with the asymmetric dynamic disjunction of Beaver 2001). In the quantificational case, the equivalence holds only if two additional assumptions are made:
-Non-Triviality: quantificational clauses should not be 'trivial' (i.e. replaceable with a tautology or a contradiction).
-Constancy: the domain is finite, and in addition restrictors should hold true of a constant number of individuals throughout the context set.
These assumptions are stated precisely in the Appendix [9] and in Schlenker 2007a. Let us just recapitulate the main conclusion:

(64) Under the assumptions of Non-Triviality and Constancy,
a. C[F] ≠ iff $Transp^i$(C, F).
b. If C[F] ≠ #, C[F] = {w ∈ C: w |= F}

We just showed that whenever local contexts exist, our reconstruction of dynamic semantics is equivalent to the Transparency theory. Furthermore, Constancy entails that in each world the domain of individuals is finite, which by results proven in the Appendix guarantees that local contexts always exist (see Section 6.1 for further discussion, and Appendix, [16]). So we obtain in this way a relatively general equivalence between the present system and standard dynamic semantics.

(65) **Equivalence with Standard Dynamic Semantics**
Let C be a context set and *F* be a formula which satisfy Non-Triviality and Constancy.Then for every presupposition trigger *$\underline{d}$d'* such that for some strings *a, b* F = a $\underline{d}$d' b, lc^i(C, $\underline{d}$d', a _ b) ≠ #. Furthermore, Sat^i(C, F) iff C[F] ≠ #

Even when local contexts do exist, there are interesting points of divergence between the Transparency theory and the present account on the one hand, and standard dynamic semantics on the other. Specifically, we sometimes make weaker predictions than dynamic semantics (which is why the additional assumptions of Non-Triviality and Constancy are needed to attain full equivalence; here we discuss the predictions of the theory *without* these assumptions). For instance, Transparency may be satisfied for *(less than three P . $\underline{Q}$Q')*, *(more than three P . $\underline{Q}$Q')* and *(exactly three P . $\underline{Q}$Q')* even if it is *not* presupposed that every P-individual is a Q-individual. To see this, consider a world w in which there are exactly two

P-individuals. We can reason in identical fashion within the context of the Transparency theory or of our reconstruction of dynamic semantics. Since there are just two P-individuals in w, any statement of the form *(less than three P . _)* will be trivially true, and any statement of the form *(more than three P . _)* or *(exactly three P . _)* will be trivially false. It immediately follows that any predicate *Q* will be transparent:

(66) For any predicative expressions *Q* and *G,*
a. $w \models$ (less than three P . (Q and G)) $\Leftrightarrow$ (less than three P . Q)
b. $w \models$ (more than three P . (Q and G)) $\Leftrightarrow$ (more than three P . Q)
c. $w \models$ (exactly three P . (Q and G)) $\Leftrightarrow$ (exactly three P . Q)

For the same reason, any restriction on *Q* will be transparent as well - including the strongest conceivable one, the empty set:

(67) For any predicative expressions *Q* and *G,* for any X of type <s, <e, t>> for which
$X(w)(d) = 0$ whenever $d \in D_w$,
$w \models$ (less than three P . $^{c'}$Q) $\Leftrightarrow$ (less than three P . Q)
$w \models$ (more than three P . $^{c'}$Q) $\Leftrightarrow$ (more than three P . Q)
$w \models$ (exactly three P . $^{c'}$Q) $\Leftrightarrow$ (exactly three P . Q)

It immediately follows that at w the presupposition *Q* of *QQ'* is satisfied *even* if it not the case that *every P-individual satisfies Q*. This is interesting because we showed earlier (in (35)) that with *(No P . QQ')* we do predict universal inferences ('every P-individual satisfies *Q*').

On superficial inspection, these predictions may seem to be quite welcome. As mentioned, Chemla 2007 shows with experimental means that *(No P . QQ')* does give rise to the strong inference that every P-individual satisfies *Q*; but he also shows that considerably weaker patterns are obtained if *no* is replaced with *less than three*, *more than three* or *exactly three*: subjects are essentially at chance with respect to universal inferences. It could be that such results will be refined (to my ear there are in some cases clear differences between *less than three* and *more than three* - the former gives rise to stronger universal inferences than the latter). But in any event, the *details* of the present predictions should moderate one's initial optimism. With the hypotheses of Non-Redundancy and Constancy, which are certainly satisfied in most standard situations (including the scenarios used in Chemla's experiment), we *do* predict universal inferences. Thus if one wishes to use Chemla's data to argue for the present theory, one must argue that subjects cannot fully integrate all aspects of the context when they decide whether to derive universal inferences. This might be plausible, but in any event further work is needed to justify such an interpretation.

5 Local Meanings

The present account has two important advantages over standard dynamic semantics: it is predictive, and it offers a natural account of symmetric readings, at least in simple cases[11]. In both respects it is similar to the Transparency theory (though the symmetric version of the analysis is slightly more natural in the present framework). But the new analysis has a more momentous advantage over the Transparency theory: it affords a natural notion of 'local meaning', understood as the meaning of an expression relative to its local context (more

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[11] As mentioned in a previous footnote and in the Appendix, the predictions of Symmetric Satisfaction are dubious for sentences that contain several presupposition triggers.

simply: the local meaning of *E* is **c' ∧ E**, where *c'* denotes the local context of *E* relative to the context set in the relevant syntactic environment). This, in turn, makes it possible to develop a general theory of triviality which fully follows Stalnaker's initial insights (Stalnaker 1978): an expression *E* is locally trivial if its local meaning is the same as the local meaning of a tautology, or in other words if *E* is entailed by its local context; and *E* is locally contradictory if its local meaning is the same as that of a tautology, which happens just in case the local context of *E* entails the negation of *E*. We will also suggest that the notion of 'local meaning' might be crucial to reach a proper understanding of how (some) presuppositions are generated, though this part of the analysis is admittedly more speculative.

5.1 Local Triviality

The following constructions are deviant, presumably because *he is sick* is in some sense redundant:

(68) a. #John has cancer and [he is sick or desperate]
a'. John has cancer and he is desperate.
b. #If John has cancer, he is sick or desperate.
b'. If John has cancer, he is desperate.

We also announced in Section 3.1 that the contrast in (69) (repeated from (43)) can be accounted for in terms of 'local triviality':

(69) a. John lives in France and he resides in Paris.
b. #John lives in Paris and he resides in France.

Be Brief as defined by the Transparency theory is entirely silent about these cases, since the offending element does not occur in the first member of a conjunction. By contrast, the theory of local contexts offers a far more general analysis. All we need are the Stalnakerian constraints in (70) (see Singh 2007 for a recent discussion):

(70) The assertive component of an expression *E* may not be trivially true or trivially false relative to its local context:
a. it may not be entailed by the local context of *E* (local triviality)
b. its negation may not be entailed by the local context of *E* (local contradiction).
(These requirements may presumably be interpreted in incremental or symmetric terms.)

This analysis immediately provides an account of the deviance of (68)a-b. In both cases, the local context of *he is sick* is C ∧ **John has cancer**, which certainly entails that John is sick - with the result that the expression is locally trivial; exactly the same analysis accounts for (69) if one adopts an *incremental* version of the prohibition against local triviality. The symmetric version of the condition predicts that (69)a should be less than perfect, although it should presumably be more acceptable than (69)b: as we observed, it seems to be harder to compute a symmetric context than an incremental one (because the former cannot be determined 'on the fly', as the sentence is processed); if so, we expect cases of triviality relative to a symmetric context to be less easily perceptible and thus to lead to weaker judgments of deviance. Further empirical research will have to determine whether these fine-grained predictions are correct (if so, we were wrong to give (69)a as fully acceptable; it should be slightly degraded).

What about violations of the constraints against expressions that *contradict* their local context? Cases in which one disjunct entails the other are known to be deviant (Hurford 1974), and they have recently been the object of highly detailed studies (Singh 2007a; Spector et al. 2008)[12]:

(71) a. #?John resides in Paris or he lives in France.
b. #John lives in France or he resides in Paris.

As was shown in (21), the incremental context of *q* in *(p or q)* uttered in a context set C is C ∧ **(not p)**. It follows that (71)b should be incrementally deviant, since for *p* = *John lives in France* it is clear that C ∧ **(not p)** entails the negation of *John resides in Paris*. By parity of reasoning, (71)a should be symmetrically deviant. What is left unaccounted for, however, is why (71)a is not as acceptable as (69)a. In the present framework this is a mystery, which I leave for future research (but see Spector et al. 2008 for an in-depth discussion).

5.2 Presupposition Generation

We will now suggest, more speculatively, that the notion of 'local meaning' might also be crucial to understand how (some) presuppositions are triggered. The argument is in two steps. We start by suggesting that in some simple examples a presupposition is generated from a bivalent meaning, but that what is crucial is the *contextual meaning* of the expression at hand, i.e. its meaning relative to the context set. We then argue that in more complex examples the notion of 'contextual meaning' is too narrow, and that *local meanings* are called for. We will thus try to establish the following conjecture:

(72) Conjecture
Some presuppositions are generated on the basis of the local (bivalent) meaning of an expression.

5.2.1 Presupposed Contextual Entailments

We start by suggesting that some expressions are 'part-time triggers': they sometimes trigger a presupposition, but only in case certain contextual conditions are satisfied. For clarity, we posit the following (underspecified) definition:

(73) Part-time triggers
An expression *E* is a part-time trigger for the presupposition *p* if:
(i) *E* does not lexically entail *p*
(ii) when *E* contextually entails *p* and certain additional conditions are satisfied, *E* triggers the presupposition that *p*.

We will by no means achieve a complete theory of presupposition generation, and thus the 'additional conditions' mentioned in (ii) will be left vague.

Consider first the following examples, which are about a group of responsible 30-year olds:

(74) a. Mary has announced to her parents that she is pregnant.
=> Mary is pregnant.
b. Mary hasn't announced to her parents that she is pregnant / I doubt that Mary has

[12] Special thanks to B. Spector for very helpful conversations on this topic and on this specific hypothesis.

announced to her parents that she is pregnant.
=> Mary is pregnant.
c. Has Mary announced to her parents that she is pregnant?
=> Mary is pregnant.
d. None of these ten women has announced to her parents that she is pregnant.
=> Each of these ten women is pregnant.

In each case, we obtain the pattern of inference that is characteristic of presuppositions. It may thus look like *announce* is a *bona fide* presupposition trigger. But this conclusion is contradicted by other examples. Suppose that we are now discussing a group of teenage patients in a mental hospital, and that we say:

(75) a. John has announced that he has met Elvis.
≠> John has met Elvis.
b. John hasn't announced that he has met Elvis.
≠> John has met Elvis.
c. Has John announced that he has met Elvis?
≠> John has met Elvis.
d. (At least,) none of these ten patients has announced that he has met Elvis
≠> Each of these ten patients has met Elvis.

Clearly, these examples do not imply that Elvis is alive. Importantly, no amount of tinkering with the notion of 'accommodation' can save the presuppositional analysis in this case. This is because any kind of accommodation (be it 'global' or 'local') should yield in (75)a an inference that Elvis is indeed alive. The contrast between (74) and (75) is thus a genuine puzzle for lexical theories of presupposition.

What is the source of the contrast? Going back to the examples in (74), we can manipulate and even reverse the judgments by modifying the context. Let it now be assumed that we are talking about a group of playful 7-year-olds (of course for this to make sense we must replace *women* with *girls* in (74)d). It seems to me that the examples can then be uttered naturally without a presupposition or entailment that the relevant individuals are pregnant. The generalization appears to be, roughly, that when background assumptions guarantee that *x announces that p* contextually entails *p*, then *x announces that p* presupposes that *p*. If Mary is a responsible 30-year old, she is unlikely to announce that she is pregnant unless she really is. By contrast, if Mary is a playful 7-year-old, that is reason enough to block the inference from *x announces that x is pregnant* to *x is pregnant*, and no presupposition - nor entailment - emerges. It should be noted that it is not necessary to manipulate the nature of the subject to modify the presuppositional facts. Suppose that we are discussing a group of men who all have mistresses, but whose reliability is otherwise unknown. It seems to me that the examples in (76) are normally understood as presuppositional, while those in (77) aren't:

(76) a. Smith has announced to his mistress that he is fired.
=> Smith is fired
b. Smith hasn't announced to his mistress that he is fired.
=> Smith is fired
c. Has Smith announced to his mistress that he is fired?
=> Smith is fired.
d. None of ten men has announced to his mistress that he is fired.
=> Each of these ten men is fired.

(77) a. Smith has announced to his mistress that he will leave his wife within a year.
≠> Smith will leave his wife within a year
b. (Wisely,) Smith hasn't announced to his mistress that he will leave his wife within a year.

≠> Smith will leave his wife within a year
c. Has Smith (foolishly) announced to his mistress that he will leave his wife within a year?
≠> Smith will leave his wife within a year
d. (Wisely,) None of ten men has announced to his mistress that he will leave his wife within a year
≠> Each of these ten men will leave his wife within a year.

Even an unfaithful man doesn't typically go around saying that he has been fired unless this is indeed so. By contrast, he may well tell his mistresses that he will leave his wife without thereby intending to do so. In this case, then, the nature of the embedded proposition suffices to yield a striking presuppositional contrast between (76) and (77). It would thus seem that *announce* fits the definition of a 'part-time trigger' given in (73). I believe that similar facts hold of *tell* (and possibly of *learn*, though the issue is more complicated[13]).

Let me speculate that *x announces that p* presupposes that *p* as soon as it contextually entails that *p* because, quite generally, (some) *presuppositions are just distinguished contextual entailments*. The modifier *distinguished* is crucial, because it is self-evident that not every contextual entailment is a presupposition. We do not attempt to determine what makes a contextual entailment a 'distinguished' one - an answer to this question would yield a solution to the triggering problem for presuppositions. Still, some properties of this analysis are worth mentioning. First, we now explain why *announce* (as well as *tell*, and possibly *learn*) sometimes triggers presuppositions: it is only in case contextual assumptions guarantee that *x announces that p* entails that *p* that the latter is turned into a presupposition. Second, a special case of contextual entailment is logical entailment - it might well be that whatever explains what a 'distinguished' entailment is will provide an account of (some) standard triggers as well. Third, we predict that two words that have the same contextual meaning should also display the same presuppositional behavior. For instance, the verbs *inform y that p* and *announce to y that p* have a similar semantics, except that *inform* is lexically veridical whereas *announce* isn't[14]. This difference is suggested by the following contrast:

(78) a. Mary announced to her parents something false.
b. ?Mary informed her parents of something false.

On the other hand, as soon as some background assumptions guarantee that *x announced to y that p* is contextually veridical, the difference between *announce* and *inform* is obliterated. It is easy to check that the presuppositions we obtained in our 'pregnancy' example are exactly those we get with *inform*:

(79) a. Mary has informed her parents that she is pregnant.
=> Mary is pregnant.
b. Mary hasn't informed her parents that she is pregnant.
=> Mary is pregnant.
c. Has Mary informed her parents that she is pregnant?
=> Mary is pregnant.
d. None of these ten women has informed her parents that she is pregnant.
=> Each of these ten women is pregnant[15].

---

[13] Thanks to I. Heim, P. Egré and especially B. Spector for arguments that *learn* might in fact be ambiguous between a factive and a non-factive reading. I leave this issue for future research.

[14] We will say that a verb *V* is *lexically veridical* if its meaning guarantees that *V p* entails that *p*; if the entailment goes through with additional assumptions that are met in C, we will say that in C *V* is contextually veridical.

[15] A more minimal pair is provided by the difference between *guess* and its closest French translation, *deviner*. The difference between the two verbs is that the latter but not the former is veridical:

Needless to say, these observations only begin to scratch the surface of presupposition generation. But they suggest that at least some presuppositions are generated from contextual meanings - a claim that was made on related grounds by Stalnaker (1974) and Simons (2001) (see also Abusch 2002 for a different account of presupposition generation)[16].

5.2.2 Local Meanings

The preceding examples are compatible with an analysis in which (some) presuppositions are generated from bivalent *contextual* meanings, seen as the meaning of an expression relative to the context set (without taking into account the syntactic environment in which the expression is found). But in more complex cases, the more sophisticated notion of a *local meaning* might have to be appealed to.

(i) a. John guessed that the ball would land on a black number (but he was wrong).
b. #Jean a deviné que la boule allait s'arrêter sur un nombre noir (mais il avait tort).
Jean has DEVINER that the ball would stop on a number black (but he was wrong)

There are cases, however, in which *x guessed that p* may be used to mean: *x correctly guessed that p*:

(ii) John guessed that I didn't like him
=> I didn't like John

As soon as *guess* displays this veridical behavior, it behaves like a presupposition trigger, just like *deviner*:
(iii) a. Did John guess that I didn't like him?
=> I didn't like John
b. John didn't guess that I didn't like him.
=> I didn't like John
a'. Est-ce que Jean a deviné que je ne l'aimais pas?
Did Jean DEVINER that I NE him liked not?
=> I didn't like Jean
b'. Jean n'a pas deviné que je ne l'aimais pas
Jean has not DEVINER that I NE him liked not
=> I didn't like Jean

[16] As stated, our analysis is incompatible with Abusch's theory of presupposition generation. For Abusch, (some) presuppositions are triggered on the basis of certain lexical alternatives, which are an *arbitrary* property of a word. Thus there could in principle be two words that have the same syntax, the same total meaning (i.e. the same truth conditions, lumping together falsity and undefinedness), but different presuppositions. She offers one beautiful minimal pair that suggests that her prediction is correct:

(i) a. John is right that Mary is pregnant.
b. John is aware that Mary is pregnant.

Abusch convincingly argues that (i)a presupposes that John thinks that Mary is pregnant, while (i)b presupposes that Mary is pregnant. Standard tests support Abusch's conclusion. Interestingly, however, there are reasons to think that the pair is not *syntactically* minimal, and that *is right* has a hidden structure, possibly *is right ~~in thinking~~ that Mary is pregnant*. The argument stems from patterns of *wh*-extraction:

(ii) a. (?)Which of these individuals is your mother aware that you invite home?
b. * Which of these individuals is your mother right that you invited home?

(ii)b suggests that there some island blocks extraction from the embedded clause - which is compatible with the hypothesis that in fact the embedded clause is itself contained within an adjunct (*right in thinking that you invited home*).

The argument is simple: all we have to do is embed our earlier examples in the consequent of conditionals, making sure that it is only in the presence of the assumption expressed by the antecedent that they have the desired local meaning.

(80) *Context:* At a costumed party, we encounter a short female with a mask and a wig. We do not know whether this is Ann, a playful (and tall) 11-year old, or Mary, a responsible (and short) 30-year old.
a. If this is Mary, the person in front of us has just announced to her parents that she is pregnant.
b. If this is Mary, the person in front of us has not announced to her parents that she is pregnant.

I believe that in both cases we obtain an inference that the adult, Mary, is in fact pregnant. In (80)a, the reason might be rather mundane: in order to believe what he says, the speaker must think that Mary has announced to her parents that she is pregnant; since Mary is a responsible adult, it is unlikely that she did so unless she really is pregnant. No presuppositional analysis is needed to obtain this result. But the situation is different in (80)b, where projection occurs from the scope of a negation. Unsurprisingly, we infer that the speaker believes that Mary has not said to her parents that she is pregnant; but there could be a variety of reasons for this negative belief, and by itself it certainly does not entail that Mary is in fact pregnant. This inference is most easily accounted for by treating it as a presupposition triggered in the consequent of the conditional, which then gets projected[17]. If we replace *Mary* with *Ann*, however, I believe that the effect disappears - one infers that Ann has been *particularly* playful, but not that she really is pregnant.

Could *contextual* meanings account for the judgments obtained in (80)? Probably not. Since we are not certain about the denotation of the description *the person in front of us*, the consequent of the conditional certainly does not contextually entail that the person we are considering is pregnant (if this person happens to be Ann, she may announce to her parents that she is pregnant, but this would only be because she is playful, not because she is pregnant). If the algorithm that triggers presuppositions takes as its input a *contextual meaning*, it should *fail* to produce a presupposition (80) - which would leave unexplained the inference we obtain in (80)b. By contrast, a more adequate prediction is derived if the triggering algorithm takes as its input the *local meaning* of the consequent of the conditional. We showed in (19) and (20) that the incremental local context of *G* in *if F, G* and *if F, not G* relative to a context set C is simply C $\wedge$ **F** (the symmetric local context has the same value in this case). In (80)a-b, then, the local meaning of the consequent is (C $\wedge$ **F**) $\wedge$ **G**, which *does* entail that Mary is pregnant. The triggering algorithm can thus apply to turn this entailment into a presupposition. (Obviously we have not explained why the algorithm *must* apply in this case, since this would require a full understanding of the triggering algorithm; but we have explained why it *may* apply, which is enough for our purposes).

If our speculations are on the right track, at least some presuppositions are generated from the meaning that certain expressions have relative to their local context. It could well

[17] Strictly speaking, we predict a conditional presupposition if we start from a standard account of presupposition projection in conditionals, together with the assumption that in this case a presupposition is generated from the local meaning of the consequent. Specifically, we predict that the sentences in (80)a-b) should presuppose that *if this is Mary, Mary is pregnant*. But as was noted above, it is well-known that in many cases conditional presupposition tend to be strengthened to unconditional ones (the so-called 'proviso problem' of Geurts 1999). The strengthening does seem to apply to the case at hand if we do not make additional assumptions - presumably because the conditional assumption would in all likelihood be made because one believes the unconditional one. It may be that the facts change if the context is slightly modified, for instance if the person we have in front of us has a rather big belly.

be that there are other sources of presupposition generation; our approach is in this respect quite versatile. But if it turns out that *all* presuppositions are generated in this way, the Projection problem will in a sense disappear. Consider for instance the clause *x stop smoking*, taken to mean *x used to smoke and doesn't smoke*. The triggering algorithm takes as its input the meaning of *stop smoking* relative to a certain local context lc. But if lc already entails that x used to smoke, the contribution of *x stop smoking* relative to lc will be identical to that *x doesn't smoke* relative to lc, and thus one would *not* expect a presupposition to be generated in the first case since none if generated in the second. In other words, in this case one does not have to ask why a presupposition is generated, and then fails to be projected; rather, *no presupposition is generated in the first place.* We leave for future research a closer investigation of these theoretical possibilities, and of the predictions they make with respect to the processing and acquisition of presuppositions

5.3 Scalar Implicatures

Our analysis of local contexts has an unexpected application: it makes it possible to explore what happens when pragmatic principles that are motivated for the global context are applied to local ones. Whether this idea is conceptually motivated in the general case remains to be seen; but it is certainly an interesting possibility to explore. In the domain of scalar implicatures, this 'localist' intuition appears to be at the heart of Chierchia's recursive account scalar implicatures (Chierchia 2004). But a rather natural implementation of this intuition within the present framework turns out to be equivalent to a different account, pioneered in Chemla 2007c.

Let us start with the basic neo-Gricean account of scalar implicatures for unembedded sentences. If I say that *John invited Ann or Mary*, the addressee will infer that I am not in a position to assert the more informative scalar alternative *John invited Ann and Mary*, presumably because I do not hold such a belief. Furthermore, if the addressee thinks that I am well-informed, he might make the further inference (called the 'epistemic step' in the literature) that I do not hold this belief because I believe that *it is not the case that John invited both Ann and Mary*.

(81) Suppose that sentence S was uttered and that S' is a scalar alternative of S. Then if S' asymmetrically entails S in the global context set, it may be inferred:
a. that the speaker does not hold the belief that S' [primary implicature]
b. if the speaker is well-informed, that he believes that (not S') [secondary implicature]

Now if S' is *less* informative by S (i.e. asymmetrically entailed by it), it is immediate that

Now let us apply these same principles to local contexts.

(82) For any context set C, for any expression *d*, for every formula *F*, for all strings *a* and *b*, if F = a d b and if lc^s(C, d, a_b) ≠ #, then:
a. *d* is symmetrically optimal at the primary level in *F* relative to the context set C and the belief set B of the speaker (abbreviation: *Opt-1^s(C, B, d, a_b)*) just in case for every scalar alternative *d'* of *d*:
if $\models^{c' \rightarrow lc^S(C, d, a\_b)}$ [(c' and d') ≤ (c' and d)] and not [(c' and d) ≤ (c' and d')]
then B $\not\models^{c' \rightarrow lc^S(C, d, a\_b)}$ c' ≤ d'
if $\models^{c' \rightarrow lc^S(C, d, a\_b)}$ (c' and d) ≤ (c' and d'),
then B $\models^{c' \rightarrow lc^S(C, d, a\_b)}$ c' ≤ d'

b. *d* is symmetrically optimal at the secondary level in *F* relative to the context set C and the belief set B of the speaker (abbreviation: *Opt-2*^{s}*(C, B, d, a_b)*) just in case for every scalar alternative *d'* of *d*:
if $\models^{c' \to lc^{S}(C, d, a\_b)}$ [(c' and d') ≤ (c' and d)] and not [(c' and d) ≤ (c' and d')]
then B $\models^{c' \to lc^{S}(C, d, a\_b)}$ c' ≤ (not d')
if $\models^{c' \to lc^{S}(C, d, a\_b)}$ (c' and d) ≤ (c' and d'),
then B $\models^{c' \to lc^{S}(C, d, a\_b)}$ c' ≤ d'

At this point it is helpful to make a general remark:

(83) For any context set C, for any expression *d*, for every formula *F*, for all strings *a* and *b*, if F = a d b and if lc^{s}(C, d, a_b) ≠ #, then for set B ⊆ C, for any expressions *d* and *d'*, B $\models^{c' \to lc^{S}(C, d, a\_b)}$ (c' and d) ≤ (c' and d')
if and only if
C ⊨ a d b ⇔ a d' b

6 Technical Extensions

We now turn to some more advanced developments of the present framework. First, we ask under what conditions local contexts are guaranteed to exist - and what should be done when they don't. Second, we return to the definition of the incremental version of the algorithms, making use of an alternative definition suggested independently by Danny Fox and Ed Stabler.

6.1 Existence of local contexts

In a nutshell, as long as the semantics is extensional and the domain of individuals in each possible world is of finite size, we can guarantee that local contexts exist. But the result fails to hold when infinite domains are considered; in such cases the satisfaction theory must be redefined in a slightly more complicated way.

6.1.1 When local contexts exist

In the propositional case, it can be shown that local contexts (both incremental and symmetric) always exist; a simple proof is given in the Appendix [16a]. In the quantificational case, local contexts may fail to exist, for reasons we will turn to shortly. But there are still broad conditions under which their existence is guaranteed. The details of the proof are laid out in the Appendix (see 16b), but the main observation is quite simple. In all cases, the set of transparent restrictions is closed under finite (generalized) conjunction: if x and x' are two transparent restrictions, then so is x ∧ x'. When the set of transparent restrictions is finite, we can ensure that it has a bottom element. For instance, if the set contains the context denotations x_1, x_2 and x_3, we start by taking the intersection of x_1 and x_2, which entails both and must be in the set; then we take its intersection with x_3 - the result is again in the set, and it entails x_1, x_2 and x_3, so it is the bottom element we were looking for. The procedure can be applied whenever the set of transparent restrictions is finite. This condition happens to be met whenever all the relevant domains of individuals are themselves finite. So we can derive a general condition that guarantees that local contexts do exist.

6.1.2 When local contexts don't exist

Interestingly, there are cases in which local don't exist. From the preceding remarks, we can already infer that the relevant examples must involve infinite domains of individuals. We start from the formula *(infinitely-many P . $^{c'}$Q)*, and consider the set of transparent values for *c'* (in this case there is no difference between the incremental and the symmetric version of the analysis). We assume that there are infinitely many elements in **P**(w), the value of *P* at a certain world w of C. Now we note that for **c'** to be transparent in *(infinitely-many P . $^{c'}$Q),* **c'**(w) must itself contains infinitely many elements. For if not, *(infinitely-many P . $^{c'}$P)* would be false at w but *(infinitely-many P . P)* would be true - and *c'* wouldn't be transparent after all. Next, we show that for any transparent value x for *c'*, we can find a 'smaller' value x' which is also transparent. Since x must contain infinitely many elements, we just take one arbitrary element out of x, obtaining an x' distinct from x which entails it (by generalized entailment). And it is clear that x', which itself contains infinitely many elements, is transparent - for the simple reason that the truth of the statement *infinitely many Ps are Qs* is utterly insensitive to whatever happens to any given finite set of elements. Since x was arbitrary, we have shown that the set of transparent restrictions does not have a bottom element.

What can be done in this case? The problem arose because in some cases there is an infinite series of increasingly stronger transparent values for *c'*, with no bottom element. One solution is to re-define the notion of satisfaction in a way that does not depend on the existence of a bottom element. This can be done by introducing a notion *Sat'* which is defined directly in terms of the set T of transparent restrictions: the presupposition is satisfied in this new sense just in case *there exists a member of T such that every element of T that entails it also entails the presupposition*[18]. As before, this notion comes in an incremental and in a symmetric version:

(84) For every v ∈ {i, s}, Sat'^{v}(C, <u>d</u>d', a_b) iff for some X ∈ tr^v(C, <u>d</u>d', a_b), for every X', if [X' ≤ X and X' ∈ tr^v(C, <u>d</u>d', a_b)], then C $\models^{c' \to X'}$ c' ≤ d

Of course when the set T of transparent restrictions has a bottom element c*, a presupposition is satisfied in the new sense just in case it is in the old sense: if c* entails the presupposition, taking X = c*, we immediately see that the condition in (84) is met. Conversely, if the condition in (84) is met, then the bottom element c* must entail the presupposition, which is thus satisfied in the old sense. However when local contexts fail to exist, we obtain new predictions, which turn out to be fully equivalent to those of the Transparency theory:

(85) **Equivalence with Transparency - General Case**
The revised definition of satisfaction yields full equivalence with the Transparency theory. Specifically, for any b ∈ {i, s}, for every formula that has the form a <u>d</u>d' b,
Sat'^{v}(C, <u>d</u>d', a _ b) iff Transp^v(C, <u>d</u>d', a _ b)

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[18] It can be noted that the problem we face and the solution we explore have counterparts in David Lewis's study of *Conterfactuals* (1973). Lewis defined a non-monotonic semantics for conditionals whose main intuition was that *if F, G* is true in world w just incase the closest F-worlds from w are also G-worlds. But Lewis argued that sometimes there is an infinite series of increasingly 'closer' F-worlds to w; for such cases the truth conditions of conditionals had to be adapted: *if F, G* was deemed true just in case for some world in the series, every world in the series below it is a G-world.

A simple proof is given in the Appendix [21]. This equivalence need not be a good thing, because in the somewhat arcane cases in which local contexts don't exist the Transparency theory (and our revised theory of satisfaction) make predictions that are arguably too weak (the example with *infinitely many* is discussed at greater length in the Appendix, [23]). It might thus prove fruitful in future research to explore alternative extensions of our primitive notion of satisfaction to derive slightly stronger predictions.

6.2 Linear vs. Structural Localism

The reader may well have been puzzled by the simplified fragment we have used throughout, which makes crucially use of parentheses in the object language, with formulas such as *(not* *p̲p')*, *(p and q̲q')*, etc. In logic, parentheses are used to ensure that the language under study is structurally unambiguous. This property extends to the present fragment, whose complex formulas all have the form *(not F)*, *(F and G)*, *(F or G), (if F . G),* or *(Q̲ P . R)*. To determine whether a string *E* is a constituent, one asks whether, when one scans *E* from left to right, (i) one first sees a left parenthesis, and (ii) the first point at which one has encountered an equal number of left and right parentheses is the last symbol of *E*. This is all very convenient, but in linguistics one does not normally take parentheses to be part of the object language; rather, they are the syntactician's way of encoding the derivational history of a sentence. Making reference to the latter is necessary due to the pervasive presence of structural ambiguities in natural language; without access to the derivational history of a sentence, we wouldn't know which truth conditions must be attributed to *John will not drink and sleep*, which may be understood as *John will [[not drink] and sleep]* or as *John will not [drink and sleep]*.

Since our analysis is based on semantic notions, we must work with derivation trees too. But this seems to run counter to the very logic of incrementalism, which requires that we consider the left-to-right order in which words appear. In the fragment we considered, the object language is sufficiently rich to make its structure unambiguous; but without the device of parentheses, we wouldn't know what to do with formulas such as (86), or the corresponding English example in (87):

(86) a. not p and q̲q'
b. ((not p) and q̲q')
c. (not (p and q̲q'))

(87) a. John will not smoke and start smoking.
b. John will [[not smoke] and [start smoking]
c. John will not [smoke and [start smoking]]

(87)b is reasonably coherent and need not yield a presupposition failure; by contrast, (87)c is incoherent and violates the presupposition of *start smoking*, which requires that its local context entail that (at the relevant time) the person should not be a smoker.

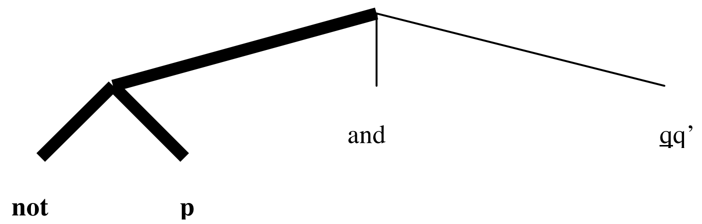
So we need to have access to the structure of our sentences. But our treatment of incrementalism also requires that we consider the left-to-right order in which words appear. In fact, in the Transparency theory, which is closely related to the present account, the metaphor that motivates the incremental version of *Be Brief* is that one 'hasn't yet' heard the end of the sentence. But if one hasn't heard the end of the sentence, how can one have access to its structure? There may be an answer, but it is not trivial. The technical solution, both in the Transparency theory and in the present account, was to enrich the object language to the point where a string fully encodes its own structure.

Danny Fox and Ed Stabler have independently suggested that it would be more satisfying to define incrementalism in a structural fashion. One way to do so is to appeals to the notion of the *left-most part of a derivation tree*: this is simply the set of nodes that are above and to the left of a certain terminal node. We abbreviate this by *left*:

(88) If *a d b* is a well-formed sentence with derivation tree *T*,
left(d, a_b, T) is the ordered set of nodes of *T* that are (reflexively[19]) to the left of some node that (reflexively) dominates d.

To give an example, consider again our example (86), in a language that has exactly the same derivation rules as our fragment, except that it doesn't include parentheses. Focusing on the derivation that corresponds to (86)b, we obtain the tree in (89), which we call *T*:

(89)



If we wish to isolate left(p, not _ and qq', T), the part of the tree to the left of *p*, we obtain the elements in bold: the mother of *p* dominates it, so it is in the leftmost-part of the tree; so does the root node, which is thus included as well. And all the nodes to their left are included too.

We can then redefine in structural terms our notion of an 'incrementally transparent restriction'. We assume that our semantics is defined on derivation trees rather than on strings. Now instead of considering all the strings that can turn an initial string into a complete sentence, we consider all the derivation trees that can turn the (relevant) left-most part of a tree into a complete derivation tree. To be somewhat more precise, we replace our earlier definition, copied in (90), with that in (91).

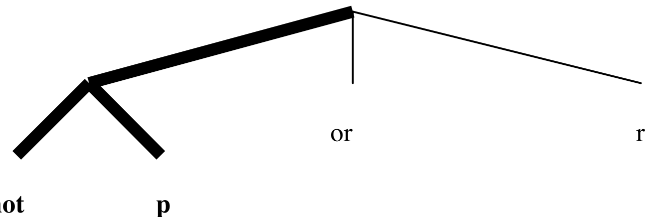
(90) tr^i(C, d, a_b, T) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, for every good final *b'*, $C \models^{c' \rightarrow x} a\ {}^{c'}d'\ b' \Leftrightarrow a\ d'\ b'$}

(91) If *a d b* is a string with the derivation tree T,
TR^i(C, d, a_b, T) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, for every derivation tree T' such that T' is a completion of left(d, a_b, T), if ${}^{c'}T'$ is obtained by replacing the occurrence *d* with ${}^{c'}d'$ in T',
$C \models^{c' \rightarrow x}\ {}^{c'}T' \Leftrightarrow T'$}

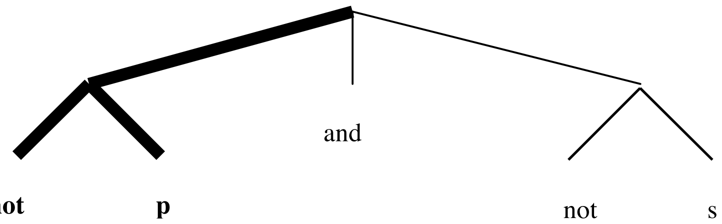
This is a mouthful, but it achieves what we desire: if we want to compute the local context of *p* in (89), we will have to consider a variety of derivation trees - for instance those represented in (92) and (93).

[19] Node n is 'reflexively' to the left of node n' if n is to the left of n' or n = n'.

(92)



(93)



This new definition has the advantage of being applicable to languages that include structurally ambiguous sentences. It is straightforward to extend it to the symmetric version of the analysis, which also stands in need of refinement if we want it to apply to structurally ambiguous sentences[20].

The definitions of local contexts and of local satisfaction can be preserved without change, except that they now derive from the relevant sets of structurally transparent restrictions. The considerable advantage of this modified analysis is that it makes it possible to work with languages that give rise to structural ambiguity.

In the case of the fragment we considered in earlier sections, it turns out that there is no difference between the linear and the structural notions. The reason is that the parentheses make it possible to construct the corresponding derivation tree deterministically as one scans a formula from left to right. To start with an example, consider again the formula in (86)b, viewed not as a derivation tree but as a sentence of the object language (whose symbols happen to include parentheses). We can build the corresponding derivation tree step by step as we scan the formula from the left to right; and crucially we never need to 'guess' or to 'backtrack' when we do so (for simplicity we treat *qq'* as a single symbol, though nothing hinges on this).

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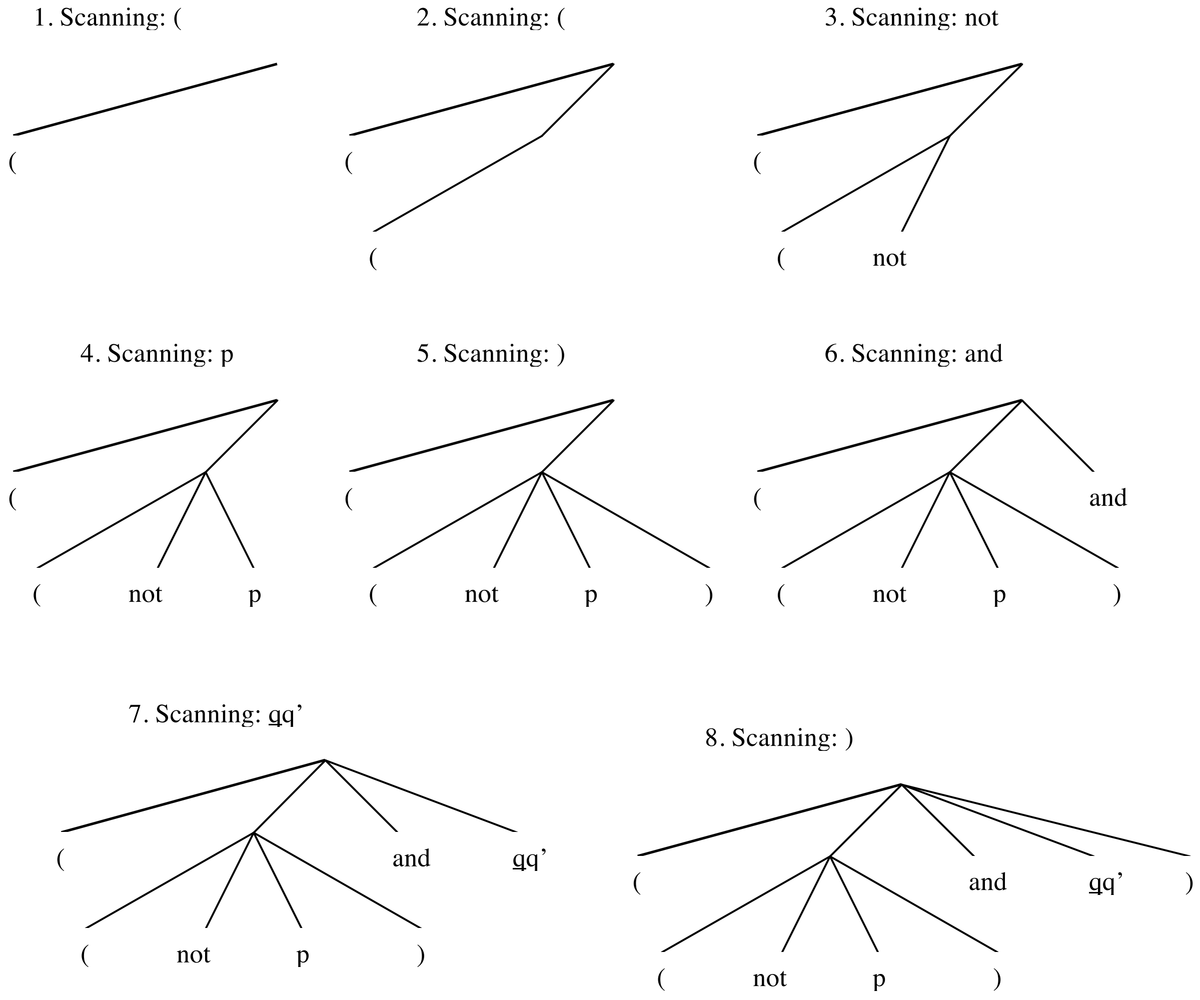
[20] Here is the symmetric definition:

(i) If *a d b* is a string with the derivation tree T,
TRs(C, d, a_b, T) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, if T' is obtained from T by replacing the occurrence *d* with *d'* and if c'T' is obtained by replacing the occurrence *d'* with *c'd'* in T',
C |=c' → x c'T' ⇔ T'}

(94) Incremental construction of the derivation tree of ((not p) and qq’)



It can be checked that if we erase all terminal nodes that include a parenthesis, we end up with the very same tree as in (89). This result is entirely as it should be, since the sole purpose of parentheses is to guarantee that derivation trees are easily recoverable. And the recovery is indeed quite easy. In the general case, it can be achieved in an incremental fashion by iterating the following procedure (after the root node has been created; it initially counts as an open branching node):

(95) -If the symbol which is scanned is (, create an open branching node under the left-most branching node that is still open, and place under it a terminal node ending with (.
-If the symbol s which is scanned is not a parenthesis, add a terminal node ending with s as the right-most daughter of the left-most open branching node.
-If the symbol which is scanned is), add a terminal node ending with) as the right most daughter of the left-most open branching node, and close that node.
(The procedure ends when the root node has been closed.)

This procedure shows that any initial string of a formula of our fragment fully determines the beginning of the derivation tree of that formula. As a result, there is simply no difference between the linear and the structural definition of local contexts. Of course this result only holds because our fragment contains parentheses that fully encode the derivational history of a formula. But this trick can in principle be applied to any context-free grammar; by systematically adding parentheses, we can ensure that our linear notions do correspond in the end to Fox's and Stabler's structural notions, which make more conceptual sense.

7 The New Presupposition Debate

The present theory is by no means alone in seeking a solution to the explanatory problem of dynamic semantics. First, there are alternative attempts to constrain standard dynamic semantics, either by imposing a template on possible lexical entries (LaCasse 2007) or by connecting it to the logic of common belief (Unger and von Eick 2007). Second, George 2007 and Fox 2007 have recently revived and considerably improved a non-dynamic trivalent analysis that was explored by Peters (1975) and Beaver and Krahmer (2001). Interestingly, these trivalent accounts depart in important respects from the predictions of dynamic semantics relative to quantified statements. Since this debate is of some import, it is worth laying out briefly.

George and Fox's analyses start from a directional version of certain trivalent logics - specifically, Strong Kleene and Supervaluations. These treat a semantic failure as an uncertainty about the value of an expression: if *pp'* is uttered while *p* is false, we just don't know whether the clause is true or false. The semantic module outputs the value # in case this uncertainty cannot be resolved - which systematically happens with unembedded atomic propositions whose presupposition is not met. But in some complex formulas it may happen that *no matter* how the value of *pp'* is resolved, one will still be in a position to determine unambiguously the value of the entire sentence. This may for instance be the case if one utters *(q and pp')* in a situation in which *q* is false and *p* is false too. *pp'* receives the 'indeterminate' value #, but no matter how the indeterminacy is resolved, this won't affect the result - the entire sentence will be false anyway. Now we can make this same reasoning with respect to every world in the context set: for any world w, the sentence will have a determinate truth value just in case either (i) *q* is false at w (so that it doesn't matter how one resolves the indeterminacy of the second conjunct); or (ii) *q* is true, and in that case the presupposition *p* of the second conjunct is satisfied. Since we are solely interested in worlds that are compatible with what the speech act participants take for granted, we derive the familiar prediction that the context set must guarantee that *if q, p*. The beauty of this family of proposals is that its underlying intuition is completely general: by treating presupposition failure as an instance of 'uncertainty' between true and false, it provides a general recipe for determining under what conditions the uncertainty in question does or does not matter for the entire sentence.

This trivalent analysis comes in several varieties; some crucial choice points are the following:

(i) Is the underlying semantics compositional or not? In the first case, we obtain a directional version of the Strong Kleene logic; in the second case, we naturally obtain a directional version of supervaluationist semantics. (Roughly speaking, supervaluations treat the semantic uncertainty triggered by an expression *pp'* type by type, so that all classical tautologies are also supervaluationist tautologies; for instance, even if *p* is false and thus *pp'* indeterminate, *(pp' or (not pp'))* is evaluated as true, because when we resolve the uncertainty *in the same way* for both tokens of *pp'* , we end up with a true statement. By contrast, Strong Kleene treats the semantic uncertainty token by token, with the result that *(pp' or (not pp'))* gets the value # when *pp'* is indeterminate).

(ii) Is the theory incremental or symmetric? Here too the various options offered by the Transparency theory and our theory of local satisfaction can be adopted.

(iii) Is the incremental principle computed linearly, structurally, or according to some other principle?

We will not attempt to do justice to the full range of possibilities. In the Appendix we state two general results that make it possible to compare the Transparency theory (and thus also our theory of local satisfaction) to one version of the Supervaluationist and Strong Kleene treatments[21]:

(96) Incremental Transparency predicts stronger presuppositions than Incremental Kleene and Incremental Supervaluations.

(97) Symmetric Transparency and Symmetric Kleene / Symmetric Supervaluations are incomparable, in the sense that:
-sometimes Symmetric Kleene and Symmetric Supervaluations predict stronger presuppositions than Symmetric Transparency.
-sometimes Symmetric Transparency predicts stronger presuppositions than Symmetric Kleene and Symmetric Supervaluations.

It is worth considering one case in which incremental trivalent approaches generally predict weaker inferences than incremental Transparency or our version of incremental satisfaction. It turns out that for the formula *(No P . QQ')* most trivalent approaches predict a weak presupposition. This is easiest to see by asking what it would take to be 'certain' that the sentence is true, or that the sentence is false (despite possible uncertainties about the value of *QQ'* as applied to some individuals); failure will simply be obtained in all other cases, i.e. those in which the formula is neither true nor false. So let us consider some arbitrary world w in the context set.

-To be certain that the sentence is true at w, we need to be in a position to determine for every object d of D_w for which **P**(w)(d) = 1 that **QQ'**(w)(d) = 1; the latter condition requires that **Q**(w)(d) = 1 and also that **Q'**(w)(d) = 1. Therefore, every P-individual in w must satisfy the presupposition *Q'*.

-To be certain that the sentence is false in w, things are much easier - all we need is to find one element d for which **P**(w)(d) = 1 and **QQ'**(w)(d) = 1. This certainly does *not* require that every P-individual of w satisfies the presupposition *Q'*.

-Overall, the sentence has a classical value just in case it is true or false, which clearly does *not* entail that in every world of the context set every P-individual satisfies the presupposition *Q'*.

This prediction might seem problematic in view of the experimental results in Chemla 2007, which suggest that the universal inference obtained in this case is rather strong. One could reply that the universal inference is the product of the presupposition *together* with the assertion; and certainly for the sentence to be *true* it must be that each *P*-individual satisfies the presupposition *Q*. But it is not clear that this strategy will apply to other cases. Arguably, the universal inference is preserved in questions, although in this case the subject has no reason to assume that the quantified statement is true:

(98) a. Did none of these ten students stop smoking?
b. Is it true that none of these ten students has stopped smoking?

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[21] In fact the result is more general - it holds whenever the Transparency theory and its supervaluationist competitor make use of the same incremental principle, no matter what it is.

Similarly, in dialogues the word *no* (or: *It's not true*, or *I doubt it*) following a statement of the form $(No\ P\ .\ \underline{Q}Q')$ does *not* appear to destroy the inference that *every P-individual is a Q-individual*:

(99) A: None of these ten students has stopped smoking.
B: No / It's not true / I doubt it.
=> B probably agrees that each of the students used to smoke.

These facts are rather expected for theories that take the presupposition to be universal (since the presupposition is presumably preserved in questions and under negation). The trivalent theory might have greater difficulties accounting for them, since in these cases the assertive component cannot be used to strengthen the inference triggered by the presupposition.

I should point out that Ben George has elegantly addressed this problem within his own trivalent system (George 2007). The basic idea is to add to the trivalent analysis a built-in bias for truth, which specifies that the sentence $(No\ P\ .\ \underline{Q}Q')$ is (incrementally) ruled out as deviant if the presupposition alone suffices to exclude the possibility that the sentence will end up true. If at some world w of the context set, some element d satisfies *P* but not *Q*, we can be certain that the sentence won't be true at w: d will make $\underline{Q}Q'$ indeterminate, which in turns means that the quantified statement may be indeterminate or false, but certainly not true. Importantly, for quantifiers that are not universal in force, some weaker presuppositions are sometimes obtained - arguably an important virtue when one considers Chemla's recent experimental results.

This only scratches the surface of the new debate on presupposition projection; to cite but one other attempt that takes a very different direction, Chemla (2007b) has recently developed a new and ambitious theory that seeks to unify presuppositions and implicatures in a non-dynamic framework, while accounting for the fine-grained patterns of projection that Chemla himself discovered[22]. A detailed comparison between these new theories of presupposition will have to await further work. But it is already clear that the presupposition debate has decisively moved beyond the traditional version of dynamic semantics.

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[22] In a nutshell, Chemla's theory is based on two assumptions.
(a) A presuppositional expression $\underline{d}d'$ evokes two alternatives: *d* and *(not d)*. (Once the alternatives of lexical elements have been specified, the alternatives of complex sentences are computed as in other theories - at least in simple cases).
(b) A sentence $a\ \underline{d}d'\ b$ is only acceptable if its alternatives *a d b* and *a (not d) b* are ruled out because *d* and *(not d)* are locally trivial, in the sense that they play exactly the same semantic role (relative to the context set) as a tautology or a contradiction that would be inserted in the same syntactic environment. More precisely, the conditions that must be met are stated in (i), where *T* is a (propositional or predicative) tautology, and *F* is a contradiction:

(i) a. $C \models a\ d\ b \Leftrightarrow a\ T\ b$
b. $C \models a\ (not\ d)\ b \Leftrightarrow a\ F\ b$

When we apply these principles to $(No\ P\ .\ \underline{Q}Q')$, we obtain the conditions in (ii):

(ii) a. $C \models (No\ P\ .\ Q) \Leftrightarrow (No\ P.\ T)$
b. $C \models (No\ P\ .\ (not\ Q)) \Leftrightarrow (No\ P.\ F)$

The right-hand side of (ii)a is trivially false (if *P* is non-empty), and thus the condition requires that $C \models (Some\ P\ .\ Q)$. This condition alone would arguably be too weak. But (ii)b comes to the rescue: the right-hand side is trivially true, so it must be that $C \models (No\ P\ .\ (not\ Q))$, or in other words: $C \models (Every\ P\ .\ Q)$ - hence the universal presupposition.

This analysis has the great virtue of predicting fine-grained and weaker-than-usual presuppositions in the case of numerical quantifiers. But one can ask what is the motivation for positing that a $\underline{Q}Q'$ evokes *both* alternative *Q* and alternative *(not Q)*. The empirical evidence is largely (though not quite solely) dependent on the very status of statements involving the quantifier *no*. Independent evidence would be welcome to further motivate the theory.

Appendix. Comparing Five Theories of Presupposition Projection

In this Appendix, we define one base language (called *L*) and study five accounts of presupposition projection in that language: dynamic semantics, the Transparency theory, our version of local satisfaction, a supervaluationist alternative, and a Strong Kleene alternative.

1. **Syntax of L**
   -Generalized Quantifiers: Q ::= Q_i
   -Predicates: P ::= P_i | P_iP_k
   -Propositions: **p ::= p_i | p_ip_k**
   -Formulas **F ::= p | (not F) | (F and F) | (F or F) | (if F. F)** | (Q_i P . P)
   To state some of our principles, the official object language is enriched with:
   (a) predicate conjunction: if *P* and *P'* are predicates, so is *(P and P')*.
   (b) restrictions of predicative and propositional types:
   if *c'* is a predicative context variable and if *P* is a predicate, $^{c'}P$ is a predicative expression*;*
   if *c'* is a propositional context variable and if *F* is a formula, $^{c'}F$ is a formula.
   Terminology: The 'propositional fragment' of L is the language defined by the expressions in bold

We start by defining a classical semantics, which we call *I*. On a technical level, we assume that:
-each propositional letter is assigned by I a function of type <s, t>
-each predicate is letter is assigned by I a function of type <s, <e, t>>
-each generalized quantifier Q_i corresponds to a 'tree of numbers' f_i, which associates a truth value to each pair of the form *(a, b)* with *a* = the number of elements that satisfy the restrictor but not the nuclear scope and *b* = the number of elements that satisfy both the restrictor and the nuclear scope.
Logical constants are given a syncategorematic semantics.
Instead of writing I(F)(w) = 1, we sometimes use the notation w |= F. We will also abbreviate I(F)(w) as $[\![F]\!]^w$ or even as F^w . When we use an extended language to state some of our principles, we will sometimes need assignment functions, and we will write w $|=^s$ F, I_s(F)(w) or $[\![F]\!]^{w,s}$ to indicate relativization of the relevant notions to the assignment function s.
When certain elements are optional, we place angle brackets (< >) around them and around the corresponding part of the semantic rules.

2. **Classical Semantics** (called *I* in what follows)
   w |= p iff $p^w = 1$
   w |= pp' iff $p^w = p'^w = 1$
   w |= (not F) iff w |≠ F
   w |= (F and G) iff w |= F and w |= G
   w |= (F or G) iff w |= F or w |= G
   w |= (if F. G) iff w |≠ F or w |= G
   w |= $(Q_i<\underline{P}>P'.<\underline{Q}>Q')$ iff $f_i(a_w, b_w)=1$ with $a_w=\{d\in D: <P^w(d) = 1 \text{ and}> P'^w(d) = 1 \text{ and } (<Q^w(d) = 0 \text{ or}> Q'^w(d) = 0)\}$, $b_w=\{d\in D: <P^w(d) = 1 \text{ and}> P'^w(d) = 1 \text{ and } <Q^w(d) = 1 \text{ and}> Q'^w(d) = 1\}$
   Additions (a) and (b) in 1 can be interpreted thanks to 5b below.

For reasons of simplicity, we will assume throughout that L is *extremely* expressive: any proposition or property can be expressed by an atomic expression:

3. **Expressivity**
   Every proposition and every property is denoted by some atomic expression of L.

We repeat from the text our definitions of generalized entailment and generalized conjunction; they are intended for much richer type-theoretic languages, but are applicable in the present framework.

4. **Generalized Entailment**
   a. If x and x' are two objects of a type τ that 'ends in t', and can take at most n arguments, x ≤ x' just in case whenever $y_1, ..., y_n$ are objects of the appropriate type, if $x(y_1) ... (y_n) = 1$, then $x'(y_1) ... (y_n) = 1$
   b. If E and E' are two expressions of a type τ that 'ends in t',
   w $|=^s$ (E ≤ E') iff $[\![E]\!]^{w,s} \leq [\![E']\!]^{w,s}$

5. **Generalized Conjunction**
   a. If x and x' are two objects of a type τ that 'ends in t', and can take at most n arguments of types $\tau_1 , ..., \tau_n$

respectively, then
$x \wedge x' = \lambda y_{1\tau_1} \quad \lambda y_{n\tau_n} \quad x(y_1) \ldots (y_n) = x'(y_1) \ldots (y_n) = 1$
b. If E and E' are two expressions of a type τ that 'ends in t',
$[\![{}^{E'}E]\!]^{w,s} = [\![(E' \text{ and } E)]\!]^{w,s} = [\![E']\!]^{w,s} \wedge [\![E']\!]^{w,s}$

We define on the basis of 2 a dynamic semantics which is in full agreement with Heim's analysis (Heim 1983), except for disjunction, which she does not discuss; here we follow Beaver 2001.

6. **Dynamic Semantics**
$C[p] = \{w \in C: p^w = 1\}$
$C[\underline{p}p'] = \#$ iff for some $w \in C$, $p^w = 0$; if $\neq \#$, $C[\underline{p}p'] = \{w \in C: p'^w = 1\}$
$C[(\text{not } F)] = \#$ iff $C[F] = \#$; if $\neq \#$, $C[(\text{not } F)] = C - C[F]$
$C[(F \text{ and } G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$); if $\neq \#$, $C[(F \text{ and } G)] = C[F][G]$
$C[(F \text{ or } G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[\text{not } F][G] = \#$); if $\neq \#$, $C[(F \text{ or } G)] = C[F] \cup C[\text{not } F][G]$
$C[(\text{if } F. G)] = \#$ iff $C[F] = \#$ or ($C[F] \neq \#$ and $C[F][G] = \#$); if $\neq \#$, $C[(\text{if } F.G)] = C\text{-}C[F][\text{not } G]$
$C[(Q_i \langle\underline{P}\rangle P'.\langle\underline{R}\rangle R')] = \#$ iff <for some $w \in C$, for some $d \in D$, $\underline{P}^w(d) = 0$> or <for some $w \in C$, for some $d \in D$, <$\underline{P}^w(d) = 1$ and> $P'^w(d) = 1$ and $\underline{R}^w(d) = 0$>. If $\neq \#$, $C[(Q_i\langle\underline{P}\rangle P'.\langle\underline{R}\rangle R')] = \{w \in C: f_i(a^w, b^w) = 1\}$ with $a^w = \{d \in D: P'^w(d) = 1 \text{ and } R'^w(d) = 0\}$, $b^w = \{d \in D: P'^w(d) = 1 \text{ and } R'^w(d) = 1\}$

The Transparency theory is based on the classical semantics in 2; it comes in an incremental version and in a symmetric version.

7. **Transparency: Principles**
a. Be Articulate
In any syntactic environment, express the meaning of an expression *<u>d</u>d'* as *(d and <u>d</u>d')*
b. Be Brief - Incremental Version
Given a context set C, a predicative or propositional occurrence of *d* is infelicitous in a sentence that begins with *α (d and* if for any expression *γ* of the same type as *d* and for any good final *β*, $C \models \alpha\ (d \text{ and } \gamma)\ \beta \Leftrightarrow \alpha\ \gamma\ \beta$.
c. Be Brief - Symmetric Version
Given a context set C, a predicative or propositional occurrence of *d* is (somewhat) infelicitous in a sentence of the form *α (d and d') β* if for any expression *γ* of the same type as *d*, $C \models \alpha\ (d \text{ and } \gamma)\ \beta \Leftrightarrow \alpha\ \gamma\ \beta$.
d. Ordering of Principles
Be Brief [in either version] >> Be Articulate

8. **Transparency: Derived Notions**
a. Incremental Transparency = Be Articulate + Incremental Version of *Be Brief*
Let C be a context set and *F* be a formula. *F satisfies Incremental Transparency relative to C* (abbreviation: *$Transp^i(C, F)$*) just in case for any presuppositional expression *<u>d</u>d'*, for any strings *α* and *β*, if $F = \alpha\ \underline{d}d'\ \beta$, then for any constituent *γ* of the same type as *d* and for any good final *β'*,
$C \models \alpha\ (d \text{ and } \gamma)\ \beta' \Leftrightarrow \alpha\ \gamma\ \beta'$
b. Symmetric Transparency = Be Articulate + Symmetric Version of *Be Brief*
Let C be a context set and *F* be a formula. *F satisfies Symmetric Transparency relative to C* (abbreviation: *$Transp^s(C, F)$*) just in case for any presuppositional expression *<u>d</u>d'*, for any strings *α* and *β*, if $F = \alpha\ \underline{d}d'\ \beta$, then for any constituent *γ* of the same type as *d*,
$C \models \alpha\ (d \text{ and } \gamma)\ \beta \Leftrightarrow \alpha\ \gamma\ \beta$

In a broad range of cases, the incremental version of the Transparency theory is equivalent to standard dynamic semantics.

9. **Incremental Transparency vs. Standard Dynamic Semantics** (from Schlenker 2007a)
a. Non-Triviality
Let $C \subseteq W$ be a context set and let *F* be a formula. <C, F> satisfies **Non-Triviality** just in case for any initial string of *F* of the form *α A*, where *A* is a quantificational clause (i.e. a formula of the form *(Q_i G. H)*), there is a good final *β* such that:
$C \not\models \alpha\ A\ \beta \Leftrightarrow \alpha\ T\ \beta$
$C \not\models \alpha\ A\ \beta \Leftrightarrow \alpha\ F\ \beta$
where *T* is a tautology and *F* is a contradiction.
b. Constancy

Let C be a context set and *F* be a formula. <C, F> satisfies **Constancy** just in case (i) the (unique) domain of individuals is of constant finite size over C, and (ii) the extension of each restrictor that appears in *F* is of constant size over C.
c. Theorem 1: Consider the propositional fragment of L. Let C ⊆ W be a context set and let *F* be a formula. Then:
(i) $Transp^i$(C, F) iff C[F] ≠ #.
(ii) If C[F] ≠ #, C[F] = {w ∈ C: w |= F}.
d. Theorem 2 [here we only state a *consequence* of Theorem 2 from Schlenker 2007a]: Let C ⊆ W be a context set and let *F* be a formula of L. Suppose that <C, F> satisfies Non-Triviality and Constancy. Then:
(i) $Transp^i$(C, F) iff C[F] ≠ #.
(ii) If C[F] ≠ #, C[F] = {w ∈ C: w |= F}.

We now turn to the definition of transparent restrictions, of local contexts and of local satisfaction.

10. **Transparent Restrictions**
Let C ⊆ W be a context set and let *a d b* be a formula, where *d* has a type that 'ends in t'; let *c'* be a variable of the same type as *d*.
a. tr^i(C, d, a_b) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, for every good final *b'*, C $|=^{c' \to x}$ a $^{c'}$d' b' ⇔ a d' b'}
b. tr^s(C, d, a_b) = {x: x is an object of the type specified by *d* and for every constituent *d'* of the same type as *d*, C $|=^{c' \to x}$ a $^{c'}$d' b ⇔ a d' b}

11. **Local Contexts**
a. lc^i(C, d, a_b) = the bottom element of tr^i(C, d, a_b), if such an element exists; # otherwise.
b. lc^s(C, d, a_b) = the bottom element of tr^s(C, d, a_b), if such an element exists; # otherwise.

12-16 are concerned with the existence of local contexts.

12. **Lemma 1. Propositional Fragment.**
For any C ⊆ W, for any formula *E*, if *a E b* is a formula of the propositional fragment, lc^i(C, E, a_b) ≠ # and lc^s(C, E, a_b) ≠ #.

Proof: We define
LC^i := λw. 1 iff for some formula g, for some good final b', w $|\neq^{f \to F}$ a ^{f}g b' ⇔ a g b', where F is a contradiction.
Step 1: LC^i ∈ tr^i(C, E, a_b)
For every w ∈ C, either LC^i(w) = 1, in which case the contextual restriction LC^i is innocuous at w and w $|=^{c' \to LC^i}$ a $^{c'}$g b' ⇔ a g b'; or LC^i(w) = 0, which means that for every formula g, for every good final b', w $|=^{f \to F}$ a ^{f}g b' ⇔ a g b', and thus w $|=^{c' \to LC^i}$ a $^{c'}$g b' ⇔ a g b'.
Step 2: For every x ∈ tr^i(C, F, a_b), LC^i entails x.
Suppose, for contradiction, that LC^i(w) = 1 and x(w) = 0. Since x ∈ tr^i(C, F, a_b), for every formula g, for every good final b', w $|=^{c' \to x}$ a $^{c'}$g b' ⇔ a g b' and since x(w) = 0 and the logic is extensional, w $|=^{f \to F}$ a ^{f}g b' ⇔ a g b'. But by the definition of LC^i this means that LC^i(w) = 0, contrary to hypothesis.
The proof of lc^s(C, F, a_b) ≠ # is similar.

13. **Lemma 2: Closure under Finite Conjunction**
For any C ⊆ W, for any formula *a d b,* tr^i(C, d, a_b) and tr^s(C, d, a_b) are closed under finite conjunction.
Proof: Assume that x', x" ∈ tr^i(C, d, a_b). We have in particular that for any admissible *d'* and for any good final *b'*,

1. C $|=^{e \to x' \wedge x'', c' \to x', c'' \to x''}$ a ^{e}d' b' ⇔ a $^{c'}$(c" and d') b'	[by the semantics of $^{c'}F$]
2. C $|=^{e \to x' \wedge x'', c' \to x', c'' \to x''}$ a $^{c'}$(c" and d') b' ⇔ a (c" and d') b'	[because x' ∈ tr^i(C, d, a_b)]
3. C $|=^{e \to x' \wedge x'', c' \to x', c'' \to x''}$ a (c" and d') b' ⇔ a $^{c''}$d' b'	[by the semantics of $^{c''}F$]
4. C $|=^{e \to x' \wedge x'', c' \to x', c'' \to x''}$ a $^{c''}$d' b' ⇔ a' d' b'	[because x" ∈ tr^i(C, d, a_b)]
5. C $|=^{e \to x' \wedge x'', c' \to x', c'' \to x''}$ a ^{e}d' b' ⇔ a' d' b'	[by 1.-4.]
6. C $|=^{e \to x' \wedge x''}$ a ^{e}d' b' ⇔ a' d' b'	[by 5., since *c'* and *c"* don't appear]
7. (x' ∧ x") ∈ tr^i(C, d, a_b)	

The proof is similar for $tr^s(C, d, a\_b)$.

14. **Lemma 3: Finite Sets**
For every $v \in \{i, g\}$, if $tr^v(C, d, a\_b)$ is finite, $lc^v(C, d, a\_b) \neq \#$.
Proof: Immediate from 13.

15. **Lemma 4: Point-wise Construction of Local Contexts**
(This lemma crucially relies on the extensionality of the fragment)
For every $v \in \{i, g\}$, if for every $w \in C$, $lc^v(\{w\}, a\ d, a\_b) \neq \#$, then $lc^v(C, d, a\_b) \neq \#$.

Proof: The idea is to construct the bottom element point-wise, i.e. world by world.
We define LC^v as $\lambda w_s . lc^v(\{w\}, d, a\_b)$ if $w \in C$; z otherwise
where z is the null object of type t if *d* is propositional, and where z is the null object of type <e, t> if *d* is predicative.
-It is immediate that $LC^v \in tr^v(C, d, a\_b)$ (because of the extensionality of the fragment).
-Suppose, for contradiction, that for some $x \in tr^v(C, d, a\_b)$, x is not entailed by LC^v. Then there is some world w such that x(w) is not entailed by $LC^v(w)$. It couldn't be that $w \notin C$, since in that case $LC^v(w) = z$, which entails everything. So $w \in C$. Clearly, since $x \in tr^v(C, d, a\_b)$, $x(w) \in tr^v(\{w\}, d, a\_b)$. But by assumption $LC^v(w)$ is the bottom element of $tr^v(\{w\}, d, a\_b)$, so it entails x(w), *contra* hypothesis.

16. **Existence Theorem: Existence of Local Contexts**
Let $C \subseteq W$ be a context set and let *a E b* be any formula.
a. If *a <u>d</u>d b* belongs to the propositional fragment, then for every $v \in \{i, s\}$, $lc^v(C, E, a\_b) \neq \#$.
b. If for every $w \in C$, the domain of individuals in w is of finite size, then for every $v \in \{i, s\}$, $lc^v(C, E, a\_b) \neq \#$.
Proof:
(a) is just Lemma 1 [12]
(b) follows from Lemma 3 [14] and Lemma 4 [15], together with the following observation:
if the domain of individuals in w, D_w, is finite, then for any intensional type τ there are only finitely many functions of type τ with $D_s = \{w\}$[23]. It follows that $tr^v(\{w\}, E, a\_b)$ is finite. By Lemma 3 [14], we construct $lc^v(\{w\}, E, a\_b)$ for every $w \in C$. By Lemma 4 [15], this makes it possible to construct $lc^v(C, E, a\_b)$.

17. **Definition of Local Satisfaction - Special Case** (when local contexts exist)
Let $C \subseteq W$ be a context set.
a. For every $v \in \{i, s\}$, for all expressions *<u>d</u>d'*, *a*, *b*, if $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, \underline{d}d', a\_b)$ just in case $lc^v(C, \underline{d}d', a\_b) \leq \mathbf{d}$
b. For every $v \in \{i, s\}$, for every formula *F* , if for all expressions *a, b, <u>e</u>e'* such that F = a <u>e</u>e' b, $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, F)$ just in case for every expression *<u>e</u>e'*, for all strings *a, b*, if F = a <u>e</u>e' b, then $Sat^v(C, \underline{e}e', a\_b)$.

18. **Definition of Local Satisfaction - General Case** (local contexts need not exist)
Let $C \subseteq W$ be a context set.
a. For every $v \in \{i, s\}$, for all expressions *<u>d</u>d'*, *a*, *b*, $Sat'^v(C, \underline{d}d', a\_b)$ iff for some $X \in tr^v(C, \underline{d}d', a\_b)$, for every X', if [$X' \leq X$ and $X' \in tr^v(C, \underline{d}d', a\_b)$], then $C \models^{c' \to X'} c' \leq d$
b. For every $v \in \{i, s\}$, for every formula *F* , $Sat'^v(C, F)$ just in case for every expression *<u>e</u>e'*, for all strings *a, b,* if F = a <u>e</u>e' b, $Sat'^v(C, \underline{e}e', a\_b)$.

19. **Lemma 5: when local contexts exist, 18 and 17 are equivalent.**
For every $v \in \{i, s\}$, if $lc^v(C, \underline{d}d', a\_b) \neq \#$, $Sat^v(C, \underline{d}d', a\_b)$ iff $Sat'^v(C, \underline{d}d', a\_b)$
Proof: First, if $Sat^v(C, \underline{d}d', a\_b)$, then by taking $X = lc^v(C, \underline{d}d', a\_b)$, we can find an $X \in tr^v(C, \underline{d}d', a\_b)$

[23] The proof is by induction on primitive types relative to w. Clearly, D_w and {0, 1} are finite. Furthermore, if E is finite, so are $D_w \to E$ and $\{0, 1\} \to E$.

such that, for every X', if [$X' \leq X$ and $X' \in tr^{v}(C, \underline{d}d', a\_b)$], then $C \models^{c' \to X'} c' \leq d$. Second, if $Sat'^{v}(C, \underline{d}d', a\_b)$, there is some X such that, for every X', if [$X' \leq X$ and $X' \in tr^{v}(C, \underline{d}d', a\_b)$], then $C \models^{c' \to X'} c' \leq d$. Since $lc^{v}(C, \underline{d}d', a\_b)$ is the bottom element of $tr^{v}(C, \underline{d}d', a\_b)$, $lc^{v}(C, \underline{d}d', a\_b) \leq X$, and therefore $C \models^{c' \to lc^{v}(C, \underline{d}d', a\_b)} c' \leq d$. By the definition $tr^{v}(C, \underline{d}d', a\_b)$ and $lc^{v}(C, \underline{d}d', a\_b)$, it must be the case that $lc^{v}(C, \underline{d}d', a\_b) \leq C$. Therefore $lc^{v}(C, \underline{d}d', a\_b) \leq \mathbf{d}$. In other words, $Sat^{v}(C, \underline{d}d', a\_b)$.

Before we go further, it is worth pointing out that the incremental version of local satisfaction systematically predicts presuppositions that are at least as strong as those predicted by the symmetric version (the same conclusion holds of the incremental vs. symmetric version of all the theories under study in this Appendix).

20. **Incremental Satisfaction predicts stronger presuppositions than Symmetric Satisfaction**
For any context set C, for all expressions *$\underline{d}$d'* and for all strings *a, b*,
a. $tr^{i}(C, \underline{d}d', a\_b) \subseteq tr^{s}(C, \underline{d}d', a\_b)$.
Furthermore, if $lc^{s}(C, d, a\_b) \neq \#$ and $lc^{i}(C, d, a\_b) \neq \#$,
b. $lc^{s}(C, d, a\_b) \leq lc^{i}(C, d, a\_b)$
c. if $Sat^{i}(C, d, a\_b)$, then $Sat^{s}(C, d, a\_b)$
Proof: Immediate.

We now turn to a comparison between Incremental Satisfaction, Incremental Transparency and Dynamic Semantics.

21. **Theorem. Equivalence with Transparency**
Let $C \subseteq W$ be a context set. Then:
(i) For any $v \in \{i, s\}$, for every formula that has the form *a $\underline{d}$d' b*, $Sat'^{v}(C, \underline{d}d', a\_b)$ iff $Transp^{v}(C, \underline{d}d', a\_b)$.
(ii) In particular, for any $v \in \{i, s\}$, for every formula *F*, $Sat'^{v}(C, F)$ iff $Transp^{v}(C, F)$.

Proof of (i):
We start with the incremental version.
=>: Suppose that $Sat'^{i}(C, \underline{d}d', a\_b)$. Then for some $X \in tr^{i}(C, \underline{d}d', a\_b)$,
$C \models^{c' \to X} c' \leq d$
For every expression *d"* of the same type as *d*, for every good final *b'*,
$C \models^{c' \to X}$ a (d and d") b' $\Leftrightarrow$ a $^{c'}$(d and d") b'
$C \models^{c' \to X}$ a $^{c'}$(d and d") b' $\Leftrightarrow$ a $^{c'}$d" b'
$C \models^{c' \to X}$ a $^{c'}$d" b' $\Leftrightarrow$ a d" b'
Hence
$C \models$ a (d and d") b' $\Leftrightarrow$ a d" b'

<=: Suppose that $Transp^{i}(C, \underline{d}d', a\_b)$.
Clearly, $\mathbf{d} \in tr^{i}(C, \underline{d}d', a\_b)$. Furthermore, for every X',
[$X' \in tr^{i}(C, \underline{d}d', a\_b)$ and $X' \leq \mathbf{d}$] => $C \models^{c' \to X'} c' \leq d$. So $Sat'^{i}(C, \underline{d}d', a\_b)$.

The argument is similar for the symmetric version of *Sat* and *Transp*.

Proof of (ii): Immediate given (i).

22. **Theorem. Equivalence with Standard Dynamic Semantics**
Let $C \subseteq W$ be a context set and *F* be a formula which satisfy Non-Triviality and Constancy. Then:
(i) for all expressions *a, b, $\underline{d}$d'* , if F = a $\underline{d}$d' b, $lc^{i}(C, \underline{d}d', a\_b) \neq \#$. Furthermore,
(ii) $Sat^{i}(C, F)$ iff $Sat'^{i}(C, F)$ iff $C[F] \neq \#$.

Proof of (i): Immediate from 16b and the fact that Constancy implies that each $w \in C$, the set of individuals in w is of finite size.
Proof of (ii): The first equivalence follows from (i) and the Lemma in 19. The second equivalence follows from (i), 9c and 21(ii).

Before going further, we consider an example in which local contexts do not exist, which makes it necessary to resort to the alternative definition of satisfaction, *Sat'*, whose incremental version yields full equivalence with Dynamic Semantics

23. **Infinitely Many**
Consider the formula F = (Infinitely-many P . $\underline{Q}$Q'). We assume that there are infinitely many elements in **P**(w).
a. *$\underline{Q}$Q'* has no local context in the context set {w}.
b. Sat'^{i}(C, F) (or equivalently Transp^i(C, F)) need not entail that C |= (Every P . Q)
Proof:
a. First, we note that if **c'** is transparent, **c'**(w) must itself contains infinitely many elements. For if not, *(infinitely-many P . $^{c'}$P)* would be false at w but *(infinitely-many P . P)* would be true - and *c'* wouldn't be transparent. Second, we show that for any transparent value x for *c'*, we can find a 'smaller' value x' which is also transparent. Since x(w) must contain infinitely many elements, we just take one arbitrary element out of x(w), obtaining an x'(w) distinct from x(w) with x'(w) ≤ x(w) (and also x ≤ x'). And it is clear that x', which itself contains infinitely many elements, is transparent (because the truth of the statement *infinitely many Ps are Qs* is insensitive to whatever happens to any given finite set of elements). Since x was arbitrary, we have shown that the set of transparent context denotations simply does not have a bottom element.
b. Assume that in w *Q* holds true of all *P*-individuals except a (non-zero) finite number. We have that for any predicative *D*, w |= (Infinitely-many P . (Q and D)) ⇔ (Infinitely-many P . D), so Transp^i(C, F), and therefore (by 21) Sat^i(C, F). Still, w |≠ (Every P . Q).

We turn to the trivalent alternatives to existing theories of presupposition; we start from a version of a supervaluationist account (somewhat adapted to the present framework), and then turn to a version of Strong Kleene (derived from the supervaluationist account). The following systems are approximations of theories pioneered by Peters, Beaver and Krahmer, and more recently George, and Fox. This entire discussion relies heavily on George's and Fox's ongoing research (George 2007, Fox 2007).

24. **Supervaluations I: Extensions of an interpretation**
Let I be an interpretation defined by 2. We start from the idea that $\underline{p}$p'(w) or $\underline{P}$P'(w)(d) are indeterminate if p(w) ≠ 1 and P(w)(d) ≠ 1. The set Ext(I) of extensions of the interpretation I is defined by the following procedure.
1. First, we treat expressions of the form *$\underline{p}$p'* or *$\underline{P}$P'* as *atomic* symbols of a classical language [rather than as complex symbols as in 2]; in effect, we obtain a new language L* which is identical to L except that expressions of the form *$\underline{p}$p'* are *$\underline{P}$P'* are part of the lexicon (in L they are syntactically complex). For simplicity, we identify formulas of L* with string identical formulas of L (despite the fact that they may be generated in distinct ways). We call I* the interpretation of L* which agrees with I, in the sense that for any expressions of the form *$\underline{p}$p'* or *$\underline{P}$P'* , I*($\underline{p}$p') = I($\underline{p}$p') and I*($\underline{P}$P') = I($\underline{P}$P'). It is immediate that such an interpretation I* can be found.
2. Second, for all interpretations i and i' of L*, we define i ∠ i' (meaning that *i' is an extension of i*) just in case for every atomic expression *E*:
-if *E* does not contain any underlined element, i(E) = i'(E)
-for every *p, p'*, if *E* = $\underline{p}$p', then for every world w, if i(p)(w)= 1, i'=($\underline{p}$p')(w) = i($\underline{p}$p')(w).
-for every *P, P'*, if *E* = $\underline{P}$P', then for every world w, for every individual d, if i(P)(w)(d) = 1, then i'($\underline{P}$P')(w)(d) = i($\underline{P}$P')(w)(d).
Note that in case i(p)(w) = 0, i'($\underline{p}$p')(w) may freely take the values 0 and 1; similarly, in case i(P)(w)(d) = 0, i'($\underline{P}$P')(w)(d) may freely take the value 0 and 1. This captures the intuition that in such cases the value of *$\underline{p}$p'* or *$\underline{P}$P'* is unknown, and could be 'resolved' in various ways.
3. Third, we define: Ext(I) := {i': i' is a classical interpretation of L* and I* ∠ i'}

25. **Supervaluations II: Truth**
As before, I is the classical interpretation defined by 2. For any formula *F* of L:
F is super-true at w (*notation*: Super(F, w) = 1) if and only if for every i ∈ Ext(I), i(F)(w) = 1
F is super-false at w (*notation*: Super(F, w) = 0) if and only if for every i ∈ Ext(I), i(F)(w) = 0
F is super-indeterminate at w (*notation*: Super(F, w) = #) if and only if *F* is neither true nor false at w.

26. **Supervaluations III: Acceptability**
a. Incremental Acceptability
F is incrementally super-acceptable in C (*notation*: Super-accept^i(C, F)) just in case for any presuppositional expression *$\underline{d}$d'*, for any strings α and β, if F = α $\underline{d}$d' β, then for any good final *β'* **which does not contain any underlined expressions**, for any world w ∈ C, Super(α $\underline{d}$d'β', w) ≠ #
b. Symmetric Acceptability

F is symmetrically super-acceptable relative to C (abbreviation: *Super-accept*^{s}*(C, F)*) just in case for any world w ∈ C, Super(α d̲d'β, w) ≠ #.

Strong Kleene logic is usually defined as a compositional trivalent logic (by contrast, supervaluationist semantics is not compositional). It will be convenient, however, to define Strong Kleene in terms of supervaluations; we will show in 40-41 that in the propositional case our definition derives the same results as the standard definition (the equivalence extends to the full quantificational fragment, but giving a proof would require that we discuss the extension of Strong Kleene to generalized quantifiers, which isn't so common to begin with).

27. **Definitions: $[s]^\#$, L** and I****

a. Let s be any string. We define $[s]^\#$ to be identical to s, except that every occurrence of the form *p̲p'* or *P̲P'* is replaced with p̲p'^{n} and P̲P'^{n} if it is the nth occurrence of its type in s (counting from left to right). We abbreviate *$[F]^\#$* as *$F^\#$* when there is no risk of ambiguity.

Examples: $[((p_1 \text{ and } \underline{p}_2p_3) \text{ or } \underline{p}_1p_2)]^\# = ((p_1 \text{ and } \underline{p}_2{p_3}^1) \text{ or } \underline{p}_1{p_2}^1)$
$[((p_1 \text{ and } \underline{p}_2p_3) \text{ or } \underline{p}_2p_3)]^\# = ((p_1 \text{ and } \underline{p}_2{p_3}^1) \text{ or } \underline{p}_2{p_3}^2)$

b. We define a set for formulas L** = L* ∪ {$F^\#$: F is a formula of L*}. L** is a subset of a language defined directly by the syntactic rules in 1, supplemented with: P ::= $P_iP_k{}^n$ and p ::= $p_ip_k{}^n$.

c. For every atomic expression *e* of L, we define $e^- = e$ if e does not contain any superscript; and if e = e'^{n} for some atomic expression *e'* and some superscript n, e^- = e'.

d. We define an interpretation I** for L** as: for every atomic expressions e of L, I**(e) = I*(e^-)

28. **Strong Kleene I: Truth from Supervaluations**

Let *F* be a formula of L, interpreted by I. I** is defined for L** as specified in 27, and we apply the definition of extensions of an interpretation and supervaluationist truth in 24(2-3) and 25, but replacing L* and I* with L** and I** respectively. Finally, we define:
F is Kleene-true at w (*notation*: Kleene(F, w) = 1) iff *$F^\#$* is super-true at w (i.e. Super($F^\#$, w) = 1).
F is Kleene-false at w (*notation*: Kleene(F, w) = 0) iff *$F^\#$* is super-false at w (i.e. Super($F^\#$, w) = 0).
F is Kleene-indeterminate at w (*notation*: Kleene(F, w) = #) iff *$F^\#$* is super-indeterminate at w (i.e. Super($F^\#$, w) = #).

29. **Strong Kleene II: Acceptability**

a. Incremental Acceptability
F is incrementally Kleene-acceptable relative to C (*notation*: Kleene-accept^i(C, F)) just in case for any presuppositional expression *d̲d',* for any strings *α* and *β*, if F = α d̲d' β, then for any good final *β'* **which does not contain any underlined expressions**, for any world w ∈ C, Kleene(α d̲d'β', w) ≠ #.
b. Symmetric Acceptability
F is symmetrically Kleene-acceptable relative to C (*notation*: Kleene-accept^s(C, F)) just in case for any world w ∈ C, Kleene(F, w) ≠ #.

30. **Lemma 6.** Let *F* be any formula of L, and assume that for every expression *d̲d'*, if *d̲d'* occurs in *F*, it occurs exactly once. Then for any world w, Super(F, w) = Kleene(F, w)

Proof: The result is trivial, since in that case *$F^\#$* is obtained from *F* by uniform replacement of certain propositional and predicative constants with other constants that have the same value.

31. **Lemma 7.** For any formula *F*, for any world w, if Kleene(F, w) ≠ #, then Super(F, w) ≠ # and Super(F, w) = Kleene(F, w).

Proof: Suppose that Kleene(F, w) ≠ #. This means that for some v ∈ {0, 1}, for all i' ∈ Ext(I**), i'($F^\#$) = v. Let E be the set {i: i is an interpretation of L** and for all integers n, k, for all underlined expressions *d̲d'*, i(d̲d'^{n}) = i(d̲d'^{k})}. For each formula *F* of L, for each interpretation i' of Ext(I*), there is an interpretation i of

E such that i($F^{\#}$) = i'(F). It follows that for all i' ∈ Ext(I**), i'(F) = v, and thus Super(F, w) ≠ # and Super(F, w) = v = Kleene(F, w).

32. **Lemma 8.** For every v ∈ {i, s}, for any formula *F*, for any context set C ⊆ W, if Kleene-accept^v(C, F), then:
(i) Super-accept^v(C, F), and
(ii) for every w ∈ C, Kleene(F, w) = Super(F, w).

Proof: This is a consequence of 31.
-Suppose that Kleene-accept^i(C, F). For any presuppositional expression *$\underline{d}$d'*, for any strings *α* and *β*, if F = α $\underline{d}$d' β, then for any good final *β'* which does not contain any underlined expressions, for any world w ∈ C, Kleene(α $\underline{d}$d'β', w) ≠ #, whence by 31 Super(α $\underline{d}$d'β', w) ≠ #. It follows that Super-accept^i(C, F).
-Suppose that Kleene-accept^s(C, F), then for every w ∈ C, Kleene(F, w) ≠ #, and by 31 Super(F, w) ≠ #. Therefore Super-accept^s(C, F).
-If Kleene-accept^i(C, F), *a fortiori* Kleene-accept^s(C, F). And if Kleene-accept^s(C, F), Kleene(F, w) ≠ # and by 31 Super(F, w) = Kleene(F, w).

33. **Definitions**
For any formula *F*, for any context set C ⊆ W, for every integer n ≥ 0:
a. Super-accept^i(C, F, n) iff for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for every good final *β'* which does not contain any underlined expressions, for every world w ∈ C, Super(α $\underline{d}$d'β', w) ≠ #.
b. Kleene-accept^i(C, F, n) iff for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for every good final *β'* which does not contain any underlined expressions, for every world w ∈ C, Kleene(α $\underline{d}$d'β', w) ≠ #.
c. Transp^i(C, F, n) iff for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for every good final *β'* which does not contain any underlined expressions, Transp^i(C, α $\underline{d}$d'β').
d. Ext(F, n) = {i: for all atomic expressions e, if e is not one the first n underlined tokens of *F*, i(e) = I**(e) and if *e* is one of the n first underlined tokens of *F*, I**(e) ∠ i*(e)}

Note: Super-accept^i(C, F, 0), Kleene-accept^i(C, F, 0) and Transp^i(C, F, 0) are all trivially true, since it is never the case that F = α $\underline{d}$d' β with at most 0 underlined tokens in *α $\underline{d}$d'*.

34. **Lemma 9**
a. Kleene-accept^i(C, F, n) iff for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for every good final *β'* which does not contain any underlined expressions, for all i, i' ∈ Ext($F^{\#}$, n), for all w ∈ C, i([α $\underline{d}$d']$^{\#}$ β') = i'([α $\underline{d}$d']$^{\#}$ β').
b. If Kleene-accept^i(C, F, n), then for all strings *α*, *β*, if F = α β and there are at most n underlined tokens in *α*, then for any good final *β'* that does not contain any underlined material, for every world w ∈ C, Kleene(α β', w) = I**(α β')(w).

Proof:
a. Immediate given the definition of Kleene-accept^i(C, F, n).
b. The case n = 0 is trivial. For n ≥ 1, if Kleene-accept^i(C, F, n), then for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for every good final *β'* which does not contain any underlined expressions, for all i, i' ∈ Ext($F^{\#}$, n), for all w ∈ C, i([α $\underline{d}$d']$^{\#}$ β') = i'([α $\underline{d}$d']$^{\#}$ β'). But it is clear that I** ∈ Ext($F^{\#}$, n), so for all i ∈ Ext($F^{\#}$, n), for all w ∈ C, i([α $\underline{d}$d']$^{\#}$ β') = I**([α $\underline{d}$d']$^{\#}$ β') = I**(α $\underline{d}$d' β') (by the definition of I**). The result follows.

35. **Lemma 10**
a. If Super-accept^i(C, F, n), then for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n underlined tokens in *α $\underline{d}$d'*, then for any good final *β'* that does not contain any underlined material, Super-accept^i(C, α $\underline{d}$d' β') and therefore Super-accept^s(C, α $\underline{d}$d' β').
b. If Kleene-accept^i(C, F, n), then for every underlined expression *$\underline{d}$d'*, for all strings *α*, *β*, if F = α $\underline{d}$d' β and there are at most n-1 underlined tokens in *α*, then for any good final *β'* that does not contain any underlined material, Kleene-accept^i(C, α $\underline{d}$d' β') and therefore Kleene-accept^s(C, α $\underline{d}$d' β').

Proof: Immediate given the relevant definitions.

36. **Theorem. Incremental Kleene = Incremental Supervaluations**
For any formula *F*, for any context set $C \subseteq W$,
a. Kleene-accept^i(C, F) iff Super-accept^i(C, F), and
b. if Kleene-accept^i(C, F), then for every world $w \in C$, Kleene(F, w) = Super(F, w).

Proof:
-Part (b) is just 32 (ii).
-Part (a): By 32 (i), if Kleene-accept^i(C, F), then Super-accept^i(C, F). To prove the converse, we establish by induction the following result for every integer $n \geq 1$:

P(n): If Super-accept^i(C, F, n), Kleene-accept^i(C, F, n).

By choosing n greater than the number of underlined expressions in *F*, the desired result will follow.

n = 0: Trivial.
n = m+1: Suppose that Super-accept^i(C, F, m+1). Then in particular Super-accept^i(C, F, m), and by the induction hypothesis Kleene-accept^i(C, F, m). Suppose that F = α $\underline{d}$d' β, where α contains m underlined expressions. We consider the set E = {D: D is a non-underlined expression of the same semantic type as *$\underline{d}$d'* and for some i' such that I** ∠ i', I**(D) = i'($\underline{d}$d')}. By our assumption of Expressivity (in 3), E fully describes the possible extensions of *$\underline{d}$d'*: {I**(D): $D \in E$} = {i'($\underline{d}$d'): I** ∠ i'}. Since Kleene-accept^i(C, F, m), for all $D \in E$, for every good finals *β'* that does not contain any underlined expression, for all $w \in W$, for all i' such that I** ∠ i', for some $v \in \{0, 1\}$, i'(α D β')(w) = v. Since *D β'* does not contain any underlined expressions, by the induction hypothesis Kleene-accept^i(C, α D β', m), hence by 34b Kleene-accept^s(C, α D β'), and thus for some v' $\in \{0, 1\}$, for all i' such that I** ∠ i', i'($\alpha^{\#}$ D β', w) = v'. By 31 v = v', and thus for all i' such that I** ∠ i', i'($\alpha^{\#}$ D β', w) = i'(α D β', w). It follows that for all $D \in E$, for every good final *β'* that does not contain any underlined material, for all i' such that I** ∠ i', for all $w \in W$, for some $v \in \{0, 1\}$, i'($\alpha^{\#}$ D β', w) = v - which shows that Kleene-accept^i(C, α $\underline{d}$d' β, m+1); in other words, Kleene-accept^i(C, F, m+1).

37. **Lemma 11**
If Kleene-accept^s(C, F), then Super-accept^s(C, F), but in general the converse is not true.

Proof:
-The *if* part follows from 32 (i).
-To see that the converse is not true, consider the formula *($\underline{p}$p' or (not $\underline{p}$p'))*: it is symmetrically super-acceptable relative to W, the set of all possible worlds, but which is not symmetrically Kleene-acceptable relative to W if *p* is non-tautologous.

38. **Theorem: Incremental Transparency predicts stronger presuppositions than Incremental Kleene and Incremental Supervaluations.**
If Transp^i(C, F), then Kleene-accept^i(C, F) (and thus also Super-accept^i(C, F)) - but in general the converse does not hold.

Proof:

a. We show by induction on $n \geq 0$ that if Transp^i(C, F, n), then Kleene-Accept^i(C, F, n). The result will follow because if Transp^i(C, F), then Transp^i(C, F, n) for n ≥ the number of underlined tokens in F. From Kleene-Accept^i(C, F, n) it then follows that Kleene-Accept^i(C, F).
n = 0: Trivial.
n = m+1: Suppose that α contains m underlined expressions and let *α $\underline{d}$d'* be an initial string of *F*. We assume that Transp^i(C, F, m+1). Thus for every *g* of the same type as *d*, for every good final *β'*,

(i) C |= α (d and g) β' ⇔ α g β'

Let w be any world of C. We define:
G = {g: g is an expression of the same type as *d* and I**(d and g)(w) = I**(d and d')(w)}
G' = {I**(g)(w): g ∈ G}
D = {i'($\underline{d}d'^{m+1}$)(w): i' ∈ Ext($F^{\#}$, m+1)}
Step 1. We establish that D' ⊆ G.

Case 1: *d* is propositional.
-Suppose I**(d)(w) = 1. Then if i' ∈ Ext($F^{\#}$, m+1), i'($\underline{d}d'^{m+1}$)(w) = I**(d')(w), and since d' ∈ G, i'($\underline{d}d'^{m+1}$)(w) ∈ G'.
-Suppose I**(d)(w) = 0. Then if i' ∈ Ext($F^{\#}$, m+1), i'($\underline{d}d'^{m+1}$)(w) may have values 0 or 1. By choosing g_0 such that w |≠ g_0 and g_1 such that w |= g_1 (which we can do by the assumption of Expressivity in 3), it is clear that I**(d and g_1)(w) = I**(d and g_2)(w) = 0 = I**(d and d')(w) [no matter what the value of *d'* is at w]. So g_0 and g_1 both belong to G, hence 0 and 1 both belong to G'. In other words, for any i' ∈ Ext($F^{\#}$, m+1), i'($\underline{d}d'^{m+1}$)(w) ∈ G'.

Case 2: *d* is predicative.
If i' ∈ Ext($F^{\#}$, m+1), i'($\underline{d}d'^{m+1}$)(w) guarantees that for any individual x, if I**(d)(w)(x) = 1, then i'($\underline{d}d'^{m+1}$)(w)(x) = I**(d')(w)(x). By Expressivity, we find *g* such that for every individual x, I**(g)(w)(x) = I**(d')(w)(x) [= i'($\underline{d}d'^{m+1}$)(w)(x)] if I**(d)(w)(x) = 1 and I**(g)(w)(x) = i'($\underline{d}d'^{m+1}$)(w)(x) if I**(d')(w)(x) = 0. It is clear that I**(g)(w) = i'(d)(w), and furthermore that for every individual x I**(d and g)(w)(x) = I**(d and d')(w)(x), i.e. that I**(d and g)(w) = I**(d and d')(w). So *g* ∈ G, and thus I**(g)(w), which is identical to i'(d)(w), belongs to G'.

Step 2. Now we show that for every string α, for every underlined expression *$\underline{d}$d'*, for every string β, if α *$\underline{d}$d'* contains at most m+1 underlined expressions and F = α $\underline{d}$d' β, then for any good final *β'* which does not contain any underlined expressions, if i' ∈ Ext($[\alpha\ \underline{d}d']^{\#}$ β', m+1), for any world w ∈ C, i'($[\alpha\ \underline{d}d']^{\#}$ β')(w) = I**(α $\underline{d}$d' β')(w); by 34 this will derive the desired result, namely that Kleene-Accept^i(C, F, m+1)
• Since D' ⊆ G, for some g ∈ G, i'($[\alpha\ \underline{d}d']^{\#}$ β')(w) = i'($\alpha^{\#}$ $\underline{g}$ β')(w) with g satisfying I**(g)(w) = i'($\underline{d}d'^{m+1}$)(w). Furthermore, by the induction hypothesis Kleene-Accept^i(C, F, m), and thus by 34 Kleene($\alpha^{\#}$ $\underline{g}$ β')(w) = I**(α $\underline{g}$ β')(w), and thus also i'($\alpha^{\#}$ $\underline{g}$ β')(w) = I**(α $\underline{g}$ β')(w).
• Since Transp^i(C, F, m+1), I**(α g β')(w) = I**(α(d and g) β')(w). By the definition of g, I**(d and g)(w) = I**(d and d')(w), hence i'($\alpha^{\#}$ $\underline{g}$ β')(w) = I**(α $\underline{g}$ β')(w) = I**(α(d and g) β')(w) = I**(α(d and d') β')(w). This completes the proof.

b. To show that Kleene-accept^i(C, F) need not entail that Transp^i(C, F), we consider the formula and the situation in (ii):

(ii) a. (No P . $\underline{Q}$Q')
b. C = {w}, and there are two P-individuals d_1 and d_2 in w: d_1 does not satisfy *Q*, but d_2 does, and d_2 also satisfies *Q'*.

It was shown in (35) that Incremental Satisfaction predicts that (iia) presupposes that *every P-individual in w satisfies Q*. But since d_2 satisfies both *Q* and *Q'*, Incremental Kleene and Incremental Supervaluations predict that (iia) should be acceptable in C.

39. **Theorem. Symmetric Transparency and Symmetric Kleene are incomparable.**
a. Sometimes Symmetric Kleene and Symmetric Supervaluations predict stronger presuppositions than Symmetric Transparency.
b. Sometimes Symmetric Transparency predicts stronger presuppositions than Symmetric Kleene and Symmetric Supervaluations.

Proof:
a. Let us consider the formula and the situation in (i):

(i) a. ($\underline{p}$p' and $\underline{q}$q')
b. C = {w_1, w_2}, w_1 |≠ p, w_1 |≠ q, w_2 |= p and w_2 |= q

Let w belong to C. If w = w_1 , Symmetric Transparency is satisfied because for all *g*:

(ii) a. w |= ((pp' and (q and g)) ⇔ (pp' and g) [both sides are false because w_1 |≠ p and thus w_1 |≠ pp']
b. w |= ((p and g) and qq') ⇔ (g and qq') [both sides are false because w_1 |≠ q and thus w_1 |≠ qq']

If w = w_2, Symmetric Transparency is trivially satisfied in w.
By contrast, both Symmetric Kleene and Symmetric Supervaluations predict the sentence to be a presupposition failure.

b. The example is the same as in 38 b (in this case there is no difference between incremental and symmetric notions, because the presupposition trigger appears at the end of the formula).

40. **Strong Kleene: Standard definition (propositional case)**
We only give rules for the atomic case, as well as *and* and *not*; the semantics of the other connectives follows from standard rules of inderdefinability (which of course hold in Supervaluationist semantics as well).
We assume once again that a classical interpretation I is defined as in 2, and we use it to define a standard Strong Kleene interpretation - with the usual convention that *pp'* is indeterminate whenever *p* is false. As before, we abbreviate $I_w(F)$ as F^w, and we define the standard Strong Kleene (called here Standard-Kleene) by specifying that for any world w and any formula *F*:
if F = pp', Standard-Kleene(F, w) = 1 iff p^w = p'^w = 1; Standard-Kleene(F, w) = 0 iff p^w = 1 and p'^w = 0; Standard-Kleene(F, w) = # otherwise.
if F = (G and H), Standard-Kleene(F, w) = 1 iff Standard-Kleene(G, w) = Standard-Kleene(H, w) = 1; Standard-Kleene(F, w) = 0 iff Standard-Kleene(G, w) = 0 or Standard-Kleene(H, w) = 0; Standard-Kleene(F, w) = # otherwise.
if F = (not G), Standard-Kleene(F, w) = 1 iff Standard-Kleene(G, w) = 0; Standard-Kleene(F, w) = 0 iff Standard-Kleene(G, w) = 1; Standard-Kleene(F, w) = # otherwise.

41. **Theorem. Equivalence between the two definitions of Strong Kleene in the propositional case.**
If *F* belongs to the propositional fragment of 1, Standard-Kleene(F, w) = Kleene(F, w)

Proof:
As before, we start from a classical interpretation I for the fragment in 1, and we call I** the classical interpretation determined for the set of formulas of the form *F* or $F^\#$, where *F* is a formula of 1. We can then study the interpretation of these formulas according to our version of Strong Kleene and Standard Strong Kleene.
Since Standard Strong Kleene is compositional, it is immediate that for any formula *F* of 1, for any world w, Standard-Kleene(F, w) = Standard-Kleene($F^\#$, w). By the definition of our version of Strong Kleene, it is also clear that Kleene(F, w) = Kleene($F^\#$, w). All that remains to show, then, is that Standard-Kleene($F^\#$, w) = Kleene($F^\#$, w). Since by construction $F^\#$ contains at most one occurrence of any underlined expression, by 30 all we have to show is that Standard-Kleene($F^\#$, w) = Super($F^\#$, w). The proof is by induction on the construction of $F^\#$.

If for some i, F = p_i, the result is immediate.
If some i, k, n, F = $p_i p_k^n$, Standard-Kleene(F, w) = # iff p_i^w = 0, iff Super(F, w) = #. It is immediate that if Standard-Kleene(F, w) ≠ #, Standard-Kleene(F, w) = F^w = Super(F, w).
If F = (not G), the result is immediate.
If F = (G and H), there are three cases to consider: Standard-Kleene(F, w) = 1, Standard-Kleene(F, w) = 0, and Standard-Kleene(F, w) = #.
• Case 1: Standard-Kleene(F, w) = 1. Then Standard-Kleene(G, w) = Standard-Kleene(H, w) = 1, hence by the induction hypothesis, for every i'∈ Ext(I), i'(G) = i'(H) = 1, and hence i'(F) = 1. Therefore Super(F, w) = 1.
• Case 2: Standard-Kleene(F, w) = 0. Then Standard-Kleene(G, w) = 0 or Standard-Kleene(H, w) = 0. Let us assume that Standard-Kleene(G, w) (the other case is similar). By the induction hypothesis, for every i' ∈ Ext(I), i'(G) = 0, and thus i'(F) = 0. Therefore Super(F, w) = 0.
• Case 3: Standard-Kleene(F, w) = #.
(a) If Standard-Kleene(G, w) = 1 and Standard-Kleene(H, w) = # (or conversely - the reasoning is the same), by the induction hypothesis we have that for every every i' ∈ Ext(I), i'(G, w) = 1 and for some j', j" ∈ Ext(I), j'(H) = 1 and j"(H) = 0. It follows that j'(F) = 1 and j"(F) = 0, hence Super(F, w) = #.

(b) If Standard-Kleene(G, w) = Standard-Kleene(H, w) = #, for some bivalent i', i", j', j" ∈ Ext(I), i'(G) = j'(H) = 1, i"(G) = j"(H) = 0. Let us call At the set of all atomic propositions, and At(F) the set of atomic propositions that occur in *F*. We can define two new interpretations of atomic elements as follows:

i'+j' = $i'_{/(At\text{-}At(G))} \cup j'_{/At(H)}$

i"+j" = $i''_{/(At\text{-}At(G))} \cup j'_{/At(H)}$

Since i', i", j', j" ∈ Ext(I), it is also clear that i'+j' ∈ Ext(I) and i"+j" ∈ Ext(I). Since At(G) ∩ At(H) = Ø, i'+j'(G) = i'(G) and i'+j'(H) = j'(H) ; and similarly i"+j"(G) = i"(G) and i"+j"(H) = j"(H). It follows in particular that

i'+j'(G) = i'+j'(H) = 1, hence i'+j'(F) = 1

i"+j"(G) = i"+j"(H) = 0, hence i"+j"(F) = 0

and thus Super(F, w) = #.

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